Topic: Composition function

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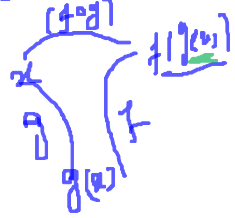
Def:

Given function f and g, the composition function of f with g, denoted by fog

Is a function defined by $(fog)(x) = f(g(x))$ Or $[g \circ f](x) = g(f(x))$



Example01: find a) $(fog)(x)$ and b) $(gof)(x)$

$f(x) = x^2 + 3$ $\quad g(x) = \sqrt{x}$

a) $[f \circ g](x) = f(g(x))$

$= f(\sqrt{x})$

$= (\sqrt{x})^2 + 3$

$= x + 3$

$[f \circ g](x) = x + 3$

Domain $= (-\infty, \infty)$

$f(x) = x^2 + 3$

$f(\sqrt{x}) = (\sqrt{x})^2 + 3$

(b) $(g \circ f)(x) = g(f(x))$

$= g(x^2 + 3)$

$= \sqrt{x^2 + 3}$

$[g \circ f](x) = \sqrt{x^2 + 3}$

Domain $x^2 + 3 \geq 0$

$x^2 \geq -3$

$x \to f(x) = y$

$f: x \to A$

$x \xrightarrow{f} f(x) = y$

$fg \neq gf$

Compositions can also be defined for three or more functions $(fogoh)(x) = f(g(h(x)))$

Example:02 $f(x) = \sqrt{x}$, $g(x) = \frac{1}{x}$, $h(x) = x^3$

$(f \circ g \circ h)(x) = f(g(h(x)))$

$= f(g(x^3))$

$= f\left(\frac{1}{x^3}\right) = \sqrt{\frac{1}{x^3}} = \frac{1}{x^{3/2}} = \frac{1}{\sqrt{x^3}}$

$\left(\frac{1}{x^3}\right)^{1/2} = \frac{(1)^{1/2}}{x^{3/2}} = \frac{1}{x^{3/2}}$

Domain $= x^3 > 0$

$= x > 0$

Example:03 Express $\sin(x^3)$ as a composition of two functions.

$f(x) = ?$ $\quad g(x) = ?$ $\Rightarrow f(g(x)) = \sin(x^3)$

$f(x) = \sin x$; $\quad g(x) = x^3$

$f(g(x)) = f(x^3)$

$= \sin(x^3)$

Q.. $(x^2+1)^{10} = f[g(x)]$

$f(x) = x^{10}$; $g(x) = x^2+1$

$f[g(x)] = f(x^2+1)$

$= [x^2+1]^{10}$

$[fg](x) = f[g(x)] = \frac{1}{x+1}$

$f(x) = \frac{1}{x}$; $g(x) = x+1$

$f[g(x)] = f(x+1)$

$= \frac{1}{x+1}$

Q:- $\tan x^5$

a :- $8+\sqrt{x} = [f \circ g](x)$

$f(x) = 8+x$

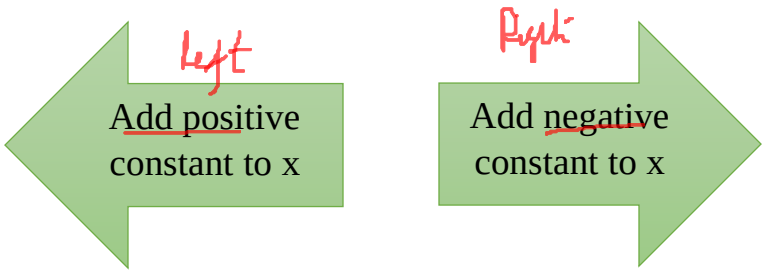
$g(x) = \sqrt{x}$

$f[g(x)] = f[\sqrt{x}]$

$= 8 + \sqrt{x}$

$f(x) \rightarrow y$

Shifting graph:

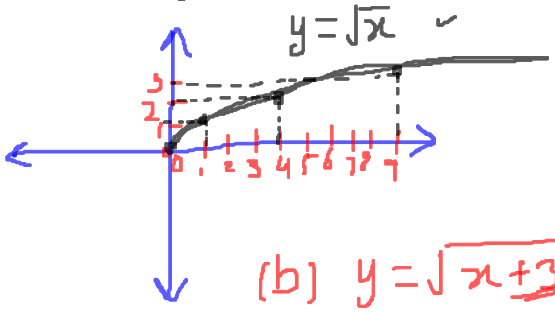


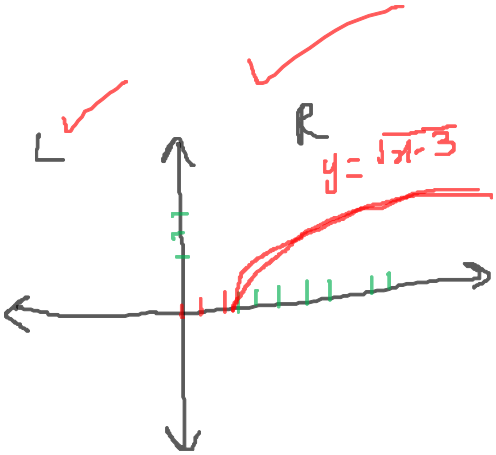
Sketch graph $y = \sqrt{x-3}$ & $y = \sqrt{x+3}$

a) $y = \sqrt{x-3}$

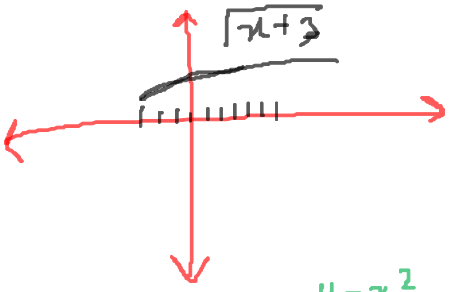
$y = \sqrt{x}$

$\sqrt{x}$	y
0	0
1	1
4	2
9	3





(b) $y = \sqrt{x+3}$



$x^2+4-2\cdot2x$

$y = x^2 - 4x + 5$

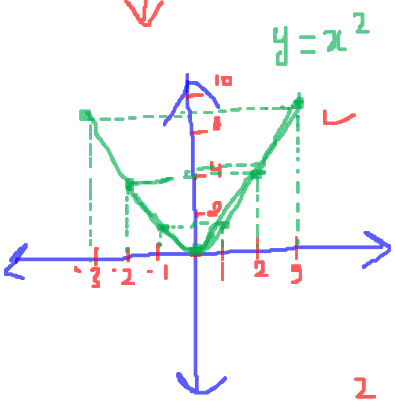
$y = (x^2 - 4x + 4) - 4 + 5$

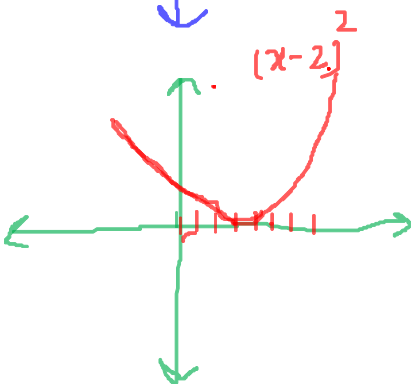
$= (x-2)^2 + 1$

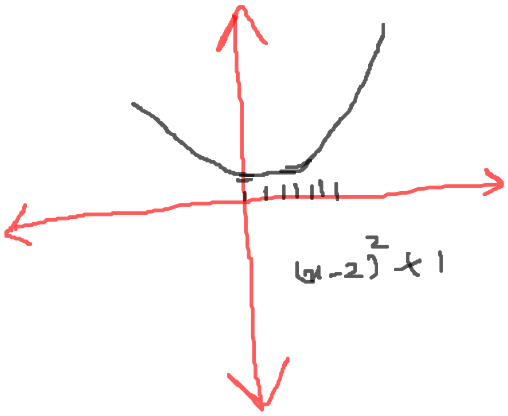
$y = (x-2)^2 + 0$

$y = x^2$

y	x
9	-3
4	-2
1	-1
0	0
1	1
4	2
9	3





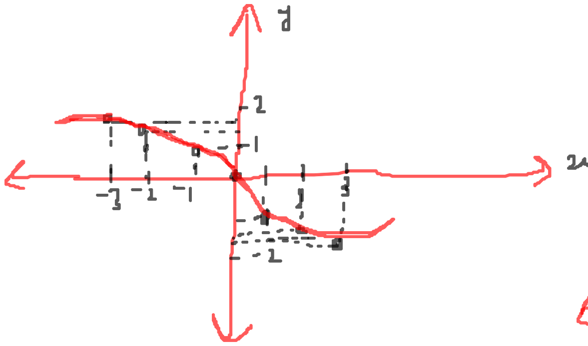


$y = \sqrt[3]{2-x}$

$y = (2-x)^{1/3}$

$y = (-x)^{1/3}$

$-x^{1/3}$	y
0	0
-1	1
-1.25	2
-1.44	3
1	-1
	-2
1.25 1.44	-3



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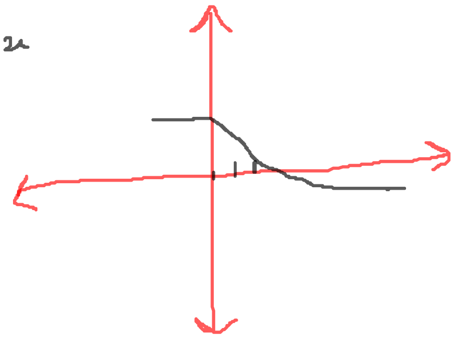
$y = f(x)$

$y = x^2$

$1 = (1)^2$

$4 = (2)^2$

$y = \sqrt[3]{-(x-2)}$



$y = 4 - |x-2|$

$y = |x| = f(x)$

$f(x) = |x|$