Applied Physics Lecture 4

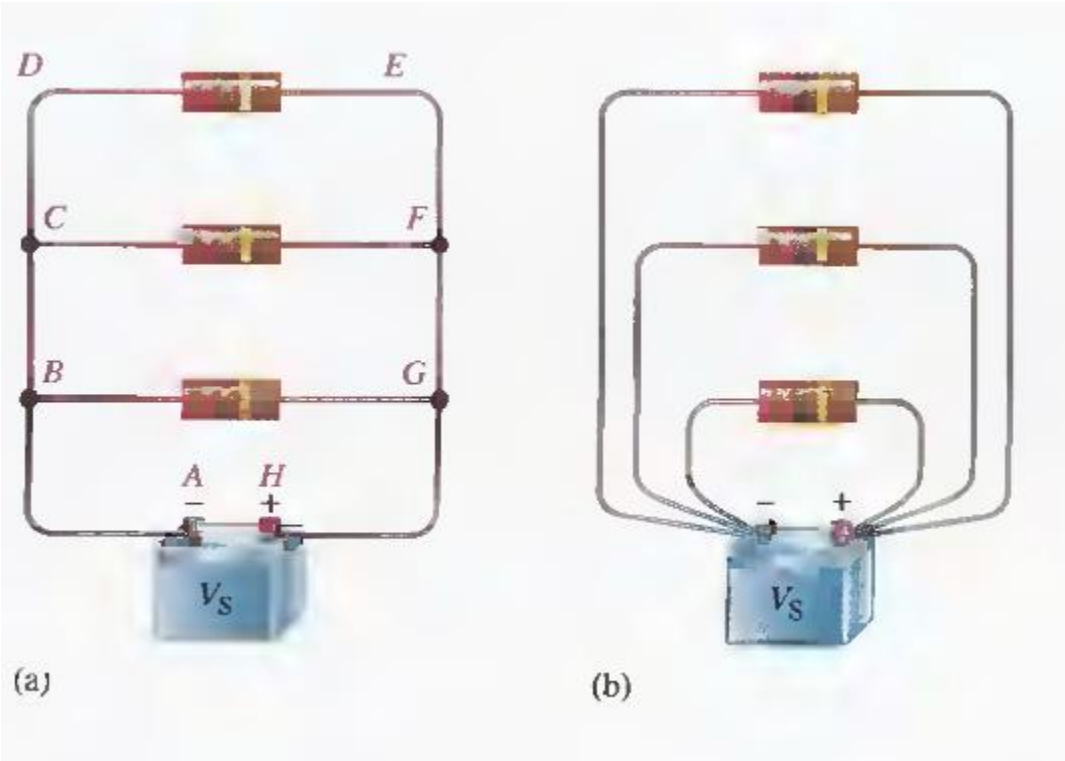
Resistive Circuits

Parallel Circuits

A branched circuit where resistors are connected side by side.

Also known as Shunt connections (another path)

All the loads operate independently.



$V = V_1 = V_2 = V_3 =$…….

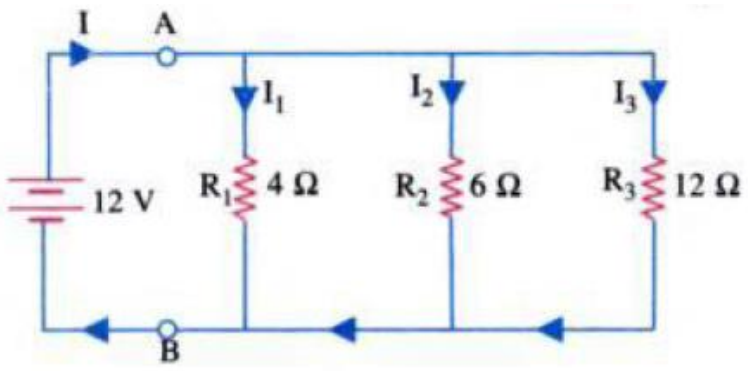
$I = I_1 + I_2 + I_3 +$…….

$I_T = V/ R_T$

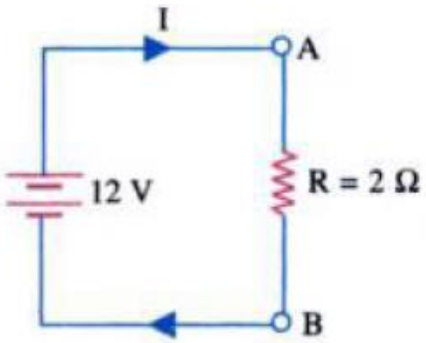
$I_1 = V/ R_1$, $I_2 = V/ R_2$, $I_3 = V/ R_3$,

$1/R = 1/R_1 + 1/R_2 + 1/R_3 +$…….

$G = G_1 + G_2 + G_3 +$…….



(a)



(b)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2} \qquad \therefore \quad R = 2\,\Omega$$

$$I_1 = \frac{V}{R_1} = \frac{12}{4} = 3\text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{12}{6} = 2\text{ A}$$

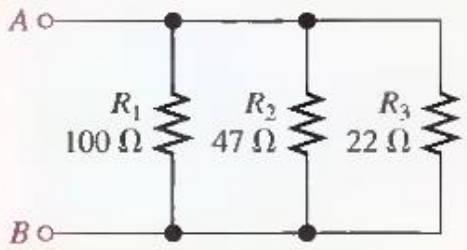
$$I_3 = \frac{V}{R_3} = \frac{12}{12} = 1\text{ A}$$

Battery current = 12/2 = 6 A

Sum of branch currents = 3 + 2 + 1 = 6 A

Calculate the total parallel resistance between points *A* and *B* of the circuit in Figure [redacted]

FIGURE [redacted]



$$G_1 = \frac{1}{R_1} = \frac{1}{100\ \Omega} = 10\ \text{mS}$$

$$G_2 = \frac{1}{R_2} = \frac{1}{47\ \Omega} = 21.3\ \text{mS}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{22\ \Omega} = 45.5\ \text{mS}$$

Next, calculate R_T by adding G_1, G_2, and G_3 and taking the reciprocal of the sum.

$$R_T = \frac{1}{G_T} = \frac{1}{10\ \text{mS} + 21.3\ \text{mS} + 45.5\ \text{mS}} = \frac{1}{76.8\ \text{mS}} = \mathbf{13.0\ \Omega}$$

For a quick accuracy check, notice that the value of R_T (13.0 Ω) is smaller than the smallest value in parallel, which is R_3 (22 Ω), as it should be.

Special Cases

Equal resistances in all branches

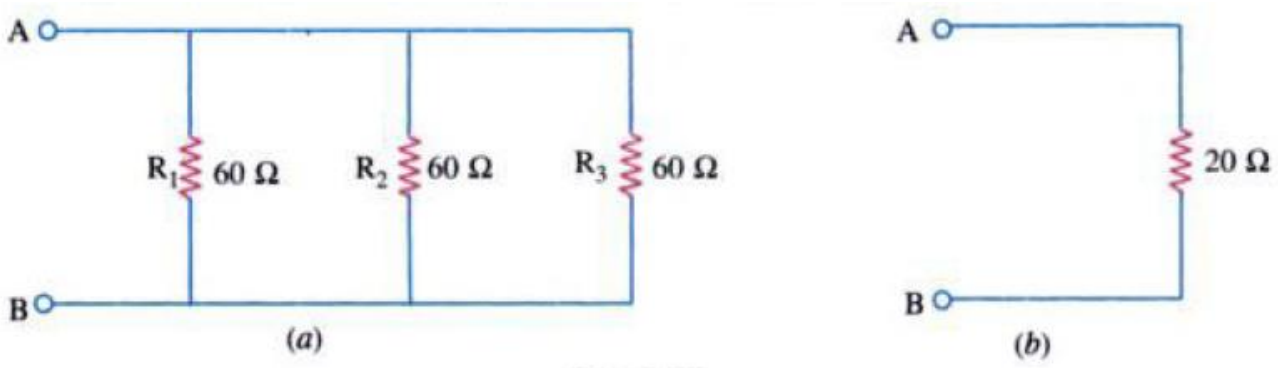
If all the resistors in parallel are equal then total resistance is equal to value of one resistance divided by number of resistors.

$R_T = R/n$

Four 8 Ω speakers are connected in parallel to the output of an amplifier. What is the total resistance across the output of the amplifier?

There are four 8 Ω resistors in parallel. Use Equation [redacted] as follows:

$$R_T = \frac{R}{n} = \frac{8\ \Omega}{4} = 2\ \Omega$$



$R_T = R/n = 60/3 = 20\ \Omega$

If n=4 then $R_T = 60/4 = 15\ \Omega$

More resistances in parallel circuit overall decreases the total resistance because overall current increases due to an additional current path and so resistance decreases (voltage is constant)

Only two branches

For only two unequal resistances, it is easy to use following formula.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Symbolically written as $R = R_1 \,||\, R_2$

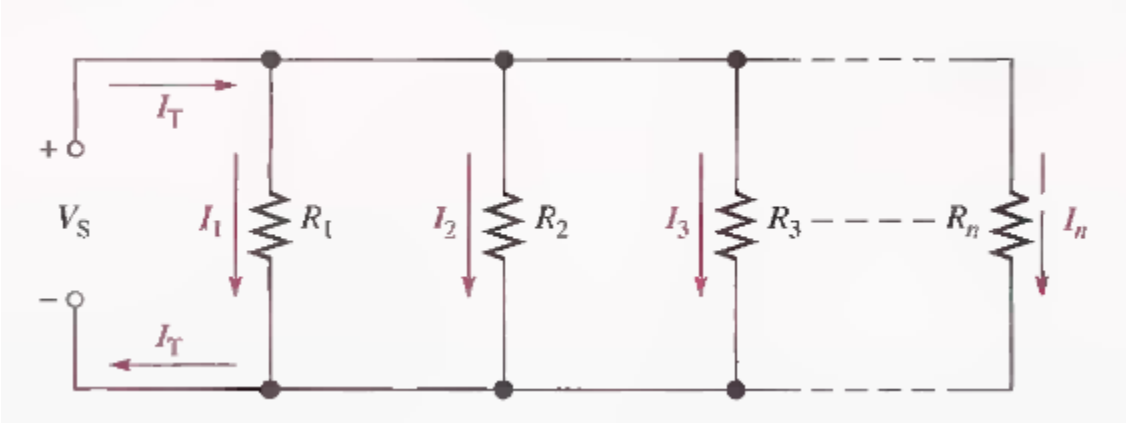
Current divider and proportional current formula

Resistors in parallel act as a current divider because

$I = I_1 + I_2 + I_3 + \ldots\ldots$

$I_1 = V/ R_1$, $I_2 = V/ R_2$, $I_3 = V/ R_3$,

Total current divides among parallel resistors into currents with values inversely proportional to the resistance values.



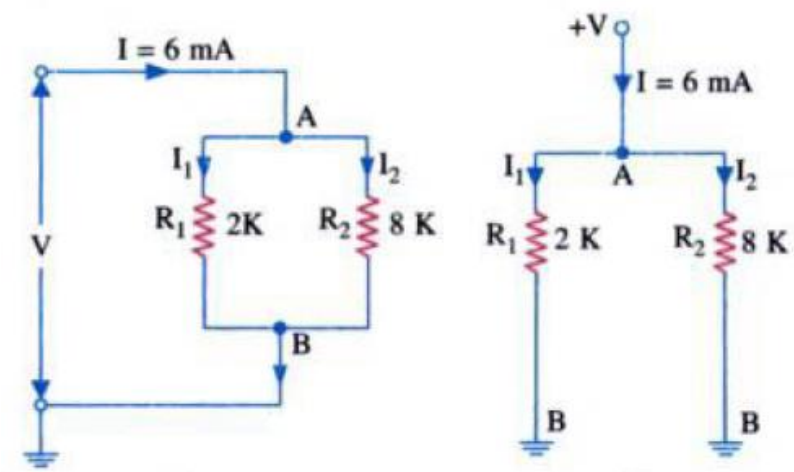
Proportional current formula

$$I_x = V/ R_x$$

$$V = I_T R_T$$

$$I_x = I_T R_T/ R_x$$

Consider an example of two resistors only



Using formula

$$I_x = I_T R_T / R_x$$

$$I_1 = I_T R_T / R_1$$

$$R_T = R_1 R_2 / R_1 + R_2 \text{ (for two resistors only)}$$

So

$$I_1 = I_T (R_1 R_2) / R_1 (R_1 + R_2)$$

$$I_1 = I_T . R_2 / R_1 + R_2$$

For I_2

$$I_2 = I_T . R_1 / R_1 + R_2$$

$$I_1 = I \cdot \frac{R_2}{R_1 + R_2}$$

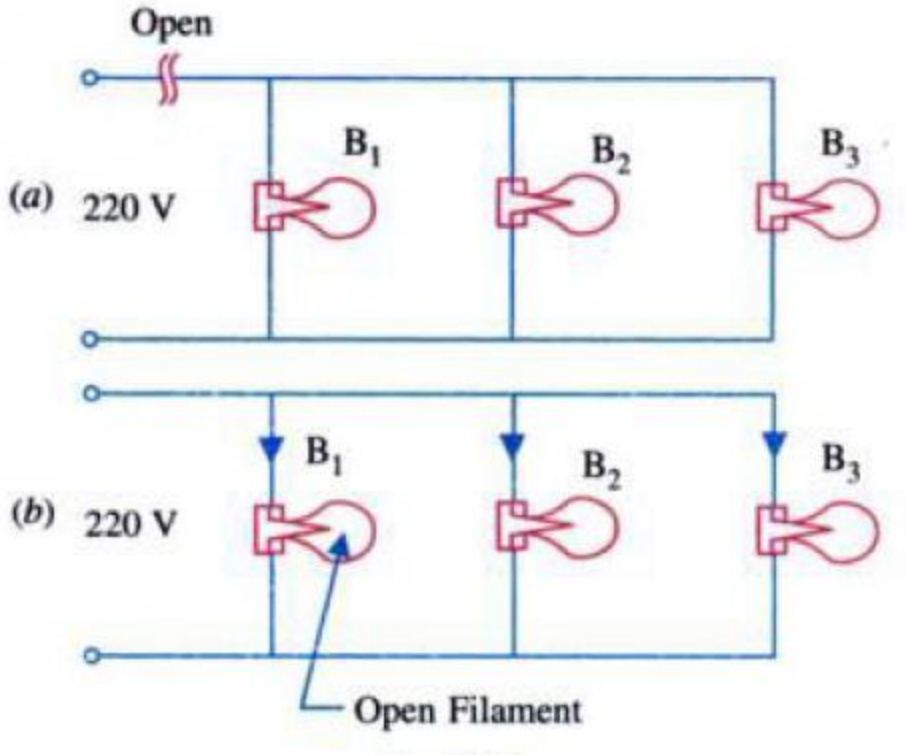
$$I_2 = I \cdot \frac{R_1}{R_1 + R_2}$$

$$I_1 = 6 \times \frac{8}{2+8} = 4.8 \text{ mA}; \quad I_2 = 6 \times \frac{2}{2+8} = 1.2 \text{ mA}$$

‘Opens’ and ‘Shorts’ in parallel circuit

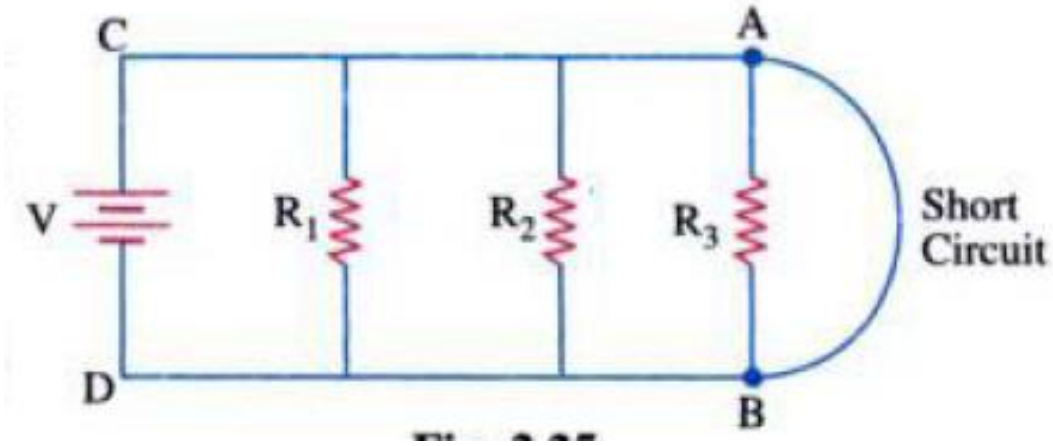
An ‘open’ means infinite resistance so no current flows.

If open occurs in main line then there will be no current in all branches. If open occurs in any one branch then other branches will remain unaffected.



If 'short' occurs then current will follow low resistance path and draws infinite current which may burn the wire.

If 'short' in one branch occurs then all other branches will also be short. There will be no current in all other branches. And components are not damaged.



Series Parallel circuit (from book 2.19-2.22, leave last topic)