Course Title: Calculus and Analytical Geometry (Math -101)

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What is calculus?

- infinitesimal differences.
- study of continuous change

- Algebra
- Probability
- Geometry

- Isaac Newton
- Gottfried Wilhelm Leibniz

$(-1)^2 = 1$

$(2)^2 = 4$

Chapter 01: Preliminaries

$W = \{0, 1, 2, 3, \dots\}$

$N = \{1, 2, 3, \dots\}$

$Z = \{\dots -1, 0, 1, \dots\}$

$Z = \{0, \pm 1, \pm 2, \dots\}$

$Q = \{p/q \mid p, q \in Z, \ q \neq 0\}$

$\frac{2}{3}, \frac{1}{1},$

$Q' \neq p/q$

$\sqrt{25}$ $\sqrt{2}, \sqrt{3}$ π

$\sqrt{(5)^2} = 5$

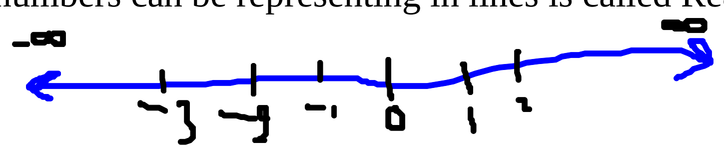
- Real Numbers: $Q \cup Q'$

Union of Rational and irrational numbers

Square is positive number $(-\infty, \infty)$, R

- Real Lines:

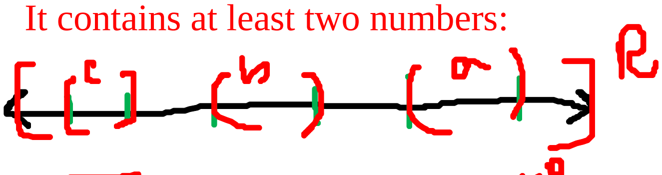
Real numbers can be representing in lines is called Real lines



- Interval:

A subset of the real lines is called interval

It contains at least two numbers:



$a \in R$

$b \in R$

$c \in R$

$[1, 5]$

$f(x) = x^2$

$f(1)$ $f(2)$ $f(3)$ $f(4)$ $f(5)$

$[a, b]$

Types

$(1, 5)$

(a, b)

$f(2)$ $f(3)$ $f(4)$

If its contains both of its points

closed

if it contains neither end points

open

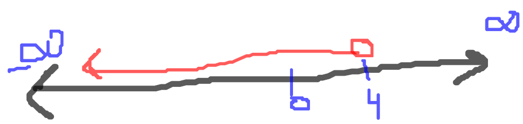
Example: 01

Solve the following inequalities and show their solution set on the real lines:

a) $2x - 1 < x + 3$

$2x - x < 3 + 1$

$x < 4$



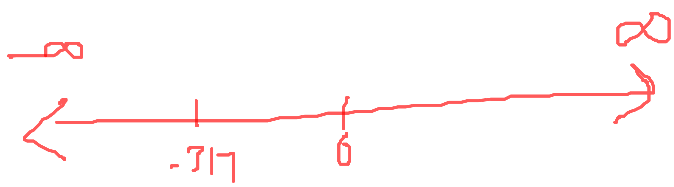
$(-\infty, \infty)$

$I = (-\infty, 4) \rightarrow$ open

or $x \leq 4$

$I = (-\infty, 4]$ half open/closed

b) $-\frac{x}{3} < 2x + 1$



$-x < (2x+1)3$

$-x < 6x + 3$

$-3 < 6x + x$

$-3 < 7x$

$\boxed{-3/7 < x}$ Or $\boxed{x > -3/7}$

$I = (-3/7, \infty)$

c) $\frac{6}{x-1} \geq 5$

$x-1=0$

$x=1$

$6 \geq 5(x-1)$

$6 \geq 5x-5$

$6+5 \geq 5x$

$\boxed{11/5 \geq x}$

Sol set $I = (1, 11/5]$

at $x=1$

$\frac{6}{0} \to \infty$

$-\infty$ 0 1 11/5 ∞

0

$I = (-\infty, 1) \cup (1, 11/5]$

- Absolute values:

The absolute value of a number denoted by $|x|$ is defined by

$$|x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$

; $\sqrt{(x)^2} = |x|$

Exp: 3

$|3| = \sqrt{(3)^2} = \sqrt{9} = 3$

0

$|0| = \sqrt{(0)^2} = 0$

-5

$|-5| = \sqrt{(-5)^2} = \sqrt{25} = 5$

Exp:- Solving Eqs with absolute values.

(a) $|2x-3| = 7$

Sol:

$$2x-3 = \pm 7$$

$2x-3 = 7$

$2x = 7+3$

$x = 10/2$

$\boxed{x = 5}$

$2x-3 = -7$

$2x = -7+3$

$\boxed{x = -2}$

$x = -4/2$

$SS = \{x_1, x_2\} = \{5, -2\}$ ✓

(b) $\left|5 - \frac{2}{x}\right| < 1$

$$\boxed{-1 < 5 - \frac{2}{x} < 1}$$

$-6 < -2/x < -4$

$3 < \frac{1}{x} < 2$

$\frac{1}{3} < x < \frac{1}{2}$

$x \in (1/3, 1/2)$ ✓

(C) $|2x-3| \leq 1$

$-1 \leq 2x-3 \leq 1$

$2 \leq 2x \leq 4$

$1 \leq x \leq 2$

$x \in [1, 2]$ ✓

(d) $|2x-3| \geq 1$

$2x-3 \geq 1$, $2x-3 \geq -1$

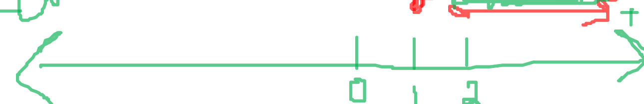
$2x \geq 4$

$x \geq 2$

$2x \geq 2$

$x \geq 1$

$-\infty$ $+\infty$

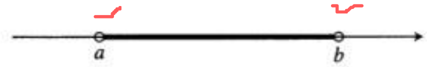


$(-\infty, 1] \cup [1, 2] \cup [2, \infty)$

$(-\infty, 1] \cup [2, \infty)$

$I = [2, \infty)$

Table 1 Types of intervals

	Notation	Set	Graph
Finite:	(a, b) [1, 2)	$\{x \mid a < x < b\}$	
	$[a, b]$	$\{x \mid a \le x \le b\}$	
	$[a, b)$	$\{x \mid a \le x < b\}$	
	$(a, b]$	$\{x \mid a < x \le b\}$	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	
	$[a, \infty)$	$\{x \mid x \ge a\}$	
	$(-\infty, b)$	$\{x \mid x < b\}$	
	$(-\infty, b]$	$\{x \mid x \le b\}$	
	R = $(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

Exercises 1

Decimal Representations

1. Express 1/9 as a repeating decimal, using a bar to indicate the repeating digits. What are the decimal representations of 2/9? 3/9? 8/9?

2. Express 1/11 as a repeating decimal, using a bar to indicate the repeating digits. What are the decimal representations of 2/11? 3/11? 9/11?

Inequalities

3. If $2 < x < 6$, which of the following statements about x are necessarily true, and which are not necessarily true?

   a) $0 < x < 4$ b) $0 < x - 2 < 4$
   c) $1 < \frac{x}{2} < 3$ d) $\frac{1}{6} < \frac{1}{x} < \frac{1}{2}$
   e) $1 < \frac{6}{x} < 3$ f) $|x - 4| < 2$
   g) $-6 < -x < 2$ h) $-6 < -x < -2$

4. If $-1 < y - 5 < 1$, which of the following statements about y are necessarily true, and which are not necessarily true?

   a) $4 < y < 6$ b) $-6 < y < -4$
   c) $y > 4$ d) $y < 6$
   e) $0 < y - 4 < 2$ f) $2 < \frac{y}{2} < 3$
   g) $\frac{1}{6} < \frac{1}{y} < \frac{1}{4}$ h) $|y - 5| < 1$

Absolute Value

Solve the equations in Exercises 13–18.

13. $|y| = 3$ **14.** $|y - 3| = 7$ **15.** $|2t + 5| = 4$

16. $|1 - t| = 1$ **17.** $|8 - 3s| = \frac{9}{2}$ **18.** $\left|\frac{s}{2} - 1\right| = 1$

Solve the inequalities in Exercises 19–34, expressing the solution sets as intervals or unions of intervals. Also, graph each solution set on the real line.

19. $|x| < 2$ **20.** $|x| \le 2$ **21.** $|t - 1| \le 3$

22. $|t + 2| < 1$ **23.** $|3y - 7| < 4$ **24.** $|2y + 5| < 1$

25. $\left|\frac{z}{5} - 1\right| \le 1$ **26.** $\left|\frac{3}{2}z - 1\right| \le 2$ **27.** $\left|3 - \frac{1}{x}\right| < \frac{1}{2}$

28. $\left|\frac{2}{x} - 4\right| < 3$ **29.** $|2s| \ge 4$ **30.** $|s + 3| \ge \frac{1}{2}$

31. $|1 - x| > 1$ **32.** $|2 - 3x| > 5$ **33.** $\left|\frac{r + 1}{2}\right| \ge 1$

34. $\left|\frac{3r}{5} - 1\right| > \frac{2}{5}$

do it.

Quadratic Inequalities

Solve the inequalities in Exercises 35–42. Express the solution sets as intervals or unions of intervals and graph them. Use the result $\sqrt{a^2} = |a|$ as appropriate.

35. $x^2 < 2$ **36.** $4 \le x^2$ **37.** $4 < x^2 < 9$