# Chapter 4 - Gas Flow and Pumping 

## 4.1- Introduction: Flow Regimes

How does the flow of gas in vacuum systems compare with familiar examples of fluid flow in the world around us? At pressures sufficiently low that the molecular mean free path is comparable with or exceeds the size of the equipment, that is for $\mathrm{K}_{\mathrm{n}}$ values > 1, it is totally different. But for vacua not so rarefied, in which mean free paths are such that gases still demonstrate fluid behavior ( $\mathrm{Kn}<0.01$ ), the concepts and measures developed to describe fluid flow at atmospheric pressure remain appropriate.

To fix ideas we can imagine gas flow through a pipe of diameter D. This will be its characteristic dimension to be used in evaluating the Knudsen number as $K_{n}=\lambda / D$ and will determine what is called the flow regime. As earlier introduced in previous chapter to describe the condition of static gas, flow regimes are de fined by

## $\mathrm{Kn}<\mathbf{0 . 0 1}$ continuum flow regime

## $\mathrm{Kn}>1$ molecular flow regime

Whereas for $0.01<\mathrm{Kn}<1$ the flow regime is described as transitional.
These defining values are not as sharp as is implied, but their general correctness is founded in experimental results, particularly those involving viscous effects. Flow has distinct characteristics, to be discussed, in each regime. Flow in the transitional regime is difficult to analyse.

When considering flow through pipes of diameter D , it is useful to be able to determine the flow regime directly in terms of the prevailing pressure. Because for air $\lambda=64 \mathrm{~mm}$ at $\mathrm{p}=10^{-3}$ mbar, so that $\lambda p=0.064 \mathrm{~mm}$ mbar, the criteria for continuum flow and molecular flow become
$\mathrm{pD}>6.4 \mathrm{mbar} \mathrm{mm}$. $\qquad$ continuum flow
pD < 0.064 mbar mm. molecular flow

Vacuum systems of typical $\sim 0.5 \mathrm{~m}$ dimension frequently operate at pressures of $10^{-5} \mathrm{mbar}$ and less so that conditions are molecular; even at $10^{-4} \mathrm{mbar}, \lambda$ has increased to be 0.64 m .

### 4.2 Measures of Flow: Throughput and Pumping Speed

Before considering details of flow in the different regimes, it is necessary to define measures of flow. Figure 4.1 shows a pipe that connects two large volumes and through which gas flows at a steady rate and at constant temperature. In the volume at the left, the pressure is taken to be higher with a value $p_{U}$, where the subscript $U$ signifies upstream and flow is from
left to right. In the other volume, the downstream pressure is $\mathrm{p}_{\mathrm{D}}$. In the pipe at a crosssectional plane 1 near the entrance, the pressure is $\mathrm{p}_{1}$; at plane 2 further downstream, it is $\mathrm{p}_{2}$.


Figure 4.1: Flow of a gas through a pipe

## Throughput in terms of Volumetric Flow Rate

The mass of gas flowing per second through plane 1 at pressure $\mathrm{p}_{1}$ would have an associated volume $V_{1}$ at that pressure. Downstream at plane 2 and the lower pressure $p_{2}$, the associated volume $V_{2}$ would be larger because, gases expand in flowing from a higher to a lower pressure through a pipe. The magnitude of the effect (gas expansion) depends on pressure difference conditions and may be quite small.

Under conditions of steady isothermal flow, and assuming that the gas behaves ideally, $\mathrm{p}_{1} \mathrm{~V}_{1}$ $=\mathrm{p}_{2} \mathrm{~V}_{2}$. Denoting the volumes per second as volumetric flow rates at the associated pressures, this becomes $p_{1} \dot{V}_{1}=p_{2} \dot{V}_{2}$. Here, the product $p \times \dot{V}$ of pressure at any cross-section multiplied by the volumetric flow rate is called the throughput Q , gives a straightforward measure of the rate at which gas flows. Thus, defining throughput,

$$
Q=p \times \dot{V}
$$

For steady flow, Q is continuous, i.e., it has the same value at every position along the pipe, reflecting the conservation of mass. In particular, $\mathrm{Q}_{\mathrm{in}}=\mathrm{Q}_{\text {out }}-$ as much gas leaves the pipe downstream as enters it upstream.

The unit of $\mathrm{Q}_{3}$ depends on the base units used. In the SI system it is the Pascal meter ${ }^{3}$ per second $\left(\mathrm{Pa} \mathrm{m}^{3} \mathrm{~s}^{-1}\right)$. The more practical unit, widely accepted in Europe, and used henceforth in the text, is the millibar liter per second (mbar $1 \mathrm{~s}^{-1}$ ). Throughput is an easily assembled and manipulated measure of flow and is extensively used.

## Throughput in terms of Mass Flow Rate

When mass flow rates need to be specified directly in units of kg per second, conversions are easily made. Let $\dot{W}$ be the mass flow rate of gas in $\mathrm{kg} \mathrm{s}^{-1}$. The mass W of a gas may be expressed as

$$
\mathrm{W}=\mathrm{n}_{\mathrm{M}} \times \mathrm{M}
$$

the product of the number of moles and the molar mass. Now

$$
\mathrm{n}_{\mathrm{M}}=\mathrm{pV} / \mathrm{R}_{0} \mathrm{~T}
$$

and the flow rate in moles may be denoted as moles per second. At a particular plane of measurement where the pressure is p , this will become

$$
\dot{n}_{\mathrm{M}}=p \dot{V} / R_{0} T=Q / R_{0} T
$$

Thus

$$
Q=\dot{n}_{M} \times R_{0} T
$$

And because $\dot{W}=\dot{n}_{\mathrm{M}} \times M$, we have

$$
\begin{gathered}
\dot{W}=\frac{M}{R_{0} T} \times Q \\
\mathrm{Q}=\mathrm{R}_{0} \mathrm{~T} / \mathrm{M} \times \dot{W}
\end{gathered}
$$

## Throughput in terms of Molecular Flow Rate

It is sometimes useful to be able to relate throughput Q to ( $\mathrm{dN} / \mathrm{dt}$ ), the number of molecules flowing per second, also called the particle flow rate. Dividing both numerator and denominator in the right-hand side of above equation by Avogadro's number $\mathrm{N}_{\mathrm{A}}$, we get

$$
\dot{W}=(m / k T) \times Q
$$

where $m$ is the mass of a molecule and $k$ is Boltzmann's constant. But also

$$
\dot{W}=m \times(d N / d t)
$$

And therefor

$$
Q=k T\left(\frac{d N}{d t}\right)
$$

The volumetric flow rate is frequently given the symbol S and called the pumping speed. This is particularly so when it refers to the intake port of a pump or the entrance to a pipe that has a pump connected to its other end. In pumping practice, typical units used are liters per second, liters per minute, and $\mathrm{m}^{3}$ per hour; $\mathrm{m}^{3}$ per second is rare!

Remembering that S is a volumetric flow rate, the defining Equation of throughput now becomes

$$
\mathrm{Q}=\mathrm{S} \times \mathrm{p}
$$

This is the usual form of the first of two basic defining equations that describe gas flow in vacuum practice. It expresses the quantity of gas flowing as the product of the pressure and the volumetric flow rate at that pressure. Allied with the condition for continuity, it is an important tool for analysis.

### 4.3 Conductance

The other fundamental equation of flow relates throughput Q to the difference between the upstream and downstream pressures $p_{U}$ and $p_{D}$ in the two volumes that the pipe connects and serves to define the quantity conductance.Thus, referring again to Figure 4.1,

$$
\mathrm{Q}=\mathrm{C}\left(\mathrm{p}_{\mathrm{U}}-\mathrm{p}_{\mathrm{D}}\right)
$$

Evidently, C has the same dimensions as S , i.e., volume per second. A typical unit is liter per second.

It may be helpful to note that although in dc electrical circuits the connection between current (analogue: Q) and potential differences (analogue: $\mathrm{p}_{\mathrm{U}}-\mathrm{p}_{\mathrm{D}}$ ) is expressed in Ohm's law by a resistance, in vacuum practice the link is made by its inverse, the conductance. Thus, conductance is a measure of ease of flow in response to a pressure difference, and the greater the conductance for a given pressure difference, the greater the throughput.

Accordingly, and pursuing circuit analogies, one may expect that there are simple rules of combination for conducting elements in series and in parallel.

It is easily shown, and intuitively reasonable, that for conductances $C_{1}, C_{2}$, etc., in parallel, the effective conductance of the combination is given by

$$
C=C_{1}+C_{2}+\text { etc } \ldots
$$

while for elements in series

$$
\frac{1}{C}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+\ldots
$$

## Effect of conductance on pumping speed

Consider, as in Figure 4.2, a vessel within which the pressure is p connected via a pipe of conductance C to a pump of speed $\mathrm{S}^{*}$. The pumping speed at the vessel is S . Let the pressure at the pipe exit and the entrance to the pump be $\mathrm{p}^{*}$. The throughput from the vessel, through the pipe and into the pump, is Q given by.

$$
Q=C\left(p-p^{*}\right)=S^{*} \times p^{*}=S \times p
$$



Figure 4.2: Effect of conductance on pumping speed

This yields for $S$, after a little algebra

$$
S=\frac{S^{*} \times C}{S^{*}+C}
$$



Figure 4.3: Variation of pumping speed at the vessel with connecting pipe conductance

The significance of this formula is illustrated in Figure 4.3 in which $S / S^{*}$, the effective pumping speed at the vessel expressed as a fraction of the pump speed $S^{*}$, is plotted against $\mathrm{C} / \mathrm{S}^{*}$, the ratio of the connecting pipe conductance to $\mathrm{S}^{*}$. From this graph we can draw following conclusions.

* Necessarily, therefore, $S$ is less than $S^{*}$.
* Clearly, only when C is appreciably greater than $S^{*}$ is the pumping speed at the vessel comparable with that of the pump.
* It is halved if the conductance and the pump speed are equal.

We may visualize this result in terms of the volumes pumped at the vessel and at the pump. Gas expands as it moves downstream through the pipe to the pump, and the volumetric flow rate that is fixed at the pump end of the pipe by the pump's speed must, therefore, be smaller at the vessel.

### 4.4 Continuum Flow

In this regime, gas behaves as a fluid, and molecule-molecule collisions with mean free path much less than the equipment size determine gas behavior.

The characteristic dynamic property of a gas is its viscosity. If a gas had zero viscosity, its steady flow through a pipe would be characterized by uniform parallel streamlines as shown in Figure 4.4(a), and the gas velocity $u$ would be constant over any cross-section. In reality, however, viscosity causes the gas at the pipe wall to be stationary, so that the velocity profile is developed as shown in Figure 4.4(b), which has a maximum value at the center and some value $u$ averaged over the section.


Fig. 4.4 Velocity profiles for streamline flow in a pipe (a) Zero viscosity (b) Finite visosity

### 4.5 Dynamical Analysis of Continuum Flow through Long Pipe

The formula of Poiseuille and Hagen, which describes, as initially formulated, the flow of liquids in long pipes, is adaptable to the gas flow but is unfortunately of limited value. This is because the usual requirement in vacuum technology is that pipes that connect work chambers to pumps shall offer as little impedance as possible to gas flow, and so they are made as short and wide-bored as possible, consistent with spatial constraints. Typical conditions do not, therefore, involve long pipes. Nevertheless, because it is the only pipe flow problem with a simple analytical solution and because its use illustrates methods of analysis, it is presented here.

Consider, as in Figure 4.5, steady flow in a long pipe of diameter $D$ in a section of length $d x$ between positions $x$ and $x+d x$, in which the pressure falls from $p$ to $p$ to $p-d p$.


Fig. 4.5 Poiseuilllle fluid flow

The volumetric flow rate of fluid through this section according to Poiseuille and Hagen law is

$$
\dot{V}=-\frac{\pi D^{4}}{128 \eta} \frac{d p}{d x}
$$

For a gas, multiplying above equation by $p$ gives the throughput $Q$ at the section $x$, which is constant along the pipe. Thus

$$
Q=p \times \dot{V}=-\frac{\pi D^{4}}{128 \eta} p \frac{d p}{d x}=-\frac{\pi D^{4}}{128 \eta} \frac{d}{d x}\left(\frac{p^{2}}{2}\right)
$$

Integrating over the length $L$ of the pipe from $x=0(p=p 1)$ to $x=L(p=p 2)$ gives

$$
Q=\frac{\pi D^{4}}{128 \eta L}\left(\frac{p_{1}^{2}-p_{2}^{2}}{2}\right)=\frac{\pi D^{4}}{128 \eta L}\left(\frac{p_{1}+p_{2}}{2}\right)\left(p_{1}-p_{2}\right)
$$

The middle term on the right-hand side is the mean pressure.
Comparing above equation with fundamental equation of throughput below

$$
Q=C\left(p_{U}-p_{\mathrm{D}}\right)
$$

we obtain

$$
C=\frac{\pi D^{4}}{128 \eta L} \bar{p}
$$

### 4.6 Molecular Flow

Molecular flow is characterized by Knudsen numbers $\mathrm{K}_{\mathrm{n}}$ greater than unity, which physically means that the mean free path associated with the prevailing number density of the contained gas molecules is greater than the size of the container, with the consequence that molecule/wall collisions dominate gas behavior.

All semblance to fluid behavior is lost because there are no molecule-molecule collisions. These are the conditions in the work chambers of many vacuum systems because, as previously noted, the mean free path for nitrogen at $10^{-4} \mathrm{mbar}$ is 0.64 m , so that for a chamber of typical size, conditions are molecular at pressures below this value.

### 4.6.1 Dynamical analysis of molecular flow through Long Pipe

In a pipe of length $L$ and diameter $D$, consider a short section between coordinates $x$ and $x+$ $d x$, across which pressure changes from $p$ to $p-d p$. There will be an associated change $n$ to $n$ $-d n$ in the number density of molecules, but $n$ may be taken as the density in the section for the purposes of the calculus. Molecular flow in the pipe may be considered to occur with a mean drift velocity, superposed on the thermal velocities, that is reduced to zero by the collisions that molecules have with the wall.

By Newton's second law, equating the rate of change of momentum due to the loss of in wall collisions to the force across the element, gives

$$
(n \bar{v} / 4)(\pi D d x)(m \bar{u})=\left(\pi D^{2} / 4\right) d p
$$

in which the first two terms on the left-hand side give the number of wall impacts per second in the element $d x$. After cancellations,

$$
n \bar{v} m \bar{u}=D(d p / d x)
$$

The number of molecules per second passing through the plane at $x$ is

$$
\frac{d N}{d t}=\left(\frac{\pi D^{2}}{4} \bar{u}\right) n=\frac{\pi D^{2}}{4} \frac{D}{m \bar{v}} \frac{d p}{d x}
$$

Substituting value of mean velocity $\bar{v}=\sqrt{8 k T / \pi m}$ and multiplying by $k T$ to convert to a throughput, and integrating over the pipe leads to

$$
Q=\frac{\pi D^{3}}{16 L} \sqrt{\frac{2 \pi k T}{m}}\left(p_{1}-p_{2}\right)
$$

The conductance is therefore

$$
C=\frac{\pi D^{3}}{16 L} \sqrt{\frac{2 \pi k T}{m}}=\frac{\pi D^{3}}{16 L} \sqrt{\frac{2 \pi R_{0} T}{M}}
$$

Because this treatment is oversimplified in its assumptions about the drift velocity, the factor $\pi / 16$ should be replaced by $1 / 6$ (see Loeb, 1961) to give

$$
C_{L}=\frac{D^{3}}{6 L} \sqrt{\frac{2 \pi R_{0} T}{M}}
$$

This formula was first proposed by Knudsen and is correct for long pipes. The geometrical dependence on $D^{3} / L$ is its most important feature and again points towards making pipes as short and fat as possible to maximize conductance.

The factor ${ }^{T / M}$ indicates the dependence on the particular gas and its temperature and is directly related to the molecular velocity. Note that, as one would expect, the conductance does not depend on pressure.

### 4.6.2 Dynamical analysis of Molecular Flow through an Aperture

Consider an aperture of area $A$ in a very thin wall separating two regions maintained at different pressures $p_{1}$ and $p_{2}$, with $p_{1}>p_{2}$ and the gas in both regions sufficiently rarefied that conditions are molecular, as shown in Figure 4.6. The molecular mean free path is greater than the diameter of the aperture, and there are no molecule-molecule collisions.


Fig. 4.6 Molecular flow through an Aperture
From each side, molecules approach the opening from all directions within a $2 \pi$ solid angle and with a range of speeds. The fluxes are represented by the arrows $J$. Corresponding to $p_{1}$ and $p_{2}$ are number densities $n_{1}$ and $n_{2}$ and associated fluxes $J_{1}$ and $J_{2}$, where $J=n \bar{v} / 4=p / \sqrt{2 \pi m k T}$. Molecules heading towards the aperture opening from both sides will pass through it, and so with $J_{1}>J_{2}$ as indicated, there will be a net flow of molecules from left to right. The number of molecules per unit time will be

$$
\frac{d N}{d t}=\left(J_{1}-J_{2}\right) A
$$

Multiplying by kT to convert to a throughput and substituting for $J$ gives

$$
Q=k T \times \frac{\left(p_{1}-p_{2}\right)}{\sqrt{2 \pi m K T}} A=\sqrt{\frac{k T}{2 \pi m}} A\left(p_{1}-p_{2}\right)=\sqrt{\frac{R_{0} T}{2 \pi M}} A\left(p_{1}-p_{2}\right)
$$

Comparing this with the basic defining equation for conductance,

$$
Q=C\left(p_{U}-p_{\mathrm{D}}\right)
$$

we obtain conductance of an aperture for molecular flow. Here, introducing the symbol $C_{A}$ for the molecular flow conductance for an aperture,

$$
C_{A}=A \sqrt{\frac{R_{0} T}{2 \pi M}}
$$

This is an important result, exploitable not only in its own right but also because the entrances into pipes and pumps can be regarded as apertures. The presence of the factor $\sqrt{T / M}^{T}$ that will also occur in other formulas for molecular flow conductance is noteworthy because it enables conductance values for other gases to be quickly computed once values for a particular gas of reference, usually nitrogen, are known.

### 4.7 The Concept of Transmission Probability and Molecular Flow

Consider, as in Figure 4.7, a pipe of length $L$, diameter $D$, and cross-sectional area $A$ connecting two regions of low pressure $p_{1}$ and $p_{2}$, such that $\lambda \gg L, D$, and conditions are molecular. The total number of molecules per second crossing the plane EN to enter the pipe is $J_{1} A$. They approach it from all directions within a solid angle $2 \pi$ in the left-hand volume.


Fig. 4.7 Molecular flow through a pipe
Relatively few molecules, like molecule (1), will be traveling in such a direction as to pass right through the pipe without touching the sides, but most will not. The majority, like molecule (2), will collide with the wall at a place such as X and return to the vacuum in a random direction as discussed earlier.

There are now three possible outcomes (a), (b), or (c) as shown by the dashed trajectories.
The molecule may (a) return to the left-hand volume, (b) go across the pipe to Y , and then another "three-outcome" event, or (c) leave the pipe through the exit plane EX into the righthand volume.

These three outcomes occur with different probabilities. Furthermore, for a molecule which goes to Y at a different distance along the pipe, the balance of probabilities for where it next goes will have changed accordingly. A little reflection shows that this three-dimensional problem is quite complex to analyze.

## Dynamical analysis of Molecular Flow through Long Pipe using concept of Transmission Probability

Considering the total number of molecules per second, $J_{1} A$, which cross plane EN and enter the pipe, and the diverse possibilities for their subsequent trajectories, it is clear that some molecules will eventually be transmitted through the exit plane EX. The remainder will return through the plane EN as shown in Fig.4.7.

The fraction that does pass through EX into the right-hand region may be defined as the transmission probability $\alpha$ of the pipe so that

$$
\text { Transmitted flux }=\alpha\left(J_{1} A\right)
$$

We expect that will be large for short pipes, and that for $L \ll D$ it will approach unity, corresponding to the flow through an aperture in a thin wall. Increase of $L$, because of the
increased number of randomizing collisions with the pipe wall, must cause $\alpha$ to decrease. For flow in the right-to-left direction, similar considerations must apply.

The transmission probability of the pipe must be the same in both directions, but the flux $J_{2}$ corresponds to the lower pressure $p_{2}$. The right-to-left flow (backward flux) is, therefore,

$$
\text { Backward flux }=\alpha\left(J_{2} A\right)
$$

and the net observable flow is the difference of the flows in each direction.

$$
\text { Net forward flow }(\text { flux })=\alpha(\mathrm{J} 1-\mathrm{J} 2) \mathrm{A}
$$

Thus, multiplying this net flow rate $\alpha(J 1-J 2) A$ by $k T$ to get a throughput gives

$$
Q=k T\left(J_{1}-J_{2}\right) A \alpha
$$

Substituting for J gives

$$
\begin{gathered}
Q=\sqrt{\frac{k T}{2 \pi m}} A \alpha\left(p_{1}-p_{2}\right)=\sqrt{\frac{R_{0} T}{2 \pi M}} A \alpha\left(p_{1}-p_{2}\right) \\
Q=\alpha C_{\mathrm{A}}\left(p_{1}-p_{2}\right)
\end{gathered}
$$

and therefore comparing with fundamental equation of throughput

$$
Q=C\left(p_{\mathrm{U}}-p_{\mathrm{D}}\right)
$$

We obtain conductance of long pipe for molecular flow

$$
C=\alpha C_{\mathrm{A}}
$$

For an aperture, clearly, $\alpha=1$

For long circular pipes

$$
\alpha=\frac{C_{L}}{C_{A}}=\frac{4 D}{3 L}
$$

Modern methods (Computer simulations) show, however, that even for $\mathrm{L} / \mathrm{D}=20$, which would normally be regarded as a safe approximation to being long pipe, this formula gives values that are $10 \%$ too high.

### 4.8 The Pumping Process, Pump-Down Time, and Ultimate Pressure

Having introduced the quantities throughput Q and pumping speed S earlier in this chapter, and the subject of vapor release and outgassing from surfaces in Chapter 3, we can now set up the basic equation of the pumping process.

Figure 4.8 is a schematic representation of a pumping system. A vessel of volume V is connected via pipe of conductance $C$ to a pump of speed $S^{*}$. The pumping speed at the vessel will be S .


Fig. 4.8 Schematic representation of a pumping system
Various sources may contribute to the gas load that has to be pumped. In addition to gas originally in the volume, outgassing from the interior surfaces will commence as soon as the pressure is reduced, as discussed in Chapter 3. Its magnitude may be represented by a throughput $\mathrm{Q}_{\mathrm{G}}$. There may be gas entry into the volume with throughput $\mathrm{Q}_{\mathrm{L}}$ by unintended leaks or, in some applications, the intentional steady inflow of a specific gas. At some stage, gas may be produced internally as a result of an operating process for which the system has been designed, and when activated, it will contribute a throughput, say, $\mathrm{Q}_{\mathrm{P}}$, to the load. These contributions are represented schematically in the figure 4.8.

Let $\mathrm{Q}_{\mathrm{T}}$ be the total of all such contributions and any others, such as vaporization, that cause the entry of gas into the volume, then

$$
Q_{\mathrm{T}}=\dot{Q}_{\mathrm{C}}+Q_{\mathrm{L}}+Q_{\mathrm{P}}^{2}+\text { etc. }
$$

In some cases, for example, for a system with no leaks in which there are no gas generating processes, $\mathrm{Q}_{\mathrm{T}}$ will simply be due to outgassing.

The pumping equation assumes the isothermal conditions normally encountered and expresses the fact that the change in the quantity of gas in the volume V , which is associated with a change dp in the pressure p in a small time-interval dt , must be the difference in the quantities entering the volume and leaving it. Thus in pressure-volume units:

$$
V d p=Q_{\mathrm{T}} d t-S p d t
$$

In the context of pumping to evacuate the vessel, the rate of exit of gas exceeds that of entry, and pressure will be falling so that dp and therefore $\mathrm{dp} / \mathrm{dt}$ are negative. The equation may be written to express the positive rate of reduction of gas in the volume, which is $-\mathrm{V}(\mathrm{dp} / \mathrm{dt})$, as

$$
-V\left(\frac{d p}{d t}\right)=S p-Q_{\mathrm{T}}
$$

This differential equation is the fundamental pumping equation, expressing the fact that the rate of change of the amount of gas in the volume at any instant is the difference between the rate of its removal $\mathrm{S} \times \mathrm{p}$ by the pump and the in flux rate $\mathrm{Q}_{\mathrm{T}}$. Although this is an exact equation true at all times $t$, integrating it to get realistic information about how pressure falls with time is often complicated for a number of reasons.

In many applications, pumping speed $S$ at the vessel depends on pressure. This may be due to either the pressure dependence of the speed $S^{*}$ of the pump itself or, unless flow is in the molecular regime, of the conductance of the connection, or both. Secondly, the gas in flux rate due to outgassing varies significantly with time, and will depend on the previous conditions of use of a system, slowly diminishing as pumping proceeds to a small and sensibly constant value, but only being dramatically reduced in normal experimental times if special procedures such as baking are adopted.

There are, nevertheless, two results of prime importance that may be obtained from above pumping equation. They relate to the lowest pressure achievable and the pump-down time when pumping speed can be considered constant. It is evident from pumping equation that when pressure eventually ceases to fall so that $\mathrm{dp} / \mathrm{dt}$ becomes zero, the steady pressure achieved in the vessel, called the system's ultimate pressure or its base pressure, and denoted pu is given by

$$
p_{s}=\frac{Q_{\mathrm{T}}}{S}
$$

This confirms common-sense thinking that, for a given pumping speed, low pressures will be achieved for small gas loads. Equally, for a given gas load, the best vacuum is obtained for the largest pumping speed. When steady state is eventually attained, the gas load and the pump's gas handling capacity are in balance.

We may note that in the early stages of pumping, starting at atmospheric pressure, because the system will be free of large leaks and the contribution of outgassing negligibly small, the term $\mathrm{S} \times \mathrm{p}$ will be very much larger than the term $\mathrm{Q}_{\mathrm{T}}$, which can be ignored. Therefore, with rearrangement.

Furthermore, many of the primary pumps used in these early stages of pumping, and particularly the rotary pump, have pumping speeds that are sensibly constant over several decades of pressure, from 1000 down to $10^{-1}$ mbar or less. Therefore, with S constant, the above equation may be straightforwardly integrated to give

$$
p=p_{0} \exp \{-(S / V) t\}
$$

where p is the pressure at time t and $\mathrm{p}_{0}$ its value at $\mathrm{t}=0$ when pumping starts. Under these conditions, therefore, pressure falls exponentially with time, $p=p_{0} \exp (-t / \tau)$, and with a time constant $\tau=\mathrm{V} / \mathrm{S}$.

Above equation may be restated as

$$
t=(V / S) \ln \left(p_{0} / p\right)
$$

So that the time taken for the pressure to fall from $\mathrm{p}_{0}$ to p may be determined
Above equation may also be used to determine the pumping speed necessary to pump down a volume to a given pressure in a specified time. Thus

$$
S=(V / t) \ln \left(p_{0} / p\right)
$$

## Chaper 4 - Exercise Problems

4.1 Below what pressure approximately will the flow of air in a pipe 25 mm in diameter cease to be continuum and become transitional? Below what pressure will it become molecular?
4.2 At a particular cross section in a pipe the volumetric flow rate of a gas is $30 \mathrm{l} \mathrm{s}^{-1}$ at a pressure $10^{-4} \mathrm{mbar}$; what is the throughput?
4.3 Convert (a) a throughput of 1 mbar liter per second to $\mathrm{Pa} \mathrm{m}^{3} \mathrm{~s}^{-1}$ and (b) a volumetric flow rate of $11 \mathrm{~s}^{-1}$ to $\mathrm{m}^{3} / \mathrm{h}$.
4.4 The throughput of oxygen in a certain process is $766 \mathrm{~Pa} \mathrm{~m}^{3} \mathrm{~s}^{-1}$. Find the mass flow rate in $\mathrm{kg} \mathrm{s}^{-1}$
4.5 In a controlled steady leak into a vacuum system, $1 \mathrm{~cm}^{3}$ of atmospheric air at 1000 mbar is sucked into the vacuum in a period of 5 seconds. What will be the volumetric flow rate internally if the pressure there is held at $10^{-1} \mathrm{mbar}$ by a pump?
4.6 A pump of speed $300 \mathrm{l} \mathrm{s}^{-1}$ holds a vacuum of $2 \times 10^{-5} \mathrm{mbar}$ in a vessel to which it is connected. What is the throughput? What is the pumping speed at a downstream location in the pumping arrangement where the pressure is $10^{-2} \mathrm{mbar}$ ?
4.7 A vacuum chamber may be connected to a pump of speed $100 \mathrm{l} \mathrm{s}^{-1}$ by pipes of conductance (a) 1000 , (b) 400 , or (c) $1001 \mathrm{~s}^{-1}$. Calculatethe pumping speed at the chamber in each case.
4.8 Air flows from a region of steady upstream pressure (a) 10 mbar , (b) 1 mbar through an aperture 1 mm square into a region where the pressure is kept below 0.1 mbar by pumping. What will be the pumping speeds and throughputs in each case?
4.9 Two conductances of value $100 \mathrm{l} \mathrm{s}^{-1}$ and $80 \mathrm{l} \mathrm{s}^{-1}$ are in parallel with each other, and the combination is in series with a conductance of $1801 \mathrm{~s}^{-1}$. What is the conductance overall?.
4.10 A component has a molecular flow conductance of $500 \mathrm{l} \mathrm{s}^{-1}$ for nitrogen. What will its conductance be for (a) hydrogen, (b) carbon dioxide?
4.11 By what factor will the molecular flow conductance of a long pipe be increased if its diameter is doubled?
4.12 The molecular flow transmission probability for a pipe whose length is equal to its diameter is 0.51 , so that only about one half of the molecules that enter it pass through. What fraction will get through for a pipe with $\mathrm{L} / \mathrm{D}=5$ ? Use this value to compute the molecular flow conductance for nitrogen gas of a pipe with entrance diameter 15 cm and 75 cm long.
4.13 The molecular flow transmission probability of a component with entrance area $4 \mathrm{~cm}^{2}$ is 0.36 . Calculate its conductance for nitrogen at (a) 295 K , (b) 600 K .
4.14 A vessel of volume $4 \mathrm{~m}^{3}$ has to be evacuated from 1000 mbar to 1 mbar in 20 min . What pumping speed (in $\mathrm{m}^{3}$ per hour) is required?
4.15 How long will it take for a vessel of volume 801 connected to a pump of speed $5 \mathrm{l} \mathrm{s}^{-1}$ to be pumped from 1000 to 10 mbar ? What is the time per decade?
4.16 What time constant is associated with the pumping of a vessel of volume 601 with a pump of speed $3001 \mathrm{~s}^{-1}$ ?
4.17 A pipe of conductance $250 \mathrm{l} \mathrm{s}^{-1}$ is attached to a pump of speed $50 \mathrm{l} \mathrm{s}^{-1}$. Calculate the ratio of the upstream to the downstream pressure and the upstream pumping speed at the pipe inlet.
4.18 Evaluate and compare the molecular flow conductances for nitrogen of (a) a circular hole of diameter 5 cm in a thin plate, (b) a pipe 20 cm long and 5 cm in diameter that therefore has the same entry diameter as (a), and (c) a pipe of this bore 1 m long.
4.19 A vacuum chamber of volume $0.03 \mathrm{~m}^{3}$ and internal area $0.6 \mathrm{~m}^{2}$ is connected by a pipe of conductance $1600 \mathrm{l} \mathrm{s}^{-1}$ to a pump of speed $400 \mathrm{l} \mathrm{s}^{-1}$. Calculate the pumping speed at the chamber. If the ultimate pressure achieved is $2 \times 10^{-8} \mathrm{mbar}$, calculate the gas throughput andhence estimate a gassing rate per $\mathrm{cm}^{2}$ of internal surface. If the pumping action suddenly ceases due to the closure of a valve above the pump, how much time will elapse before the pressure rises to(a) $10^{-6}$ (b) $10^{-5} \mathrm{mbar}$ ? What time constant may be associated with the pumping when it is restored?

