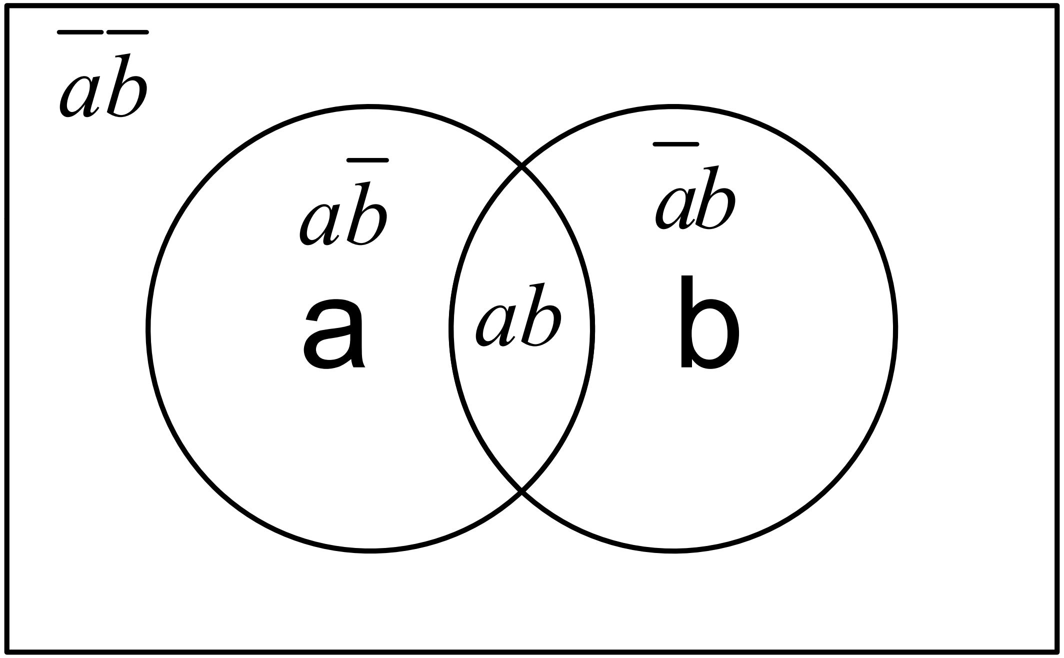
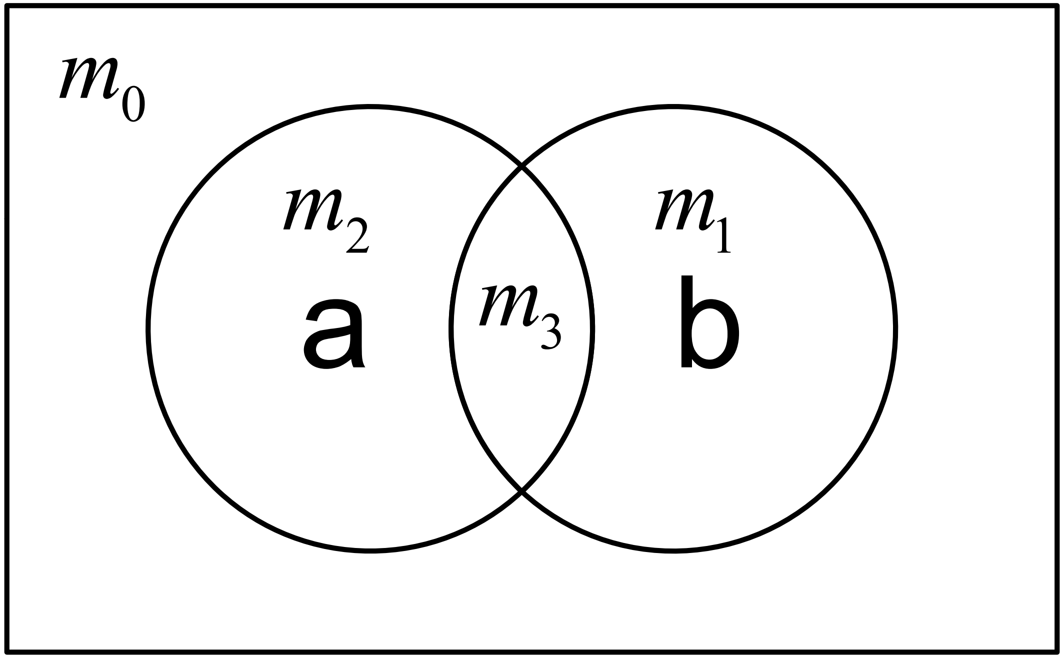
Karnaugh Maps (K-Map)

A K-Map is a graphical representation of a logic function's truth table

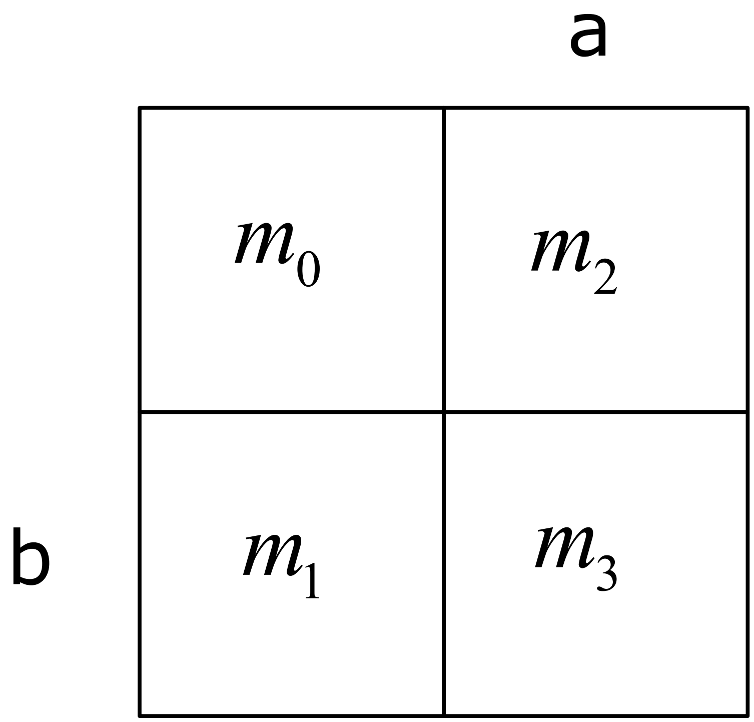
Relationship to Venn Diagrams



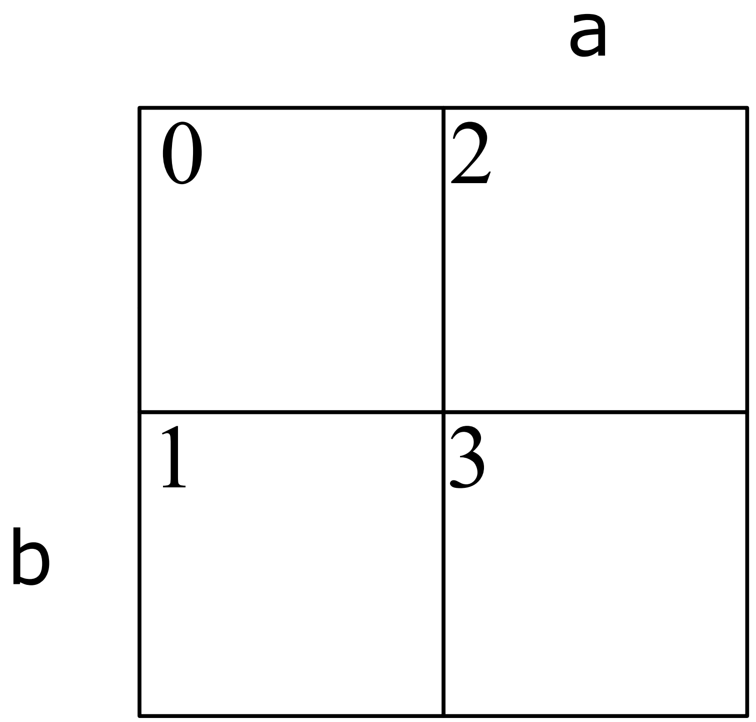
Relationship to Venn Diagrams



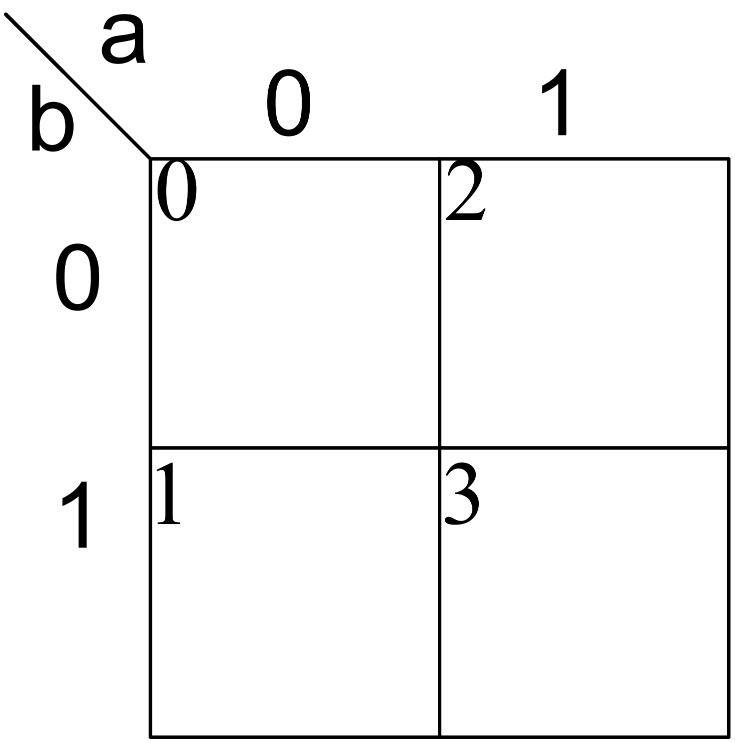
Relationship to Venn Diagrams



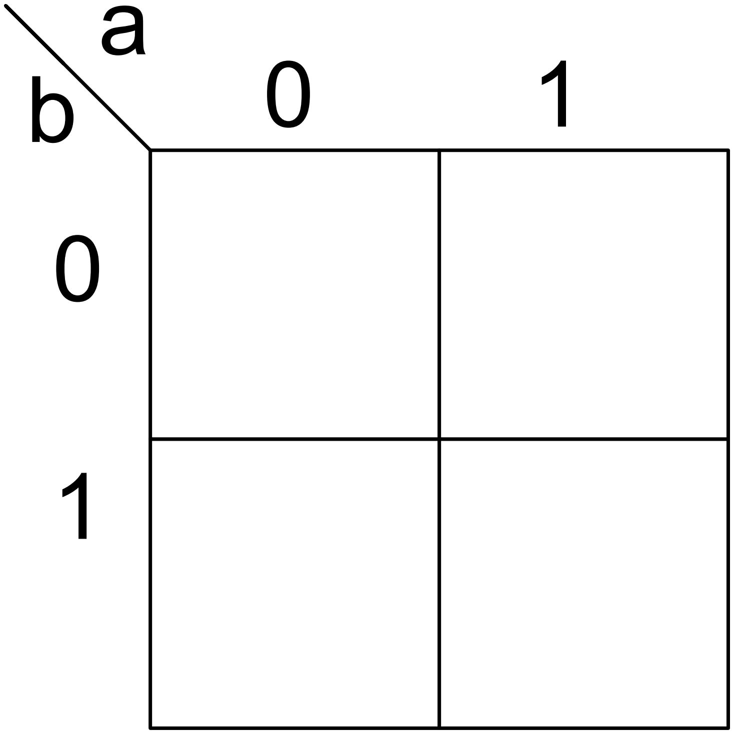
Relationship to Venn Diagrams



Relationship to Venn Diagrams



Two-Variable K-Map



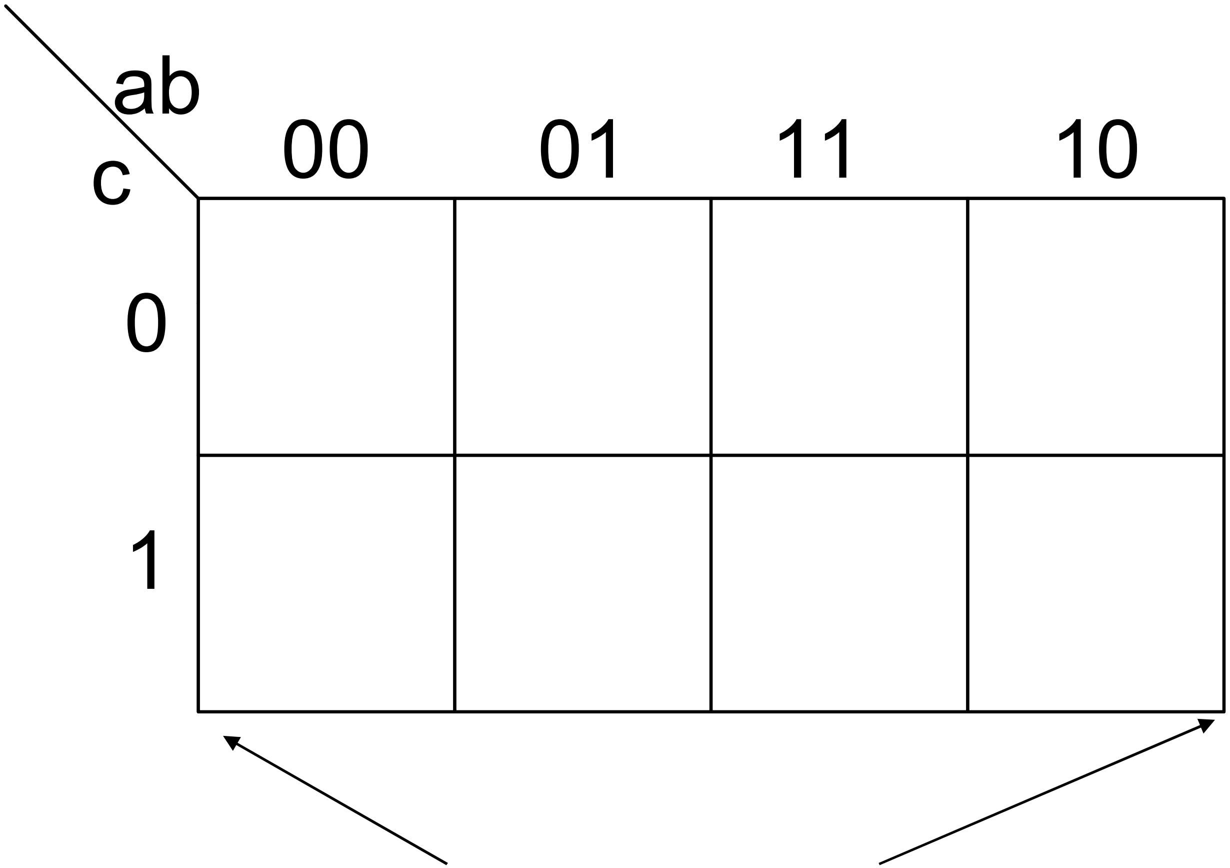
Three-Variable K-Map

c \ ab	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

Three-Variable K-Map

c \ ab	00	01	11	10
0				
1				

Three-Variable K-Map



Edges are adjacent

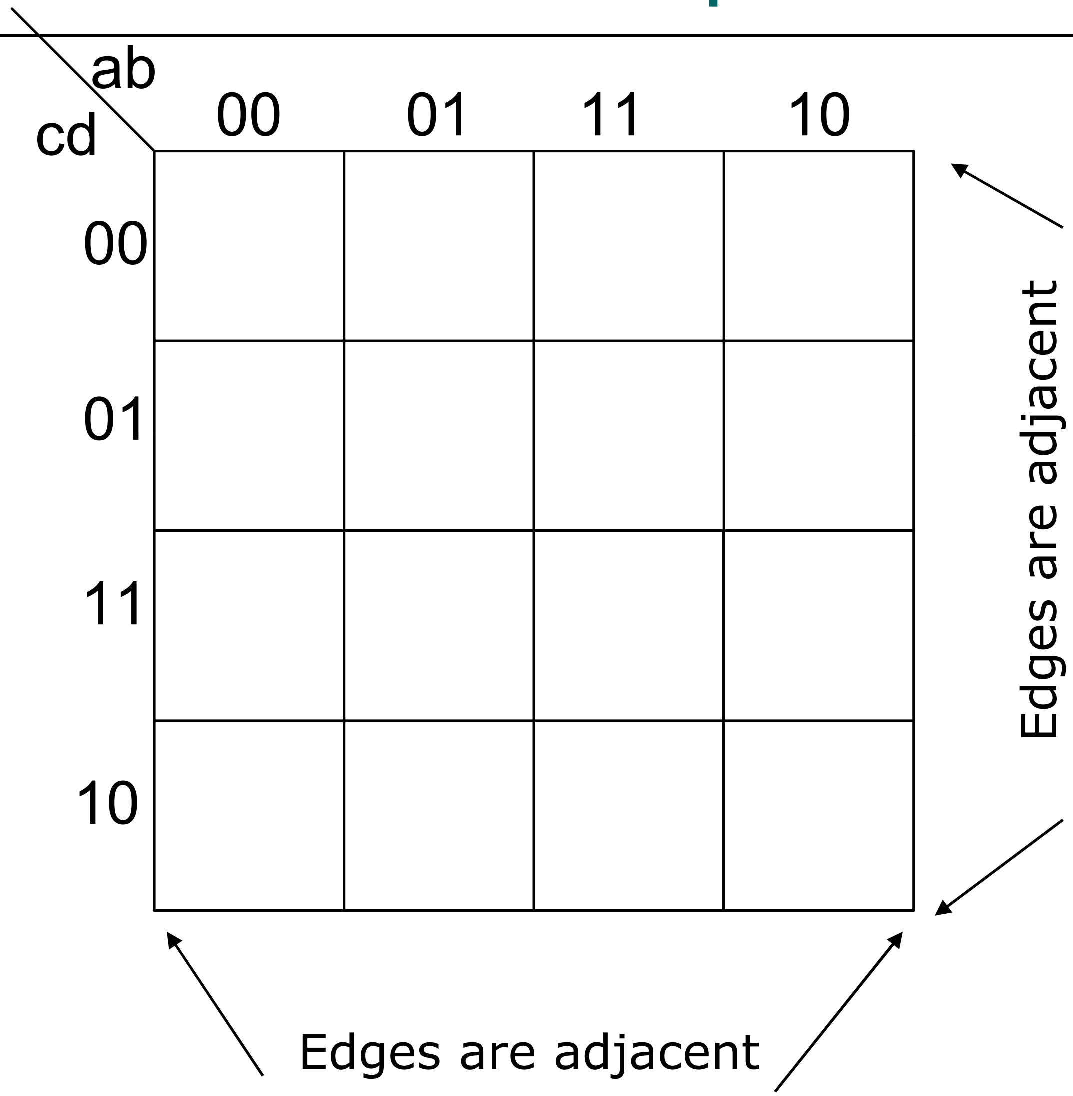
Four-variable K-Map

cd \ ab	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

Four-variable K-Map

cd \ ab	00	01	11	10
00				
01				
11				
10				

Four-variable K-Map



Plotting Functions on the K-map

SOP Form

Canonical SOP Form

Three Variable Example

$$F = AB\overline{C} + \overline{A}BC + \overline{A}\,\overline{B}C + A\overline{B}C$$

using shorthand notation

$$F = m_6 + m_3 + m_1 + m_5$$

$$F(A,B,C) = \sum m(1,3,5,6)$$

Three-Variable K-Map Example

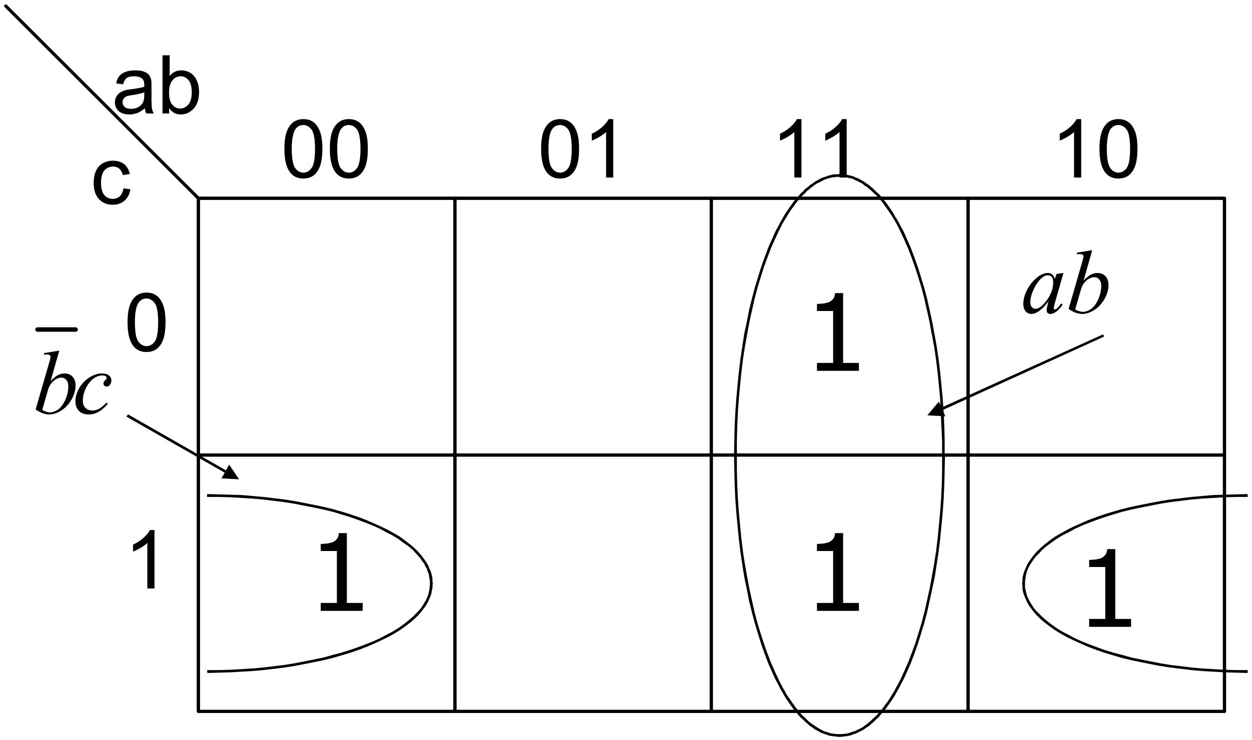
Plot 1's (minterms) of switching function

c \ ab	00	01	11	10
0			1	
1	1	1		1

$$F(a,b,c)=\sum m(1,3,5,6)$$

Three-Variable K-Map Example

Plot 1's (minterms) of switching function



$$F(a,b,c) = ab + \bar{b}c$$

Four-variable K-Map Example

cd \ ab	00	01	11	10
00	1		1	
01				1
11				
10	1		1	

$$F(a,b,c,d)=\sum m(0,2,9,12,14)$$

Karnaugh Maps (K-Map)

Simplification of Switching Functions using K-MAPS

Terminology/Definition

- Literal
  - A variable or its complement
- Logically adjacent terms
  - Two minterms are logically adjacent if they differ in only one variable position
  - Ex:

$$ab\bar{c} \text{ and } \bar{a}b\bar{c}$$

m6 and m2 are logically adjacent

Note: $ab\bar{c}+\bar{a}b\bar{c}=\left(a+\bar{a}\right)b\bar{c}=b\bar{c}$

Or, logically adjacent terms can be combined

Terminology/Definition

- Implicant
  - Product term that could be used to cover minterms of a function
- Prime Implicant
  - An implicant that is not part of another implicant
- Essential Prime Implicant
  - An implicant that covers at least one minterm that is not contained in another prime implicant
- Cover
  - A minterm that has been used in at least one group

Guidelines for Simplifying Functions

- Each square on a K-map of n variables has n logically adjacent squares. (i.e. differing in exactly one variable)
- When combing squares, always group in powers of 2^m , where m=0,1,2,….
- In general, grouping 2^m variables eliminates m variables.

Guidelines for Simplifying Functions

- Group as many squares as possible. This eliminates the most variables.
- Make as few groups as possible. Each group represents a separate product term.
- You must **cover** each minterm at least once. However, it may be covered more than once.

K-map Simplification Procedure

- Plot the K-map
- Circle **all** prime implicants on the K-map
- Identify and select all essential prime implicants for the cover.
- Select a minimum subset of the remaining prime implicants to complete the cover.
- Read the K-map

Example

- Use a K-Map to simplify the following Boolean expression

$$F(a,b,c) = \sum m(1,2,3,5,6)$$

Three-Variable K-Map Example

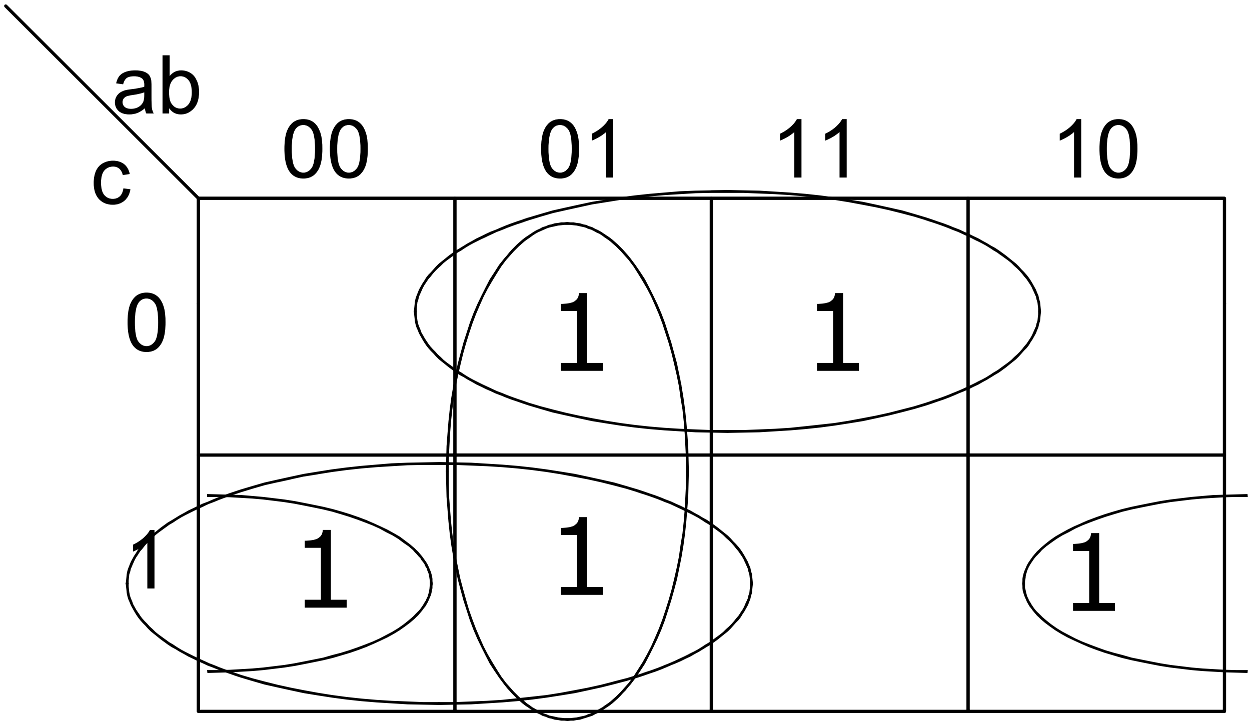
Step 1: Plot the K-map

c \ ab	00	01	11	10
0		1	1	
1	1	1		1

$$F(a,b,c)=\sum m(1,2,3,5,6)$$

Three-Variable K-Map Example

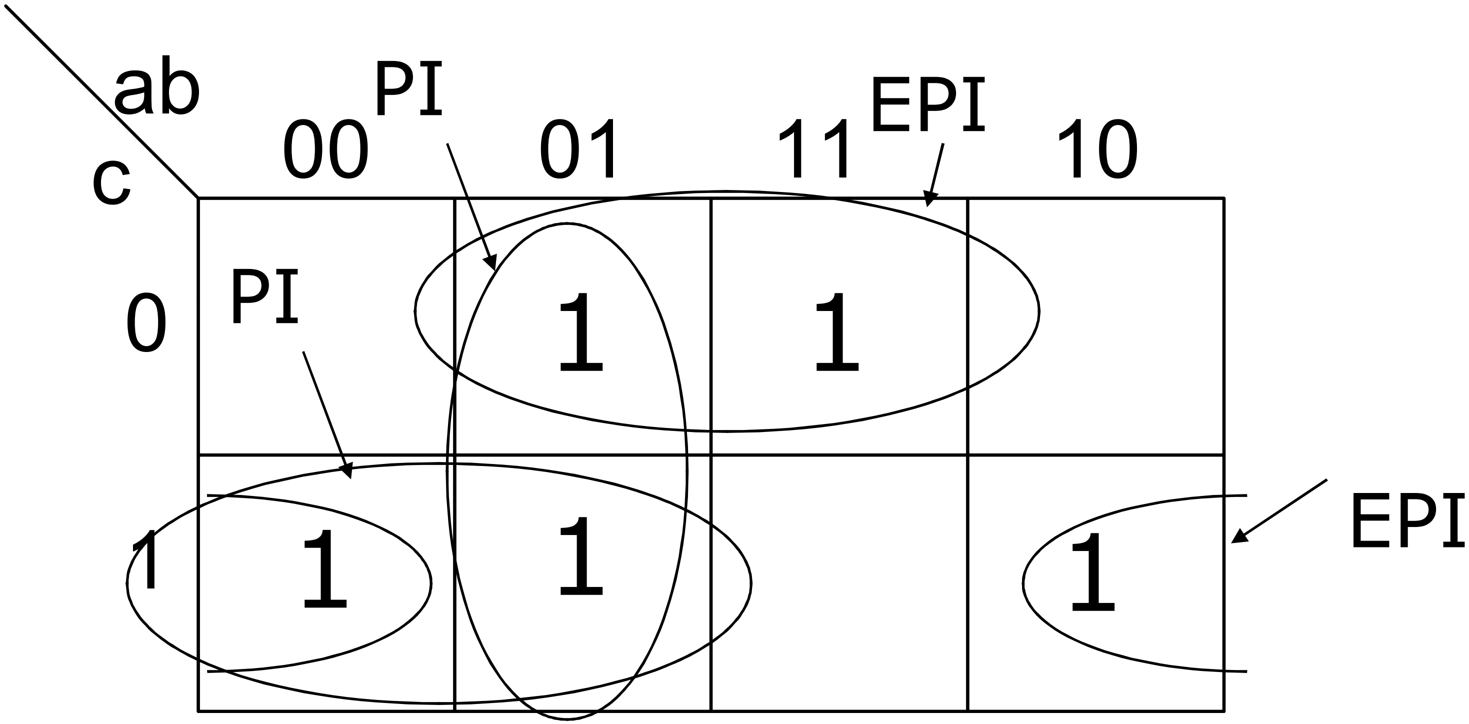
Step 2: Circle **ALL** Prime Implicants



$$F(a,b,c)=\sum m(1,2,3,5,6)$$

Three-Variable K-Map Example

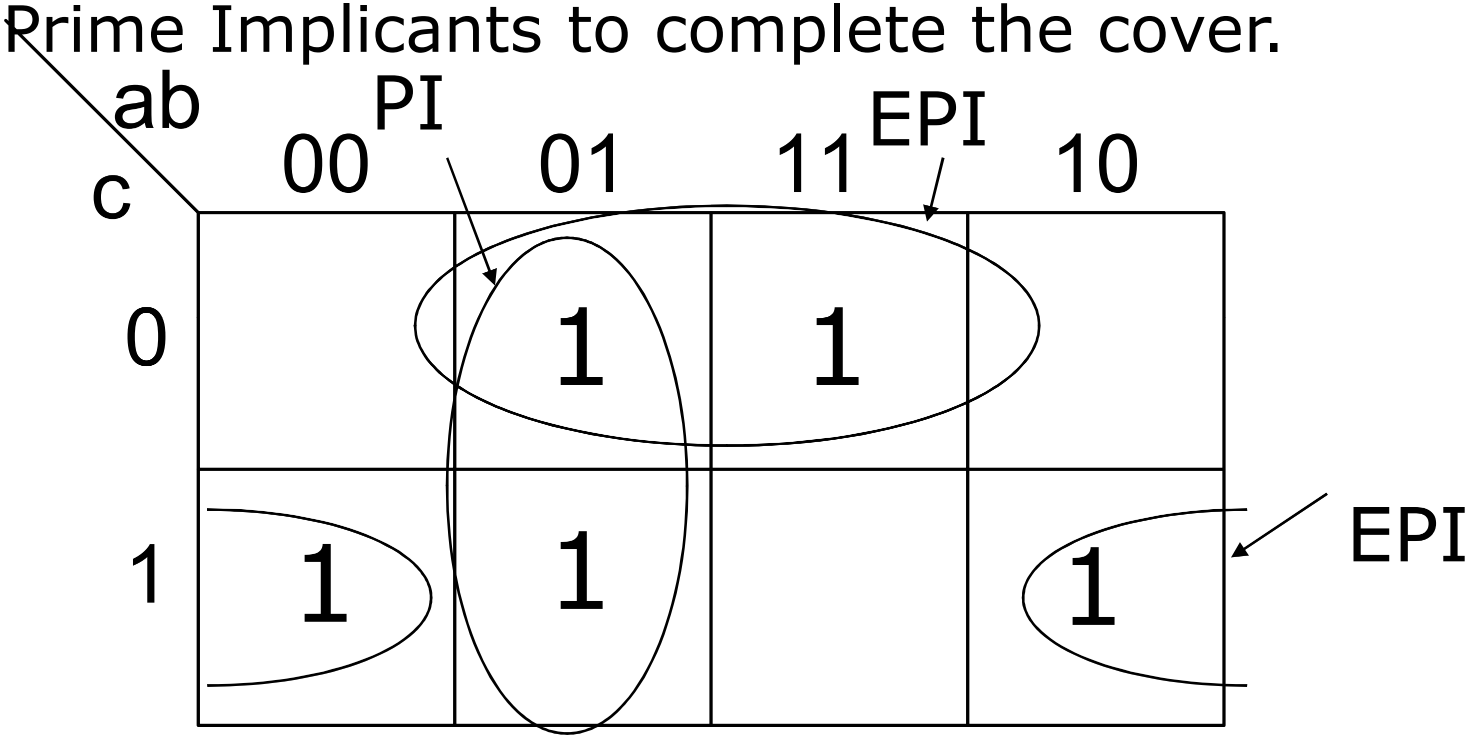
Step 3: Identify Essential Prime Implicants



$$F(a,b,c)=\sum m(1,2,3,5,6)$$

Three-Variable K-Map Example

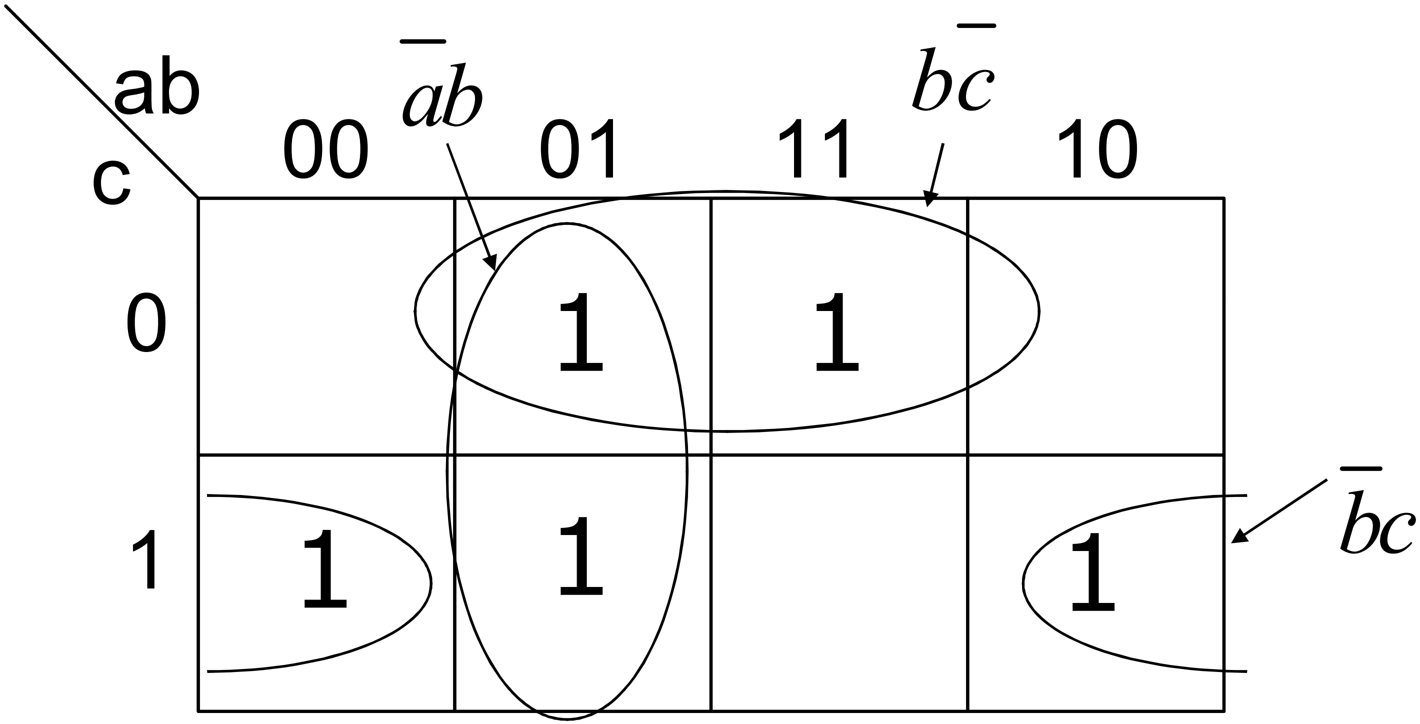
Step 4: Select minimum subset of remaining Prime Implicants to complete the cover.



$$F(a,b,c)=\sum m(1,2,3,5,6)$$

Three-Variable K-Map Example

Step 5: Read the map.



$$F(a,b,c)=\sum m(1,2,3,5,6)$$

Solution

$$F(a,b,c) = \overline{a}b + b\overline{c} + \overline{b}c = \overline{a}b + b \oplus c$$

Example

- Use a K-Map to simplify the following Boolean expression

$$F(a,b,c) = \sum m(2,3,6,7)$$

Three-Variable K-Map Example

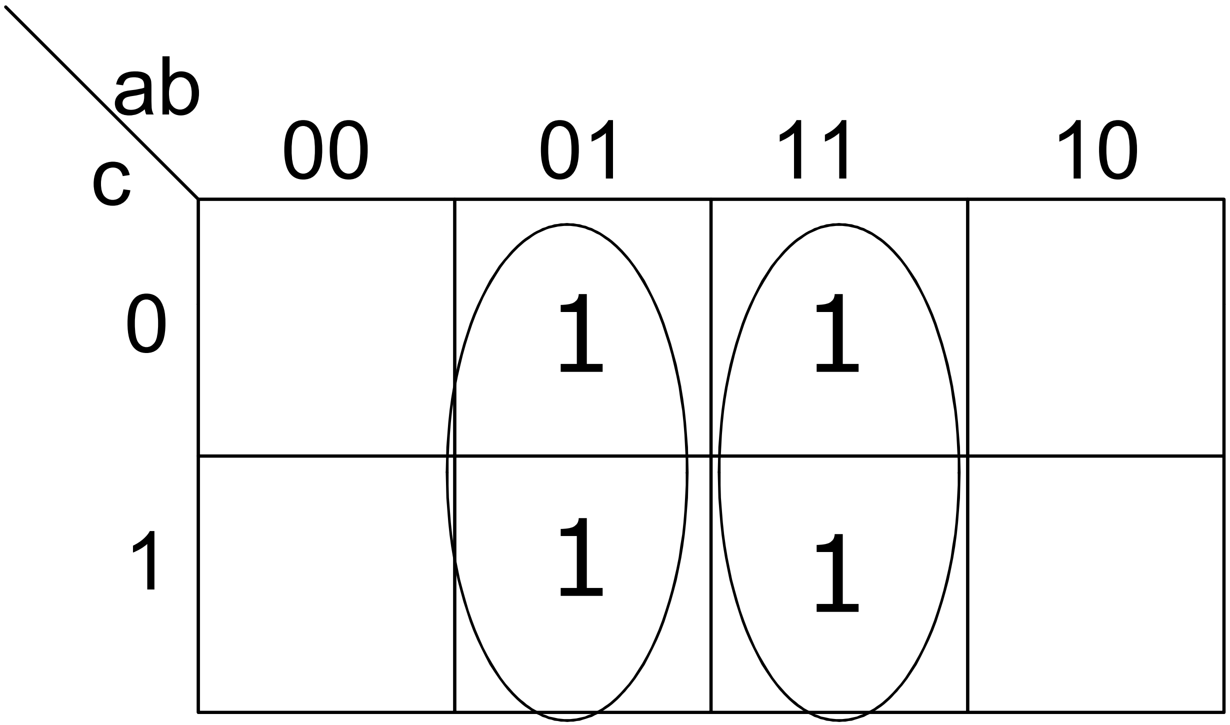
Step 1: Plot the K-map

c \ ab	00	01	11	10
0		1	1	
1		1	1	

$$F(a,b,c) = \sum m(2,4,5,7)$$

Three-Variable K-Map Example

Step 2: Circle Prime Implicants

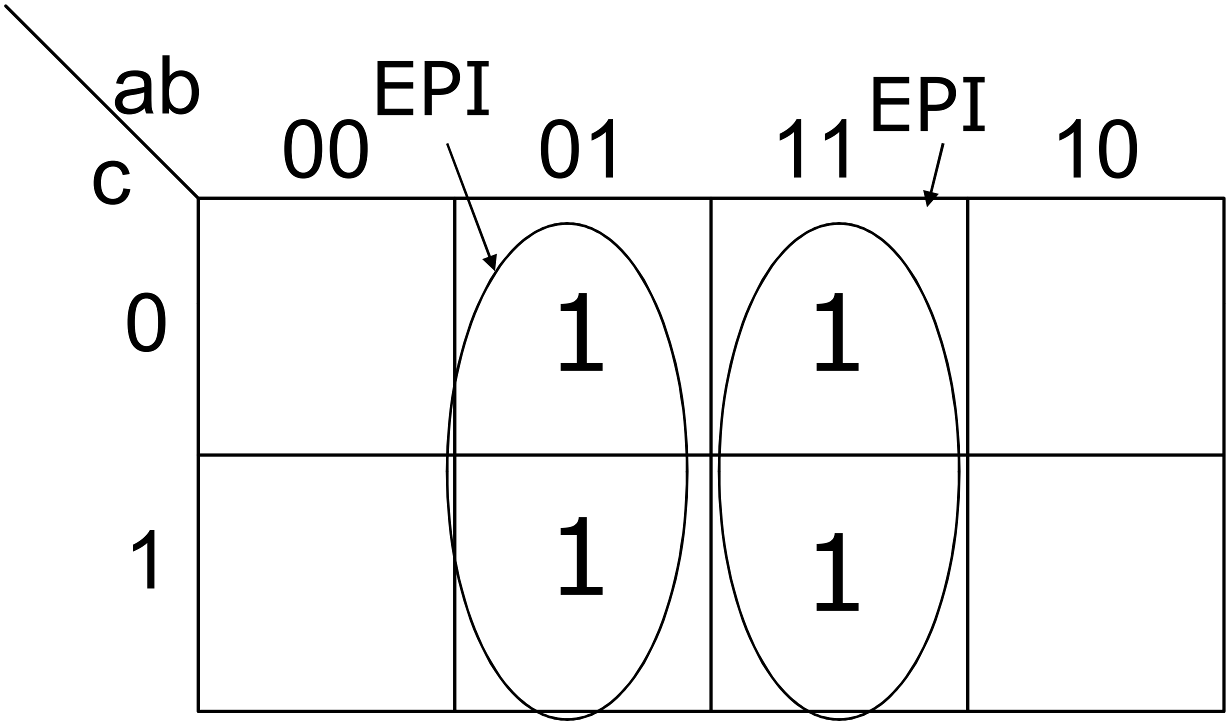


Wrong!!
We really
should draw
A circle around
all four 1's

$$F(a,b,c)=\sum m(2,3,6,7)$$

Three-Variable K-Map Example

Step 3: Identify Essential Prime Implicants

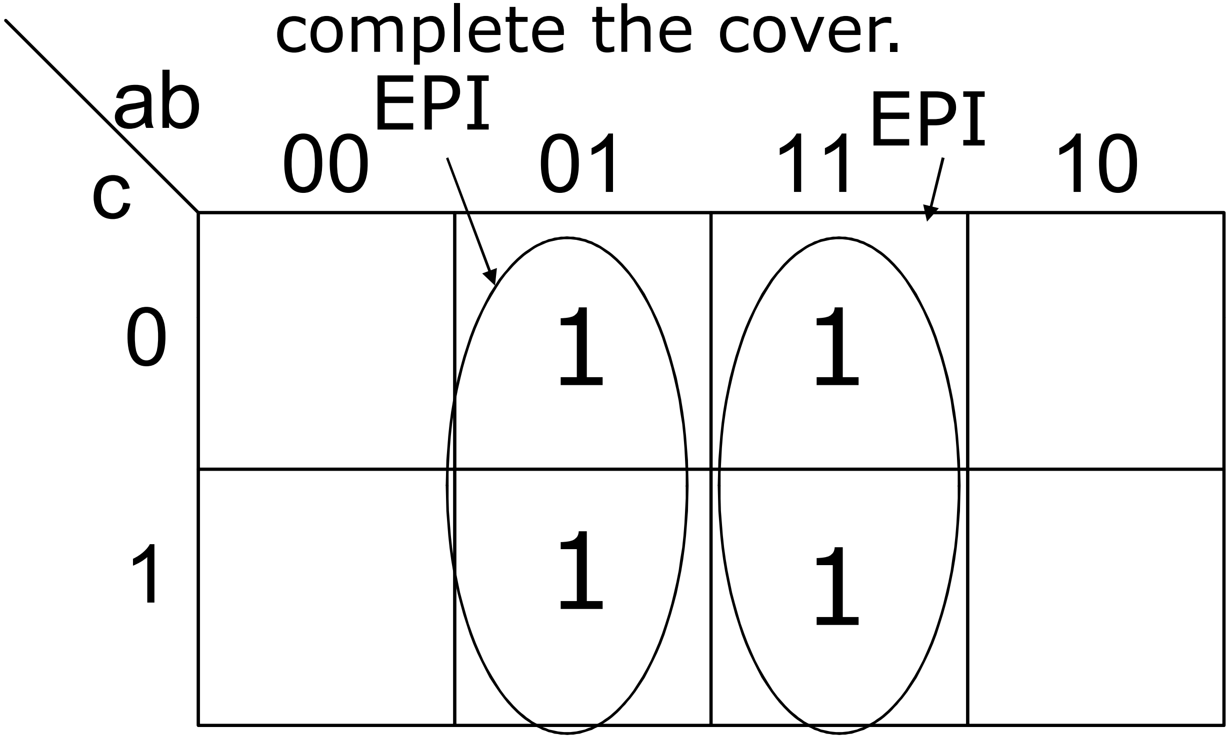


Wrong!!
We really
should draw
A circle around
all four 1's

$$F(a,b,c)=\sum m(2,3,6,7)$$

Three-Variable K-Map Example

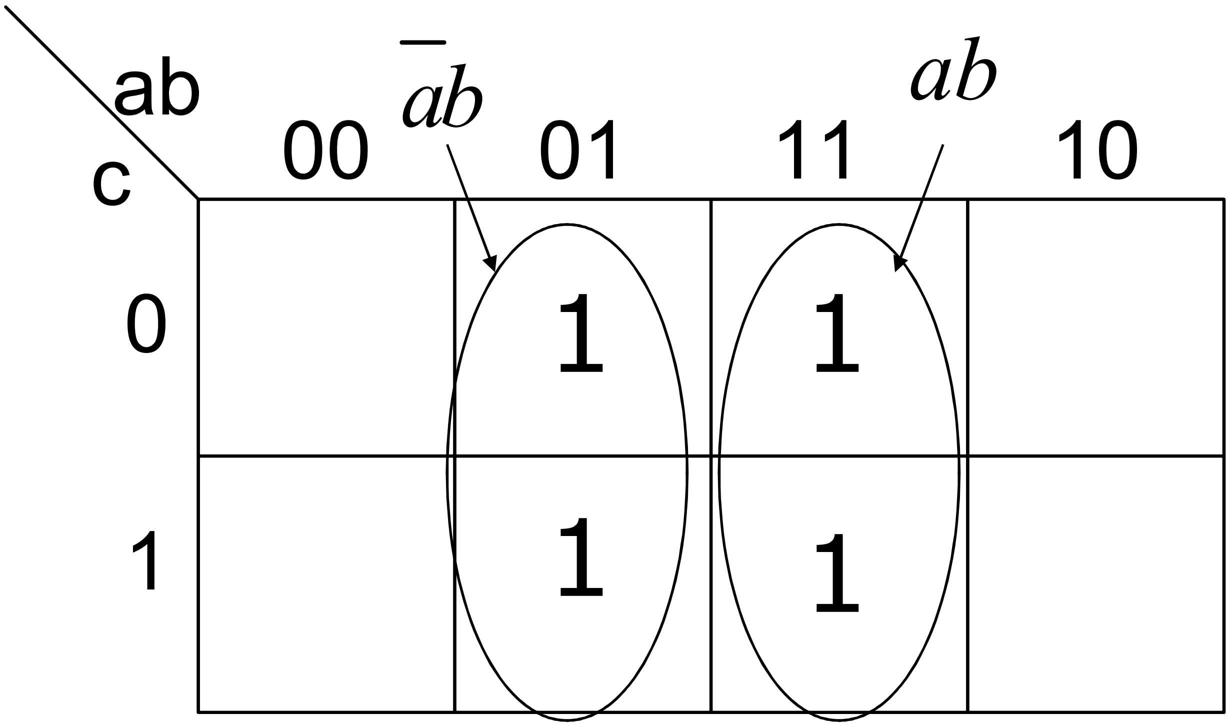
Step 4: Select Remaining Prime Implicants to complete the cover.



$$F(a,b,c)=\sum m(2,3,6,7)$$

Three-Variable K-Map Example

Step 5: Read the map.



$$F(a,b,c)=\sum m(2,3,6,7)$$

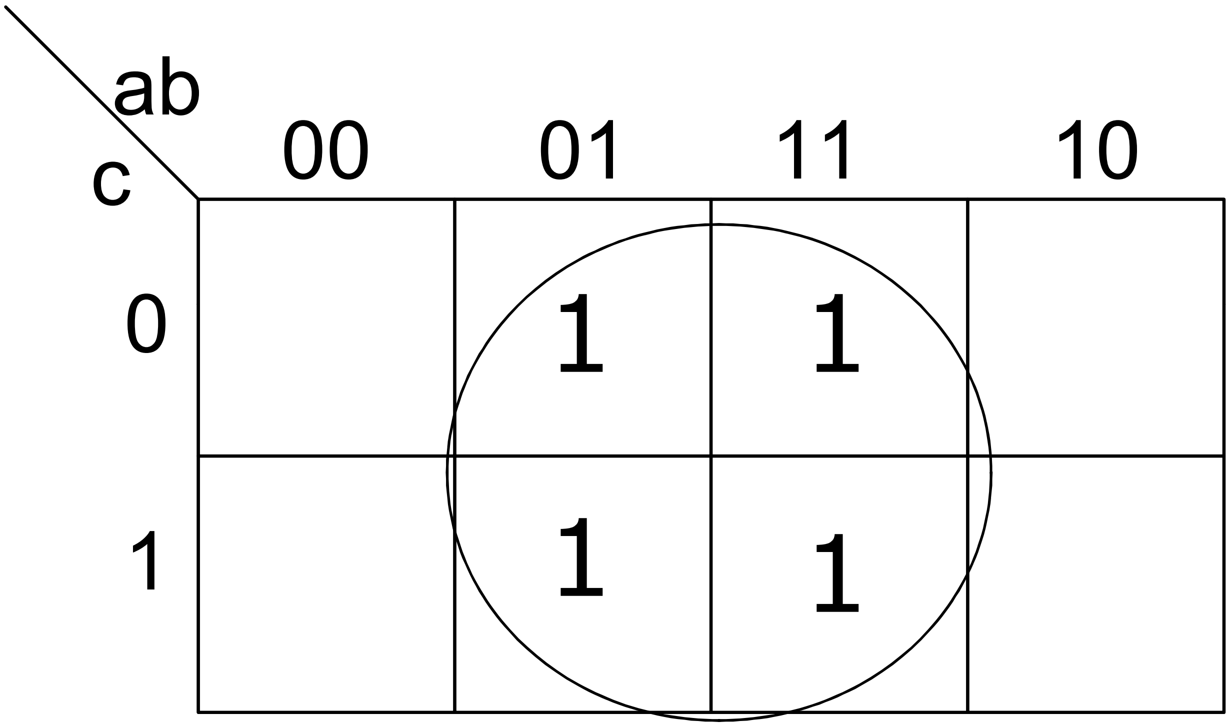
Solution

$$F(a,b,c) = \overline{a}b + ab = b$$

Since we can still simplify the function this means we did not use the largest possible groupings.

Three-Variable K-Map Example

Step 2: Circle Prime Implicants

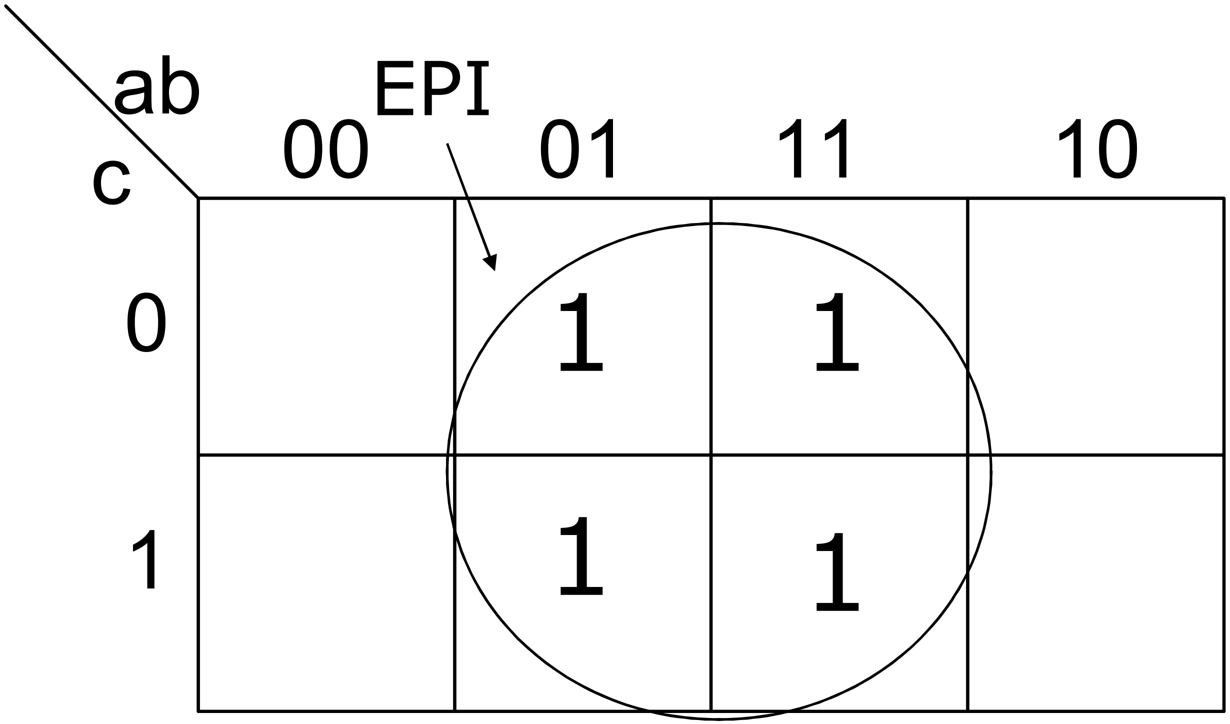


Right!

$$F(a,b,c)=\sum m(2,3,6,7)$$

Three-Variable K-Map Example

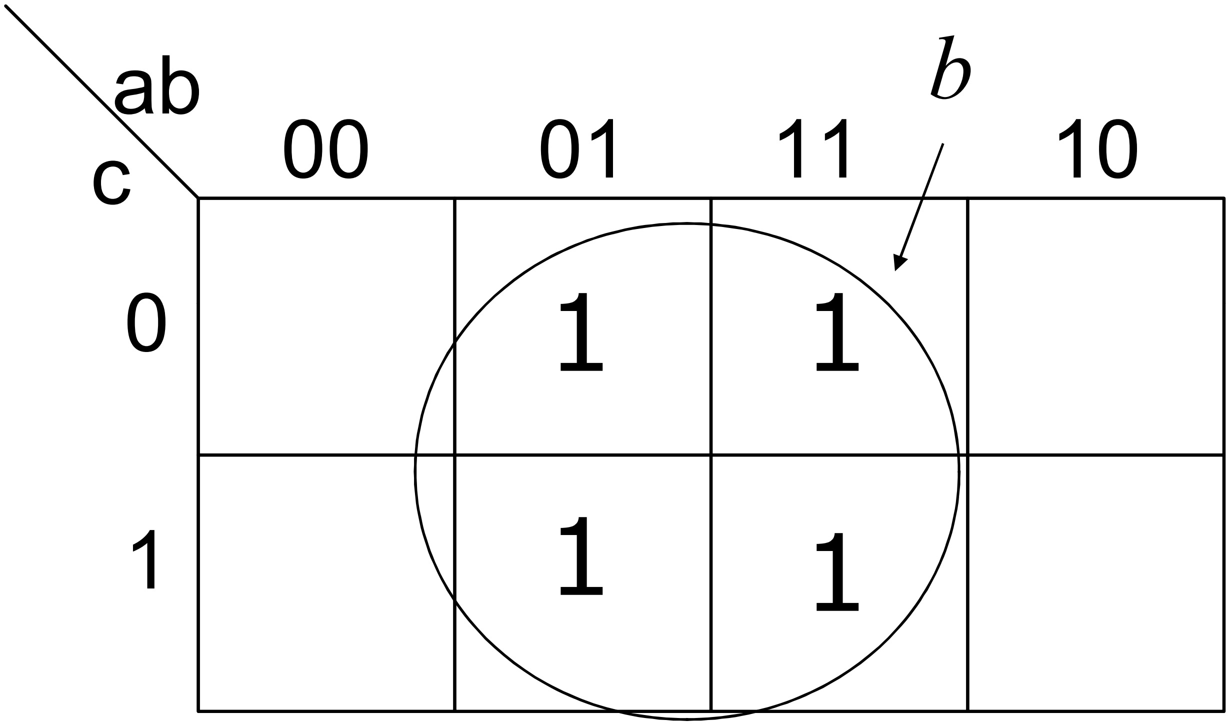
Step 3: Identify Essential Prime Implicants



$$F(a,b,c)=\sum m(2,3,6,7)$$

Three-Variable K-Map Example

Step 5: Read the map.



$$F(a,b,c)=\sum m(2,3,6,7)$$

Solution

$$F(a,b,c) = b$$

Special Cases

Three-Variable K-Map Example

c \ ab	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$$F(a,b,c)=1$$

Three-Variable K-Map Example

c \ ab	00	01	11	10
0				
1				

$$F(a,b,c) = 0$$

Three-Variable K-Map Example

c \ ab	00	01	11	10
0		1		1
1	1		1	

$$F(a,b,c) = a \oplus b \oplus c$$

Four Variable Examples

Example

- Use a K-Map to simplify the following Boolean expression

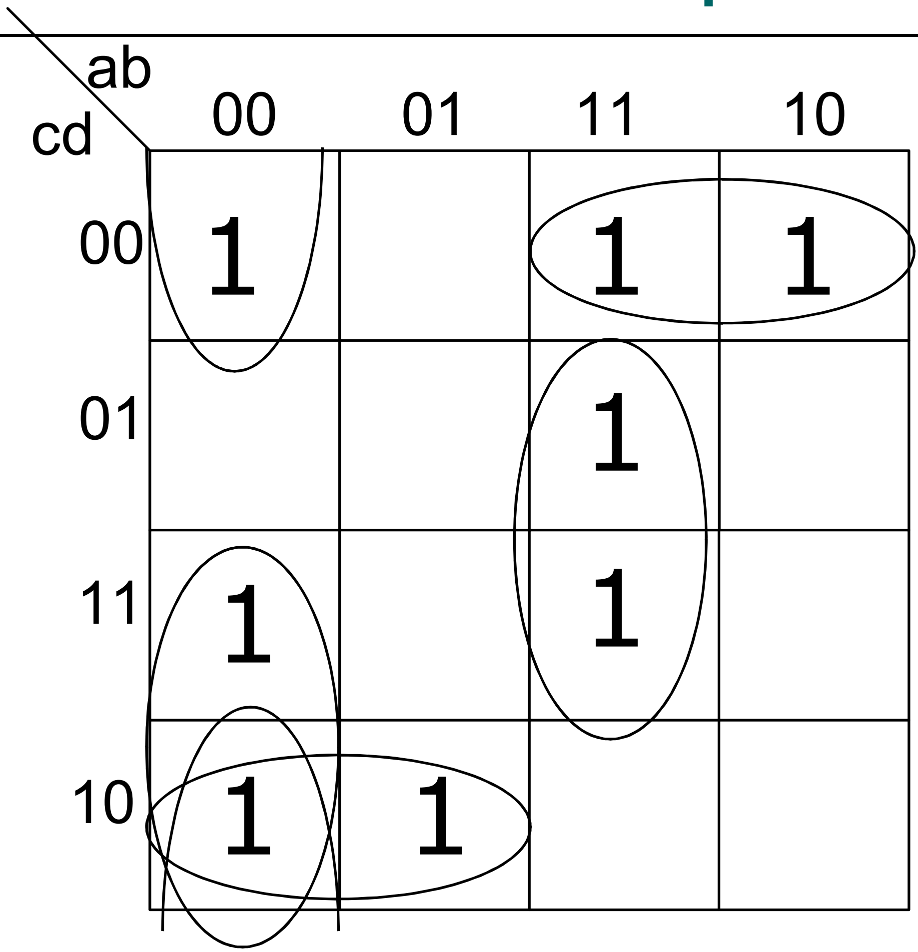
$$F(a,b,c,d) = \sum m(0,2,3,6,8,12,13,15)$$

Four-variable K-Map

cd \ ab	00	01	11	10
00	1		1	1
01			1	
11	1		1	
10	1	1		

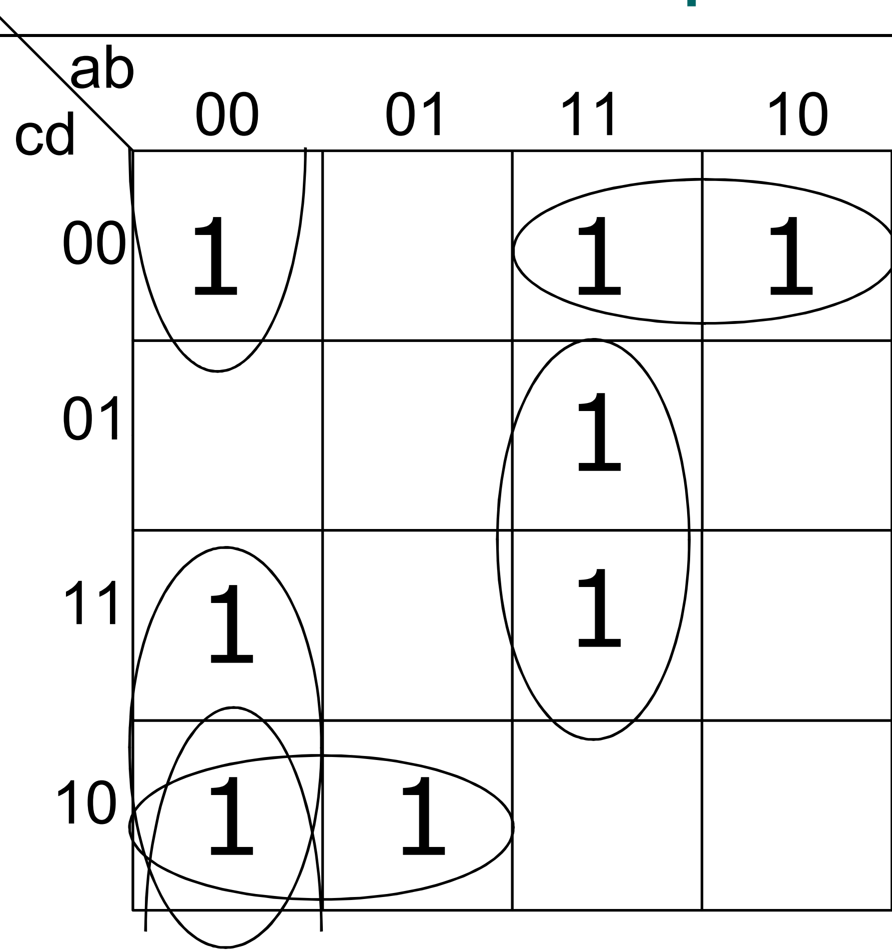
$$F(a,b,c,d) = \sum m(0,2,3,6,8,12,13,15)$$

Four-variable K-Map



$$F = \sum m\left(0,2,3,6,8,12,13,15\right)$$

Four-variable K-Map



$$F = \bar{a}\bar{b}\bar{d} + \bar{a}\bar{b}c + \bar{a}c\bar{d} + ab\bar{d} + a\bar{c}\bar{d}$$

Example

- Use a K-Map to simplify the following Boolean expression

$$F(a,b,c,d)=\sum m(0,2,6,8,12,13,15)$$

$$+d(3,9,10)$$

D=Don’t care (i.e. either 1 or 0)

Four-variable K-Map

cd \ ab	00	01	11	10
00	1	d	1	1
01			1	d
11	d		1	
10	1	1		

$$F(a,b,c,d) = \sum m(0,2,6,8,12,13,15) + d(3,4,9)$$

Four-variable K-Map

cd \ ab	00	01	11	10
00	1	d	1	1
01			1	d
11	d		1	
10	1	1		

$$F = a\bar{c} + \bar{a}\bar{d} + abd$$

Five Variable K-Maps

$$F(a,b,c,d,e)$$

Five variable K-map

Use two four variable K-maps

A=1

A=0

Use Two Four-variable K-Maps

A=0 map

de \ bc	00	01	11	10
00				
01				
11				
10				

A=1 map

de \ bc	00	01	11	10
00				
01				
11				
10				

Five variable example

$$F(a,b,c,d,e)=\sum m(5,7,13,15,21,23,29,31)$$

Use Two Four-variable K-Maps

A=0 map

de \ bc	00	01	11	10
00				
01		1	1	
11		1	1	
10				

A=1 map

de \ bc	00	01	11	10
00				
01		1	1	
11		1	1	
10				

$$F(a,b,c,d,e) = \sum m(5,7,13,15,21,23,29,31)$$

Use Two Four-variable K-Maps

A=0 map

de \ bc	00	01	11	10
00				
01		1	1	
11		1	1	
10				

$$F_1 = \overline{a}(ce)$$

A=1 map

de \ bc	00	01	11	10
00				
01		1	1	
11		1	1	
10				

$$F_2 = a(ce)$$

Five variable example

$$F = F_1 + F_2 = \bar{a}(ce) + a(ce) = ce$$

Plotting POS Functions

K-map Simplification Procedure

- Plot the K-map for the function $\overline{F}$
- Circle **<u>all</u>** prime implicants on the K-map
- Identify and select all essential prime implicants for the cover.
- Select a minimum subset of the remaining prime implicants to complete the cover.
- Read the K-map
- Use DeMorgan's theorem to convert $\overline{F}$ to F in POS form

Example

- Use a K-Map to simplify the following Boolean expression

$$F(a,b,c) = \prod M(1,2,3,5,6)$$

Three-Variable K-Map Example

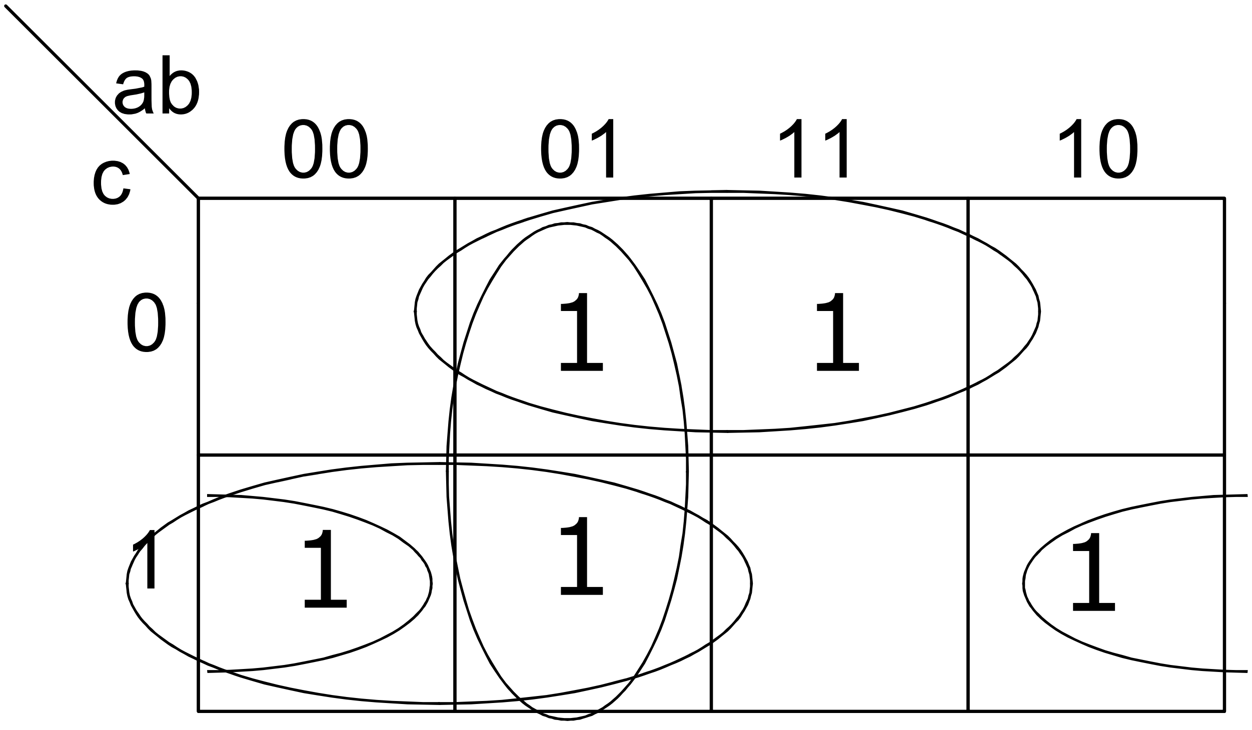
Step 1: Plot the K-map of $\overline{F}$

c \ ab	00	01	11	10
0		1	1	
1	1	1		1

$$F(a,b,c)=\prod M(1,2,3,5,6)$$

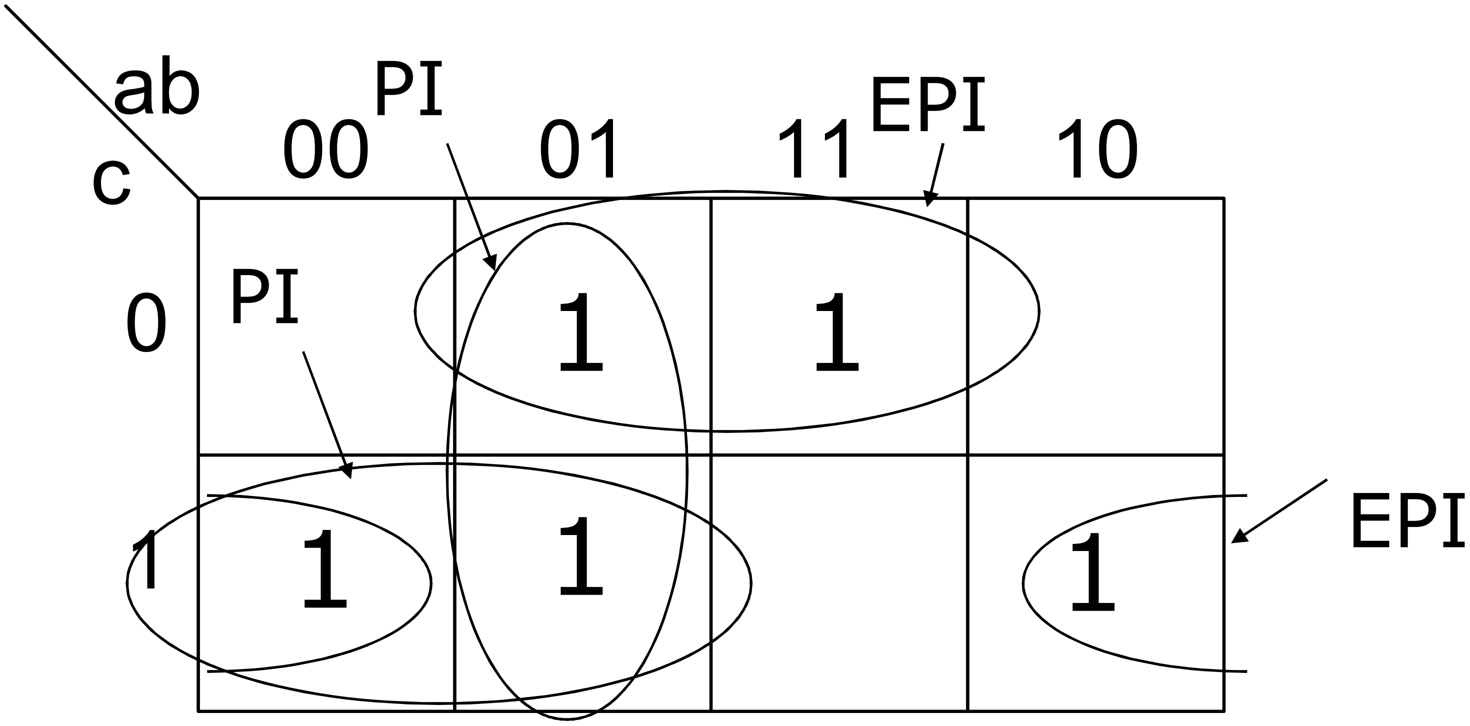
Three-Variable K-Map Example

Step 2: Circle **ALL** Prime Implicants



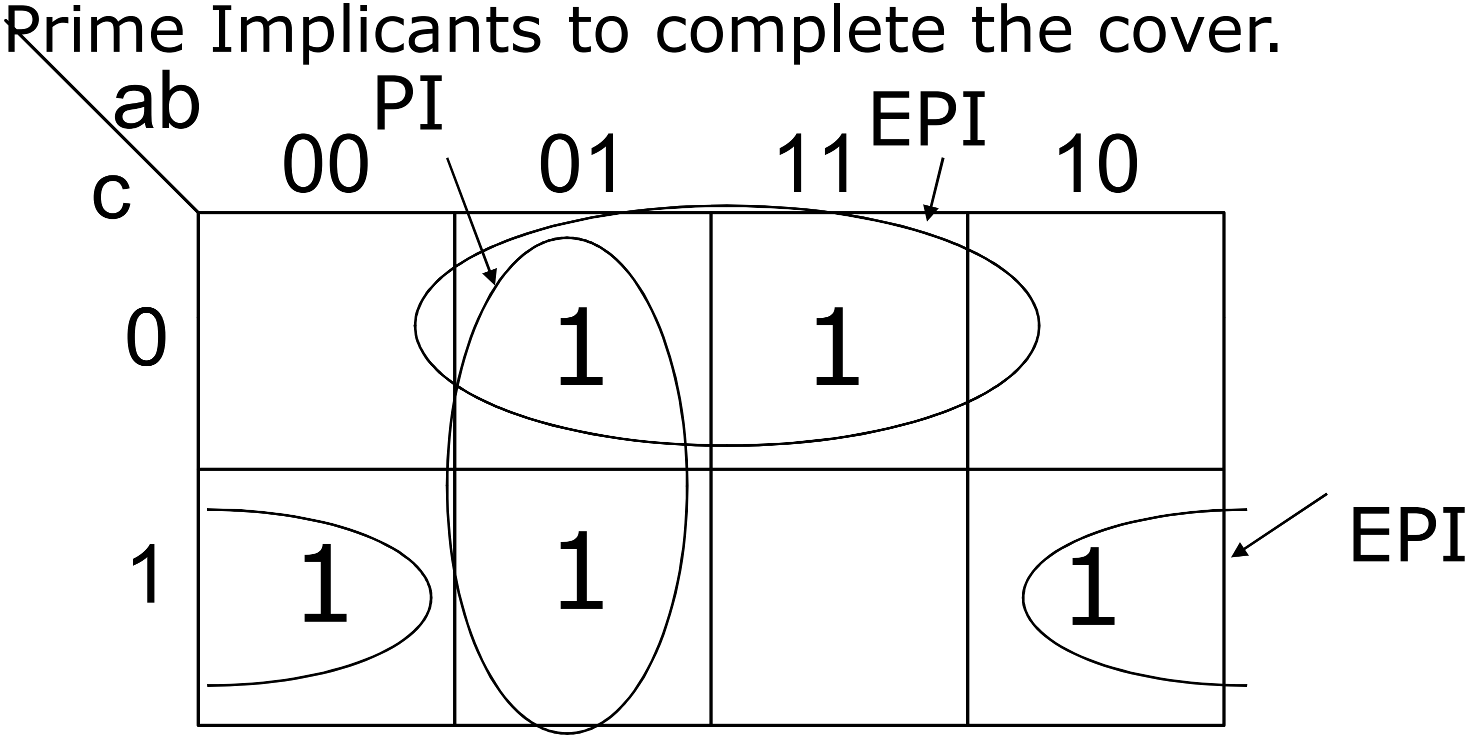
Three-Variable K-Map Example

Step 3: Identify Essential Prime Implicants



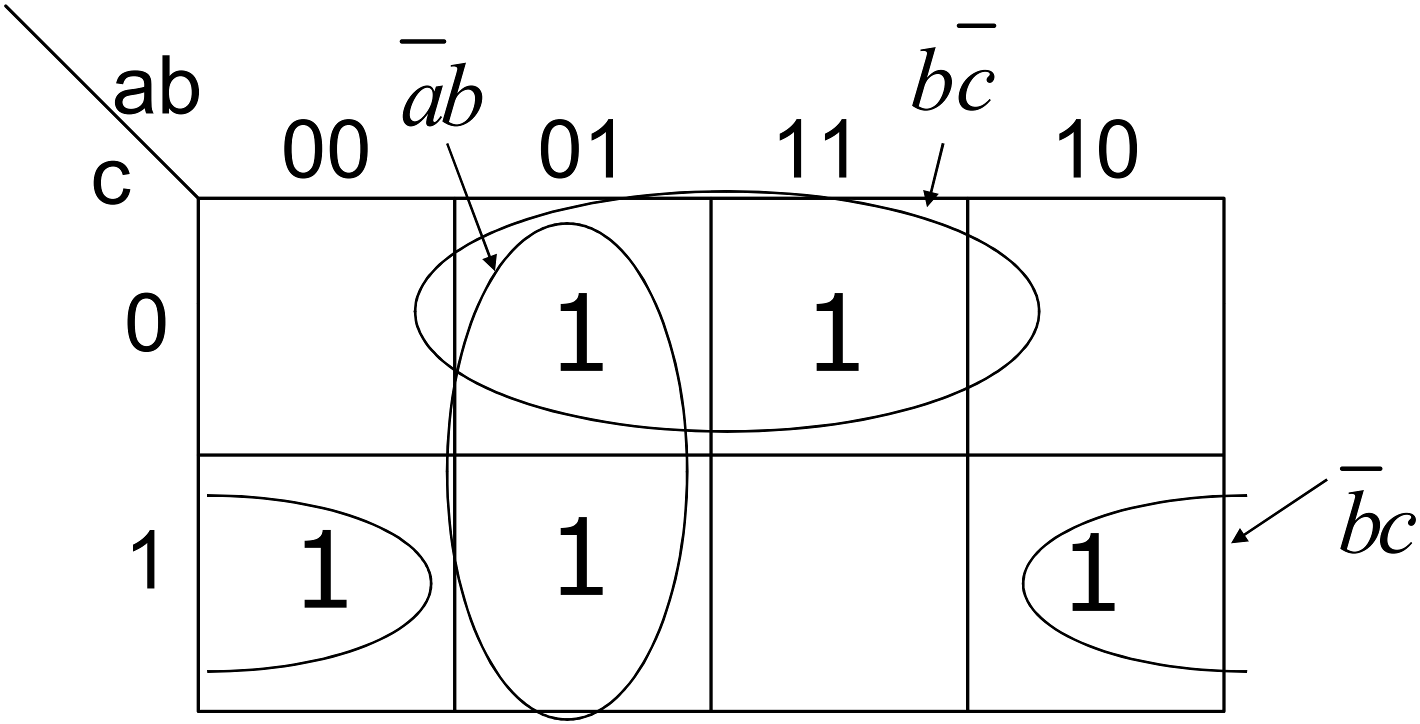
Three-Variable K-Map Example

Step 4: Select minimum subset of remaining Prime Implicants to complete the cover.



Three-Variable K-Map Example

Step 5: Read the map.



Solution

$$\overline{F} = \overline{a}b + b\overline{c} + \overline{b}c$$

$$F = \overline{\overline{a}b + b\overline{c} + \overline{b}c}$$

$$= \left(a + \overline{b}\right)\left(\overline{b} + c\right)\left(b + \overline{c}\right)$$

$$F\left(a,b,c\right) = \prod M\left(1,2,3,5,6\right)$$

TPS Quiz