Number's:

Mathematical symbols which are used to count things and in mathematical operations are called numbers.

Example:

$0, 1, 2, \pi, \sqrt{2}, Log 3$ are examples of number's.

Rational number's: (Q)

Numbers that can be written as quotient of integer i-e in the form of P/q, $(q \neq 0)$

or

Number's whose decimal form or representation is terminating or recuring are called rational number's.

Example:

1) $\frac{1}{9} = 0.1111----$ Recuring form
2) $\frac{1}{2} = 0.5$ Terminating form
3) $\frac{22}{7} = 3.142857142857---$ Recuring form

Irrational number's: (Q')

Numbers that cannot be written as quotient of integer i-e in form of P/q.

or Numbers whose decimal form or representation is non-terminating and non-recuring.

Example:

1) π, $\sqrt{2}$, $\log 3$ are irrational numbers.

2) If $n>0$ and n is not a perfect square then $\sqrt{n}$ is always irrational number.

Real numbers:

The union of rational numbers and irrational numbers is called real numbers. $\Rightarrow [\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}']$

or The numbers which can be written as decimal are called real numbers.

Examples:

1) $-\frac{3}{4} = -0.750$

2) $1/3 = 0.333----$

3) $\sqrt{2} = 1.41423$

Result: Some real numbers may have two representations.

Proof:- Real number with infinite decimal $0.999---$, also represents the real number 1.

Let $x = 0.999\ldots$

$10x = 9.999\ldots$

$10x = 9 + 0.999\ldots$

$10x = 9 + x$

$10x - x = 9$

$9x = 9$

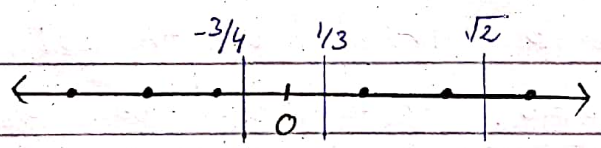
$x = 1$

Real Line:

A number line which is used to represent the real number geometrically.

Example:

Real number's $-\frac{3}{4}$, $\frac{1}{3}$ and $\sqrt{2}$ are geometrically represented as

Real Line 

Interval:

A subset of real line is called an interval if it contains at least two Points/ real number's and all real no's lying b/w them.

Example:

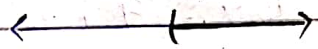
$[1, 2]$, $(1, 2)$, $[1, 2)$, $(1, 2]$, $(0, \infty)$ and $[2, \infty)$

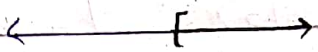
Geometrical Representation:

Geometrically intervals are corresponds

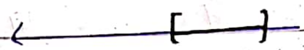
to ray or a line segment.

Example:

i) $(0, \infty)$ 

ii) $[0, \infty)$ 

(iii) $(1, 2)$ 

(iv) $[1, 2]$ 

Length of an interval:

Consider interval whose end points are a and b such that $a<b$ then length of interval is defined by $b-a$.

Example:

i) Length of interval $(2,3) = 3-2 = 1$

ii) Length of interval $[3,4] = 4-3 = 1$

Remember:

i) If $a=b$, then length of $(a,a) = \{\ \}$

ii) If $a=b$ then length of $[a,a] = \{a\}$

Types of Interval

Finite Interval	Infinite Interval
1) Closed	1) closed
2) open	2) open
3) Half open or Half closed	

Finite Interval: An Interval corresponding to a line segment is called a Finite interval.

Example:

$(1,2)$, $[1,2)$, $(1,2]$, $[1,2]$

1) $[1,2]$ 1   2   closed

2) $(1,2)$ 1   2   open

3) $[1,2)$ 1   2   Half open/Half closed

$(1,2]$ 1   2   Half open/Half closed

Infinite Interval: An interval correspond to a ray or real line is called an infinite interval.

Example:

1) $[1,\infty)$ closed

2) $(1,\infty)$ 1   open

Remember: Entire real line is an infinite interval of both types i-e open or closed.

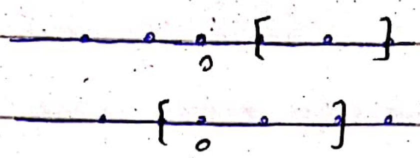
$\mathbb{R}$ $(-\infty,\infty)$

0

$\mathbb{R} = (-\infty,\infty)$ has no end points so it is open infinite interval. Since $\mathbb{R}$ has no end points so we say it vacuously contain all of them so it is closed.

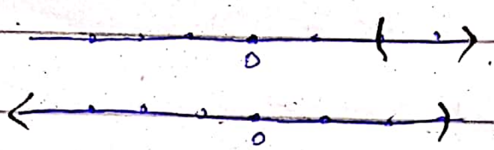
Intersection of Intervals

1) $[1,3]$ and $[-1,2]$



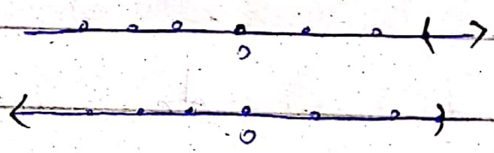
Intersection $[1,2]$

2) $(-\infty,3)$ and $(2,\infty)$



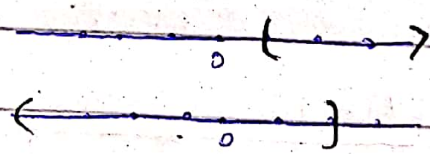
Intersection $(2,3)$

3) $(-\infty,3)$ and $(3,\infty)$



Intersection $= \phi$

4) $(1,\infty)$ and $(-\infty,2]$



Intersection $(1,2]$

Absolute Value of a Real Number

Absolute value of a number $x \in \mathbb{R}$ is denoted by $|x|$ and defined as.

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Example:

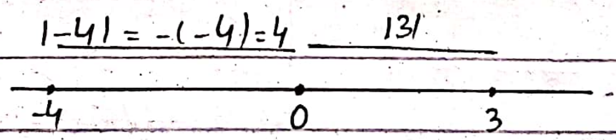
Absolute value of $|5| = 5$

Absolute value of $|-5| = -(-5) = 5$

Absolute value of $|0| = 0$

Geometrically:

Geometrically, absolute value of x is the distance from x to zero on real number line. As distance is always +ve so absolute value of x is always +ve or zero. $|x| \geq 0 \ \forall \ x \in \mathbb{R}$.



Another def of $|x|$: As we know that $\sqrt{x}$ is always denote non-negative square root of x.

So we define $|x|$ as

$$|x| = \sqrt{(x)^2}$$

Remember: $\sqrt{a^2} = |a|$

$$\sqrt{a^2} = a \quad \text{when} \quad a \geq 0$$

Properties of Absolute values:

1) $|x| = |-x|$

Example: $|5| = 5$

$|-5| = -(-5) = 5$

2) $|ab| = |a||b|$

Example:

$a=2$, $b=3$, $ab=6$	$a=2$, $b=-3$, $=-6$
$\lvert ab\rvert = \lvert 6\rvert = 6$	$\lvert ab\rvert = \lvert -6\rvert = -(-6) = 6$
$\lvert a\rvert = \lvert 2\rvert = 2$	$\lvert a\rvert = \lvert 2\rvert = 2$
$\lvert b\rvert = \lvert 3\rvert = 3$	$\lvert b\rvert = \lvert -3\rvert = -(-3) = 3$
$\lvert a\rvert\lvert b\rvert = 2.3 = 6$	$\lvert a\rvert\lvert b\rvert = 2.3 = 6$

3) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

Example:

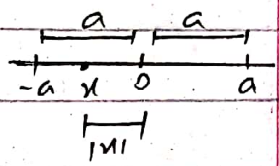
$a=6$, $b=3$, $\frac{a}{b}=2$	$a=6$, $b=-3$, $\frac{a}{b}=-2$
$\left\lvert\frac{a}{b}\right\rvert = \lvert 2\rvert = 2$	$\left\lvert\frac{a}{b}\right\rvert = \lvert -2\rvert = -(-2) = 2$
$\lvert a\rvert = 6$, $\lvert b\rvert = 3$	$\lvert a\rvert = 6$, $\lvert b\rvert = \lvert -3\rvert = -(-3) = 3$
$\frac{\lvert a\rvert}{\lvert b\rvert} = \frac{6}{3} = 2$	$\frac{\lvert a\rvert}{\lvert b\rvert} = \frac{6}{3} = 2$

4) $|a+b| \leq |a| + |b|$

Example:

$a=2$, $b=3$; $a+b=5$	$a=-2$, $b=3$, $a+b=-2+3=1$
$\lvert a+b\rvert = \lvert 5\rvert = 5$	$\lvert a+b\rvert = \lvert 1\rvert = 1$
$\lvert a\rvert = \lvert 2\rvert = 2$	$\lvert a\rvert = \lvert -2\rvert = -(-2) = 2$
$\lvert b\rvert = \lvert 3\rvert = 3$	$\lvert b\rvert = \lvert 3\rvert = 3$
$\lvert a\rvert + \lvert b\rvert = 2+3 = 5$	$\lvert a\rvert + \lvert b\rvert = 2+3 = 5$
$\lvert a+b\rvert = \lvert a\rvert + \lvert b\rvert$	$\lvert a+b\rvert < \lvert a\rvert + \lvert b\rvert$

(5) $|x| \leq a$ For $a \geq 0$ $\iff -a \leq x \leq a$



Absolute Values & Intervals

Let a is any +ve integer.

$|x| = a$ iff $x = \pm a$

$|x| < a$ iff $-a < x < a$

$|x| \leq a$ iff $-a \leq x \leq a$

$|x| > a$ iff $x > a$ or $x < -a$

$|x| \geq a$ iff $x \geq a$ or $x \leq -a$

Example: Solve $|2x - 3| = 7$

$$2x - 3 = \pm 7$$

$2x - 3 = 7$	$2x - 3 = -7$
$2x = 7 + 3$	$2x = -7 + 3$
$2x = 10$	$2x = -4$
$x = 5$	$x = -2$

$$S.S = \{-2, 5\}$$

Example: Solve $\left|5 - \frac{2}{x}\right| < 1$

$$-1 < 5 - \frac{2}{x} < 1$$

$$-5 - 1 < -2/x < 1 - 5$$

$$-6 < -2/x < -4$$

$$-3 < -1/x < -2$$

$$3 > 1/x > 2$$

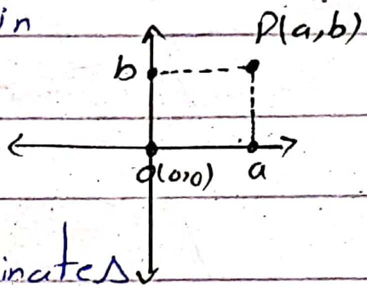
$$\frac{1}{3} < x < \frac{1}{2}$$

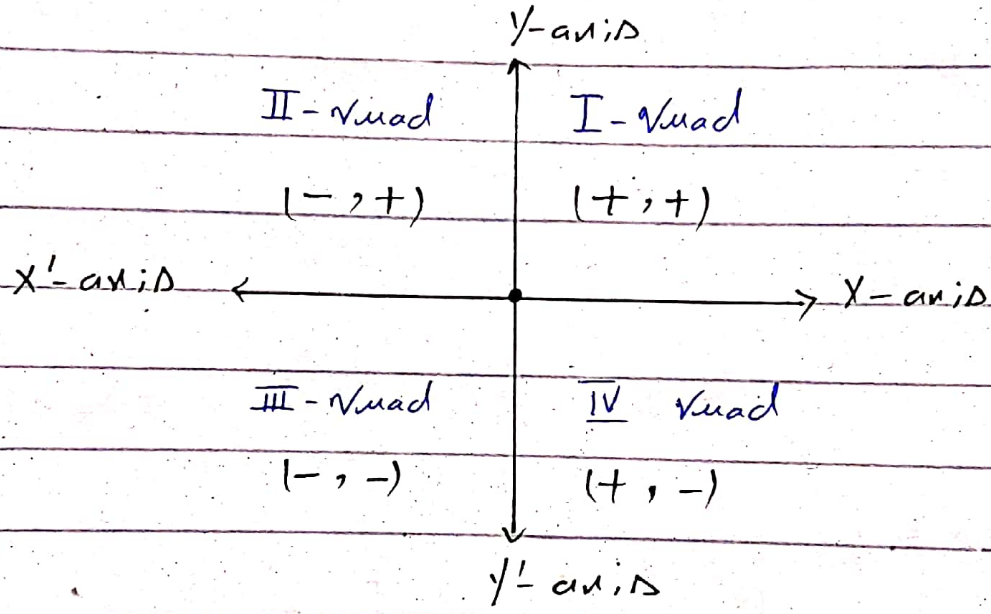
$$S.S = \left\{\frac{1}{3}, \frac{1}{2}\right\}$$

Coordinates

Cartesian Coordinates:-

If P is any point in the plane, we can draw lines through P perp. to two coordinates of P. also called coordinate axis.





Origin: The coordinate system is the point in the plane where x and y are both zero.

Quadvent: Coordinates axis divide into four regions is called quadvent.

Coordinate Pair: The ordered pair (a,b) is assigned to the point P.

Increment's and Distance

Increment: When Particles moves From one Point to another Point, the net change is called increments.

$$\Delta x = x_2 - x_1$$

Example:

Find the Coordinate increment From $A(\overset{x_1}{7}, \overset{y_1}{2})$ to $B(\overset{x_2}{5}, \overset{y_2}{3})$.

Here

$\Delta x = x_2 - x_1$
$= 5 - 7$
$= -2$

$\Delta y = y_2 - y_1$
$= 3 - 2$
$= 1$

Distance:

Distance Formula for Point in the Plane $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Find the distance b/w $P(\overset{x_1}{-1}, \overset{y_1}{2})$ and $Q(\overset{x_2}{3}, \overset{y_2}{4})$

$$= \sqrt{(3-(-1))^2 + (4-2)^2}$$

$$= \sqrt{(4)^2 + (2)^2}$$

$$= \sqrt{20} = 2\sqrt{5}$$

Graph:-

Involving the variable x and y is the set of all Points $p(x, y)$ whose coordinate satisfy the equation and Inequality.

Circle centered at origion then

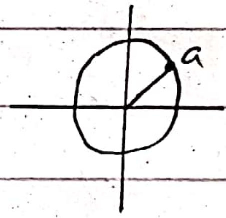
a) If $a>0$

then equation

$$x^2+y^2=a^2$$

$$\sqrt{x^2+y^2}=\sqrt{a^2}=a$$

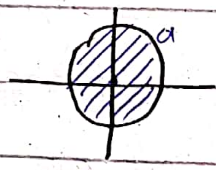
radius $= a$



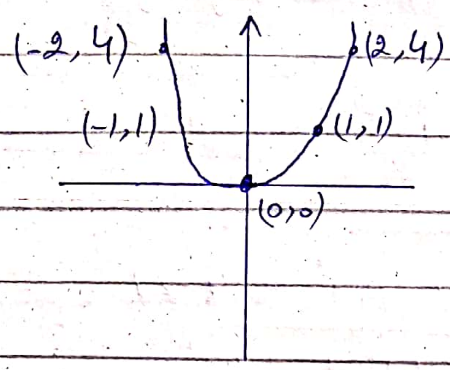
b) $x^2+y^2 \le a^2$

$$\sqrt{x^2+y^2} \le \sqrt{a^2} \le a$$

rad $= a$



Example: $y^4 = x^2$ $y = \pm x^2$



Functions:

If value of one variable quantity, which we might call variable y, depend on the value of other variable x then, we say, y is a Function of x.

Input values is called Domain of function and Output values are called Range of Function.

i-e

$$y = f(x)$$

$y \downarrow$ dependent variable (Range)

$x \downarrow$ Independent variable (domain)

Example:

$F(t) = 2(t-1)+3$

Evaluate F at I/P 0, 2, $x+2$ and $F(2)$

$F(0) = 2(0-1)+3 = -2+3 = 1$

$F(x+2) = 2(x+2-1)+3$

$= 2x+4-2+3$

$= 2x+5$

Example:

	Function	Domain (x)	Range (y)
$y=$	$\sqrt{1-x^2}$	$[-1,1]$	$[0,1]$
$y=$	$1/x$	$[-\infty,0)\cup(0,\infty]$ $\mathbb{R}-0$	$\mathbb{R}-0$

Graphs of Functions:

$y^2 = x^2$ interval $[-2, 2]$

x	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4