Lecture:03: Calculus and Analytical Geometry

Chapter 01: Before Calculus

## Quadratic inequalities:

$\sqrt{x^2} = |x|$

$x^2 = 2$

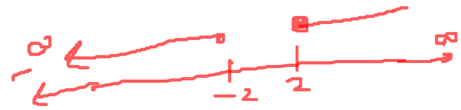
$x = \sqrt{2}$

a) $x^2 < 2$

$|x| < \sqrt{2}$

$-\sqrt{2} < x < \sqrt{2}$

$I = (-\sqrt{2}, \sqrt{2})$




(b) $4 \leq x^2$

$\pm 2 \leq x$

$\boxed{2 \leq x}$ ; $\boxed{-2 \leq x}$

$I = (-\infty, -2] \cup [2, \infty)$

$\boxed{2 \leq x}$

$x \geq 2$

$[2, \infty)$

$-2 \leq x$

$\boxed{x \leq -2}$

$x > 2$

$(2, \infty)$

$R = (-\infty, \infty)$

(c) $\frac{1}{9} < x^2 < \frac{1}{4}$

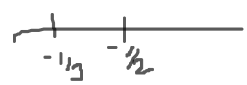
$\frac{1}{3} < \pm x < \frac{1}{2}$

$\frac{1}{3} < x < \frac{1}{2}$

$\frac{1}{3} < -x < \frac{1}{2}$

$-\frac{1}{3} < x < -\frac{1}{2}$

$\boxed{-\frac{1}{2} < x < -\frac{1}{3}}$




$I = \left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(-\frac{1}{2}, -\frac{1}{3}\right)$

## Even and Odd Function:

$3,\ x^{-5},\ x^2+1,\ x^2+x,\ x^3+x,\ x^4+3x^2+1,\ \frac{1}{x^2-1},\ \frac{x}{x^2-1},\ |t^3|,\ 2|t|+1$

a) $f(x) = 3$

$f(-x) = 3 = f(x)$

$\boxed{f(-x) = f(x)}$

$f(x)$ is even

(b) $f(x) = x^2 + 1$

$f(-x) = (-x)^2 + 1$

$f(-x) = (-1)^2(x)^2 + 1$

$= x^2 + 1$

$f(x)$

$\boxed{f(-x) = f(x)}$

$\Rightarrow f(x)$ is even

$f(-x) = f(x) =$ even

$f(-x) = -f(x) =$ odd

(c) $f(x) = x^3 + x$

$f(x) = (-x)^3 + (-x)$

$= (-1)^3 x^3 - x$

$= -x^3 - x$

$= -(x^3 + x)$

$f(-x) = -f(x)$

$\boxed{f(-x) = -f(x)}$

$\Rightarrow f(x)$ is odd

$f(x) = \frac{x}{x^2-1}$

$f(-x) = \frac{-x}{(-x)^2-1}$

$= \frac{-x}{(x^2-1)}$

$\boxed{f(-x) \neq f(x) \\ f(-x) \neq -f(x)}$

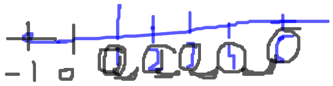
neither even nor odd

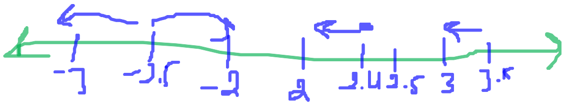
$f(t) = |t^3|$

$f(-t) = |(-t)^3| = |-t^3| = |t^3|$

$f(t)$ is even

$$\boxed{f(t) = 2|t|+1}$$



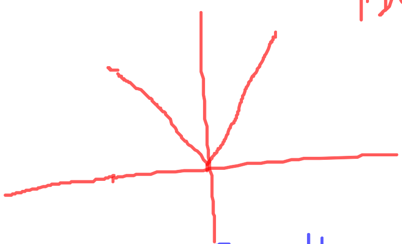


- **Greatest integer function**

The function whose value at any number x is the smallest greater or equal to x is called greatest integer function.

$[x] \to [[x]]$ $y = [[x]]$

$|x|$



$[2] = 2$

$[2.4] = 2$

$[3.5] = 3$

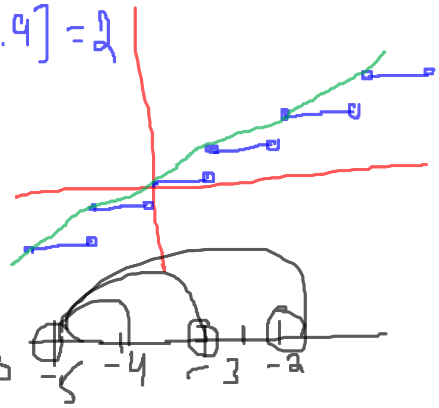
$[-2] = -2$

$[-2.5] = -3$

$[-3.5] = -4$

$[2.4] = 2$

$[-2.4] = -3$



- **Piece wise defined functions:**

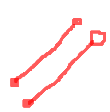
diff. functs $\Rightarrow$ diff domain

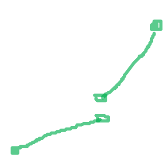
Exp: $y = f(x) = \begin{cases} x, & \boxed{0 \le x \le 1} \\ 2x - 1 & 1 < x \le 2 \end{cases}$

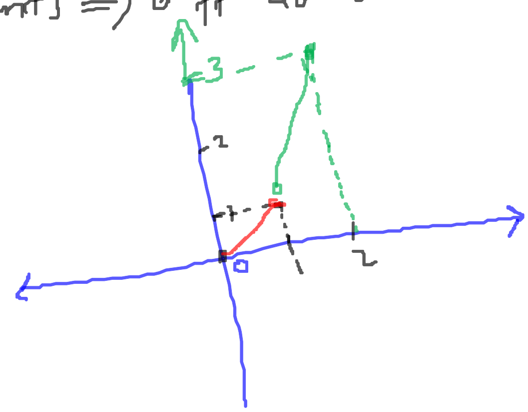
$0 \le x < 1$

| $x$ | $f(x) = x$ |
|---|---|
| 0 | 0 |
| 1 | 1 |

| $x$ | $f(x) = 2x - 1$ |
|---|---|
| 1 | $2(1) - 1 = 1$ |
| 2 | $2(2) - 1 = 3$ |



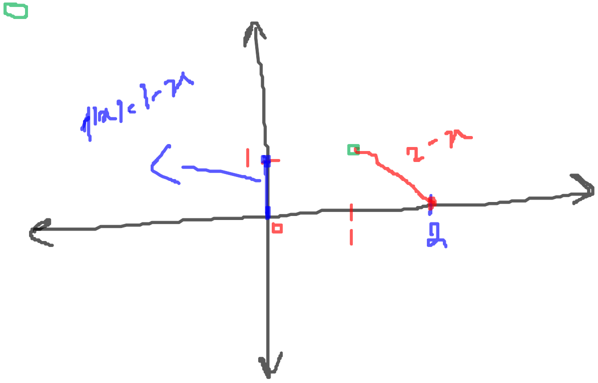




$$f(x)=\begin{cases}1-x, & 0\le x\le 1\\ 2-x, & 1<x\le 2\end{cases}$$

| $x$ | $f(x)=1-x$ |
|---|---|
| 0 | $1-0=1$ |
| 1 | $1-1=0$ |

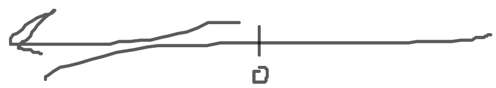
| $x$ | $f(x)=2-x$ |
|---|---|
| 1 | $2-1=1$ |
| 2 | $2-2=0$ |




0

$$f(x)=\begin{cases}\dfrac{1}{x}, & \boxed{x<0}\\ x, & \boxed{0\le x}\end{cases}$$

$\frac{1}{0} \to \infty$




$f(x)=\frac{1}{x}$

$f(x)=x$

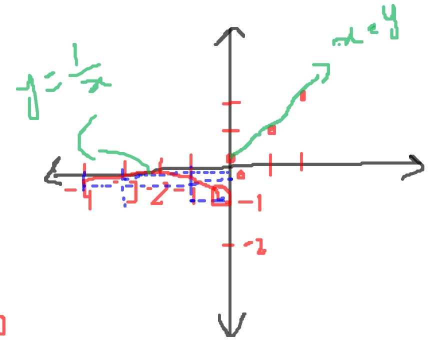
$0\le x$

$x\ge 0$

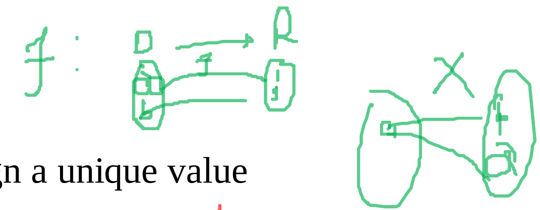
$\boxed{x<0}$

| $x$ | $f(x)=\frac{1}{x}$ |
|---|---|
| -1 | -1 |
| -4 | -0.25 |
| -3 | -0.333 |

| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |



# Function, Domain & Range



- Function:

Function is the set of rules that assign a unique value

$f(x) = x$

$\boxed{f(1) = 1}$

$f(1) = \pm 1$

- Domain:

All possible input values are called domain

- Range:

All possible output values are called ~~domain~~ Range

$f(x) = x \quad 0 \le x \le 1$

$D = [0, 1]$

$f(0) = 0$ smallest

$f(1) = 1$ largest

$[0, 1]$

**Remembering point to find domain**

① $f(x) = \text{polynomial}$

Domain $= (-\infty, \infty)$

$f(x) = x^2 + 1$

$D = -\infty, \infty$

② $f(x) = \frac{1}{\text{polynomial}}$

Domain $= \text{poly} \neq 0$

$\frac{1}{0}$

$f(x) = \frac{1}{x}$

$x \neq 0$

$D = (-\infty, 0) \cup (0, \infty)$

$f(x) = \frac{1}{(x-1)}$ ; $\frac{1}{0}$

$x - 1 \neq 0$

$\boxed{x \neq 1}$

② $f(x) = \text{squaroot}$

$D :\Rightarrow f(x) \geq 0$

$f(x) = \sqrt{x}$

$x \geq 0$

$D = [0, \infty)$

③ $f(x) = \frac{1}{\sqrt{f(x)}}$

$f(x) > 0$

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