CHAPTER 20

Index Numbers

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STATISTICS *in* PRACTICE

U.S. DEPARTMENT OF LABOR, BUREAU OF LABOR STATISTICS

WASHINGTON, D.C.

The U.S. Department of Labor, through its Bureau of Labor Statistics, compiles and distributes indexes and statistics that are indicators of business and economic activity in the United States. For instance, the Bureau compiles and publishes the Consumer Price Index, the Producer Price Index, and statistics on average hours and earnings of various groups of workers. Perhaps the most widely quoted index produced by the Bureau of Labor Statistics is the Consumer Price Index. It is often used as a measure of inflation.

In March 2009, the Bureau of Labor Statistics reported that the Consumer Price Index (CPI) increased by .5% in February. The February level of 212.2 was .3% higher than in February 2008. On a seasonally adjusted basis, the CPI increased .4% in February after rising .3% in January. The 8.3% increase in the gasoline price index seemed to cause most of the increase. The food index actually declined .1%. Some economists considered the CPI increase good news because it reduced the likelihood of a deflationary period.

The Bureau of Labor Statistics, one day earlier, had reported that the Producer Price Index (PPI) increased by .1% in February, seasonally adjusted. The increase followed a .8% increase in January and a 1.9% decline in December. The PPI measures price changes in wholesale markets and is often seen as a leading indicator of changes in the Consumer Price Index. The slower rate of increase in February was heavily influenced by the declining rate of increase in energy goods. The energy goods index rose by 1.3% in February after rising by 3.7% in Janurary.

Gasoline prices are a component of the Consumer Price Index. © Randy Green/Alamy Limited.

In this chapter we will see how various indexes, such as the Consumer and Producer Price Indexes, are computed and how they should be interpreted.

Each month the U.S. government publishes a variety of indexes designed to help individuals understand current business and economic conditions. Perhaps the most widely known and cited of these indexes is the Consumer Price Index (CPI). As its name implies, the CPI is an indicator of what is happening to prices consumers pay for items purchased. Specifically, the CPI measures changes in price over a period of time. With a given starting point or *base period* and its associated index of 100, the CPI can be used to compare current period consumer prices with those in the base period. For example, a CPI of 125 reflects the condition that consumer prices as a whole are running approximately 25% above the base period prices for the same items. Although relatively few individuals know exactly what this number means, they do know enough about the CPI to understand that an increase means higher prices.

Even though the CPI is perhaps the best-known index, many other governmental and private-sector indexes are available to help us measure and understand how economic conditions in one period compare with economic conditions in other periods. The purpose of this chapter is to describe the most widely used types of indexes. We will begin by constructing some simple index numbers to gain a better understanding of how indexes are computed.

20.1

Price Relatives

TABLE 20.1

REGULAR GASOLINE (ALL FORMULATIONS) COST

Year	Price per Gallon ($)
1990	1.30
1991	1.10
1992	1.09
1993	1.07
1994	1.08
1995	1.11
1996	1.22
1997	1.20
1998	1.03
1999	1.14
2000	1.48
2001	1.42
2002	1.34
2003	1.56
2004	1.85
2005	2.27
2006	2.57
2007	2.80
2008	3.25
2009	2.35
2010	2.78
2011	3.52

Source: U.S. Energy Information Administration.

The simplest form of a price index shows how the current price per unit for a given item compares to a base period price per unit for the same item. For example, Table 20.1 reports the cost of one gallon of regular gasoline for the years 1990 through 2011. To facilitate comparisons with other years, the actual cost-per-gallon figure can be converted to a **price relative**, which expresses the unit price in each period as a percentage of the unit price in a base period.

$$\text{Price relative in period } t = \frac{\text{Price in period } t}{\text{Base period price}}(100) \qquad \textbf{(20.1)}$$

For the gasoline prices in Table 20.1 and with 1990 as the base year, the price relatives for one gallon of regular gasoline in the years 1990 through 2011 can be calculated. These price relatives are listed in Table 20.2. Note how easily the price in any one year can be compared with the price in the base year by knowing the price relative. For example, the price relative of 85.4 in 1995 shows that the price of gasoline in 1995 was 14.6% below the 1990 base-year price. Similarly, the 2002 price relative of 103.1 shows a 3.1% increase in the gasoline price in 2002 from the 1990 base-year price. And the 2011 price relative of 270.8 shows a 170.8% increase in the price of regular gesoline from the 1990 base-year price. Price relatives, such as the ones for regular gasoline, are extremely helpful in terms of understanding and interpreting changing economic and business conditions over time.

20.2

Aggregate Price Indexes

TABLE 20.2

PRICE RELATIVES FOR ONE GALLON OF REGULAR GASOLINE

Year	Price Relative (Base 1990)
1990	(1.30/1.30)100 = 100.0
1991	(1.10/1.30)100 = 84.6
1992	(1.09/1.30)100 = 83.8
1993	(1.07/1.30)100 = 82.3
1994	(1.08/1.30)100 = 83.1
1995	(1.11/1.30)100 = 85.4
1996	(1.22/1.30)100 = 93.8
1997	(1.20/1.30)100 = 92.3
1998	(1.03/1.30)100 = 79.2
1999	(1.14/1.30)100 = 87.7
2000	(1.48/1.30)100 = 113.8
2001	(1.42/1.30)100 = 109.2
2002	(1.34/1.30)100 = 103.1
2003	(1.56/1.30)100 = 120.0
2004	(1.85/1.30)100 = 142.3
2005	(2.27/1.30)100 = 174.6
2006	(2.57/1.30)100 = 197.7
2007	(2.80/1.30)100 = 215.4
2008	(3.25/1.30)100 = 250.0
2009	(2.35/1.30)100 = 180.8
2010	(2.78/1.30)100 = 213.8
2011	(3.52/1.30)100 = 270.8

Although price relatives can be used to identify price changes over time for individual items, we are often more interested in the general price change for a group of items taken as a whole. For example, if we want an index that measures the change in the overall cost of living over time, we will want the index to be based on the price changes for a variety of items, including food, housing, clothing, transportation, medical care, and so on. An **aggregate price index** is developed for the specific purpose of measuring the combined change of a group of items.

Consider the development of an aggregate price index for a group of items categorized as normal automotive operating expenses. For illustration, we limit the items included in the group to gasoline, oil, tire, and insurance expenses.

Table 20.3 gives the data for the four components of our automotive operating expense index for the years 1990 and 2011. With 1990 as the base period, an aggregate price index for the four components will give us a measure of the change in normal automotive operating expenses over the 1990–2011 period.

An unweighted aggregate index can be developed by simply summing the unit prices in the year of interest (e.g., 2011) and dividing that sum by the sum of the unit prices in the base year (1990). Let

$$P_{it} = \text{unit price for item } i \text{ in period } t$$
$$P_{i0} = \text{unit price for item } i \text{ in the base period}$$

TABLE 20.3 DATA FOR AUTOMOTIVE OPERATING EXPENSE INDEX

Item	Unit Price ($) 1990	2011
Gallon of gasoline	1.30	3.52
Quart of oil	2.10	6.25
Tire	130.00	145.00
Insurance policy	820.00	1040.00

An unweighted aggregate price index in period t, denoted by I_t, is given by

$$I_t = \frac{\Sigma P_{it}}{\Sigma P_{i0}}(100) \qquad \textbf{(20.2)}$$

where the sums are for all items in the group.

An unweighted aggregate index for normal automotive operating expenses in 2011 ($t = 2011$) is given by

$$I_{2011} = \frac{3.52 + 6.25 + 145.00 + 1040.00}{1.30 + 2.10 + 130.00 + 820.00}(100)$$
$$= \frac{1194.77}{953.4}(100) = 125.3$$

From the unweighted aggregate price index, we might conclude that the price of normal automotive operating expenses has only increased 25.3% over the period from 1990 to 2011. But note that the unweighted aggregate approach to establishing a composite price index for automotive expenses is heavily influenced by the items with large per-unit prices. Consequently, items with relatively low unit prices such as gasoline and oil are dominated by the high unit-price items such as tires and insurance. The unweighted aggregate index for automotive operating expenses is too heavily influenced by price changes in tires and insurance.

If quantity of usage is the same for each item, an unweighted index gives the same value as a weighted index. In practice, however, quantities of usage are rarely the same.

Because of the sensitivity of an unweighted index to one or more high-priced items, this form of aggregate index is not widely used. A weighted aggregate price index provides a better comparison when usage quantities differ.

The philosophy behind the **weighted aggregate price index** is that each item in the group should be weighted according to its importance. In most cases, the quantity of usage is the best measure of importance. Hence, one must obtain a measure of the quantity of usage for the various items in the group. Table 20.4 gives annual usage information for each item of automotive operating expense based on the typical operation of a midsize automobile for approximately 15,000 miles per year. The quantity weights listed show the expected annual usage for this type of driving situation.

TABLE 20.4 ANNUAL USAGE INFORMATION FOR AUTOMOTIVE OPERATING EXPENSE INDEX

Item	Quantity Weights*
Gallons of gasoline	1000
Quarts of oil	15
Tires	2
Insurance policy	1

*Based on 15,000 miles per year. Tire usage is based on a 30,000-mile tire life.

Let Q_i = quantity of usage for item i. The weighted aggregate price index in period t is given by

$$I_t = \frac{\Sigma P_{it} Q_i}{\Sigma P_{i0} Q_i}(100) \qquad \textbf{(20.3)}$$

where the sums are for all items in the group. Applied to our automotive operating expenses, the weighted aggregate price index is based on dividing total operating costs in 2011 by total operating costs in 1990.

Let $t = 2011$, and use the quantity weights in Table 20.4. We obtain the following weighted aggregate price index for automotive operating expenses in 2011.

$$I_{2011} = \frac{3.52(1000) + 6.25(15) + 145.00(2) + 1040.00(1)}{1.30(1000) + 2.10(15) + 130.00(2) + 820.00(1)}(100)$$
$$= \frac{4943.75}{2411.5} = 205.0$$

From this weighted aggregate price index, we would conclude that the price of automotive operating expenses has increased 105% over the period from 1990 through 2011.

Clearly, compared with the unweighted aggregate index, the weighted index provides a more accurate indication of the price change for automotive operating expenses over the 1990–2011 period. Taking the quantity of usage of gasoline into account helps to offset the smaller percentage increase in insurance costs. The weighted index shows a larger increase in automotive operating expenses than the unweighted index. In general, the weighted aggregate index with quantities of usage as weights is the preferred method for establishing a price index for a group of items.

In the weighted aggregate price index formula (20.3), note that the quantity term Q_i does not have a second subscript to indicate the time period. The reason is that the quantities Q_i are considered fixed and do not vary with time as the prices do. The fixed weights or quantities are specified by the designer of the index at levels believed to be representative of typical usage. Once established, they are held constant or fixed for all periods of time the index is in use. Indexes for years other than 2011 require the gathering of new price data P_{it}, but the weighting quantities Q_i remain the same.

In a special case of the fixed-weight aggregate index, the quantities are determined from base-year usages. In this case we write $Q_i = Q_{i0}$, with the zero subscript indicating base-year quantity weights; formula (20.3) becomes

$$I_t = \frac{\Sigma P_{it} Q_{i0}}{\Sigma P_{i0} Q_{i0}} (100) \qquad \textbf{(20.4)}$$

Whenever the fixed quantity weights are determined from base-year usage, the weighted aggregate index is given the name **Laspeyres index**.

Another option for determining quantity weights is to revise the quantities each period. A quantity Q_{it} is determined for each year that the index is computed. The weighted aggregate index in period t with these quantity weights is given by

$$I_t = \frac{\Sigma P_{it} Q_{it}}{\Sigma P_{i0} Q_{it}} (100) \qquad \textbf{(20.5)}$$

Note that the same quantity weights are used for the base period (period 0) and for period t. However, the weights are based on usage in period t, not the base period. This weighted aggregate index is known as the **Paasche index**. It has the advantage of being based on current usage patterns. However, this method of computing a weighted aggregate index presents two disadvantages: The normal usage quantities Q_{it} must be redetermined each year, thus adding to the time and cost of data collection, and each year the index numbers for previous years must be recomputed to reflect the effect of the new quantity weights. Because of these disadvantages, the Laspeyres index is more widely used. The automotive operating expense index was computed with base-period quantities; hence, it is a Laspeyres index. Had usage figures for 2011 been used, it would be a Paasche index. Indeed, because of more fuel efficient cars, gasoline usage decreased and a Paasche index differs from a Laspeyres index.

Exercises

Methods

SELF test

1. The following table reports prices and usage quantities for two items in 2009 and 2011.

	Quantity		Unit Price ($)	
Item	2009	2011	2009	2011
A	1500	1800	7.50	7.75
B	2	1	630.00	1500.00

a. Compute price relatives for each item in 2011 using 2009 as the base period.
b. Compute an unweighted aggregate price index for the two items in 2011 using 2009 as the base period.
c. Compute a weighted aggregate price index for the two items using the Laspeyres method.
d. Compute a weighted aggregate price index for the two items using the Paasche method.

2. An item with a price relative of 132 cost $10.75 in 2011. Its base year was 1994.
   a. What was the percentage increase or decrease in cost of the item over the 17-year period?
   b. What did the item cost in 1994?

Applications

SELF test

3. A large manufacturer purchases an identical component from three independent suppliers that differ in unit price and quantity supplied. The relevant data for 2009 and 2011 are given here.

		Unit Price ($)	
Supplier	Quantity (2007)	2009	2011
A	150	5.45	6.00
B	200	5.60	5.95
C	120	5.50	6.20

   a. Compute the price relatives for each of the component suppliers separately. Compare the price increases by the suppliers over the two-year period.
   b. Compute an unweighted aggregate price index for the component part in 2011.
   c. Compute a 2011 weighted aggregate price index for the component part. What is the interpretation of this index for the manufacturing firm?

4. R&B Beverages, Inc., provides a complete line of beer, wine, and soft drink products for distribution through retail outlets in central Iowa. Unit price data for 2008 and 2011 and quantities sold in cases for 2008 follow.

		Unit Price ($)	
Item	2008 Quantity (cases)	2008	2011
Beer	35,000	17.50	20.15
Wine	5,000	100.00	118.00
Soft drink	60,000	8.00	8.80

   Compute a weighted aggregate index for the R&B Beverage sales in 2011, with 2008 as the base period.

5. Under the last-in, first-out (LIFO) inventory valuation method, a price index for inventory must be established for tax purposes. The quantity weights are based on year-ending inventory levels. Use the beginning-of-the-year price per unit as the base-period price and develop a weighted aggregate index for the total inventory value at the end of the year. What type of weighted aggregate price index must be developed for the LIFO inventory valuation?

		Unit Price ($)	
Product	Ending Inventory	Beginning	Ending
A	500	.15	.19
B	50	1.60	1.80
C	100	4.50	4.20
D	40	12.00	13.20

20.3 Computing an Aggregate Price Index from Price Relatives

In Section 20.1 we defined the concept of a price relative and showed how a price relative can be computed with knowledge of the current-period unit price and the base-period unit price. We now want to show how aggregate price indexes like the ones developed in Section 20.2 can be computed directly from information about the price relative of each item in the group. Because of the limited use of unweighted indexes, we restrict our attention to weighted aggregate price indexes. Let us return to the automotive operating expense index of the preceding section. The necessary information for the four items is given in Table 20.5.

One must be sure prices and quantities are in the same units. For example, if prices are per case, quantity must be the number of cases and not, for instance, the number of individual units.

Let w_i be the weight applied to the price relative for item i. The general expression for a weighted average of price relatives is given by

$$I_t = \frac{\sum \frac{P_{it}}{P_{i0}}(100)w_i}{\Sigma w_i} \tag{20.6}$$

The proper choice of weights in equation (20.6) will enable us to compute a weighted aggregate price index from the price relatives. The proper choice of weights is given by multiplying the base-period price by the quantity of usage.

$$w_i = P_{i0}Q_i \tag{20.7}$$

Substituting $w_i = P_{i0}Q_i$ into equation (20.6) provides the following expression for a weighted price relatives index.

$$I_t = \frac{\sum \frac{P_{it}}{P_{i0}}(100)(P_{i0}Q_i)}{\Sigma P_{i0}Q_i} \tag{20.8}$$

With the canceling of the P_{i0} terms in the numerator, an equivalent expression for the weighted price relatives index is

$$I_t = \frac{\Sigma P_{it}Q_i}{\Sigma P_{i0}Q_i}(100)$$

Thus, we see that the weighted price relatives index with $w_i = P_{i0}Q_i$ provides a price index identical to the weighted aggregate index presented in Section 20.2 by equation (20.3). Use

TABLE 20.5 PRICE RELATIVES FOR AUTOMOTIVE OPERATING EXPENSE INDEX

	Unit Price ($)			
Item	1990 (P_0)	2011 (P_t)	Price Relative $(P_t/P_0)100$	Annual Usage
Gallon of gasoline	1.30	3.52	270.8	1000
Quart of oil	2.10	6.25	297.6	15
Tire	130.00	145.00	111.5	2
Insurance policy	820.00	1040.00	126.8	1

TABLE 20.6 AUTOMOTIVE OPERATING EXPENSE INDEX (1990–2011) BASED ON WEIGHTED PRICE RELATIVES

Item	Price Relatives $(P_{it}/P_{i0})(100)$	Base Price ($) P_{i0}	Quantity Q_i	Weight $w_i = P_{i0}Q_i$	Weighted Price Relatives $(P_{it}/P_{i0})(100)w_i$
Gasoline	270.8	1.30	1000	1300.00	352,040.00
Oil	297.6	2.10	15	31.50	9,374.40
Tire	111.5	130.00	2	260.00	28,990.00
Insurance	126.8	820.00	1	820.00	103,976.00
			Totals	2411.50	494,380.40

$$I_{2011} = \frac{494{,}380.40}{2411.50} = 205$$

of base-period quantities (i.e., $Q_i = Q_{i0}$) in equation (20.7) leads to a Laspeyres index. Use of current-period quantities (i.e., $Q_i = Q_{it}$) in equation (20.7) leads to a Paasche index.

Let us return to the automotive operating expense data. We can use the price relatives in Table 20.5 and equation (20.6) to compute a weighted average of price relatives. The results obtained by using the weights specified by equation (20.7) are reported in Table 20.6. The index number 205 represents a 105% increase in automotive operating expenses, which is the same as the increase identified by the weighted aggregate index computation in Section 20.2.

Exercises

Methods

SELF test

6. Price relatives for three items, along with base-period prices and usage are shown in the following table. Compute a weighted aggregate price index for the current period.

		Base Period	
Item	Price Relative	Price	Usage
A	150	22.00	20
B	90	5.00	50
C	120	14.00	40

Applications

SELF test

7. The Mitchell Chemical Company produces a special industrial chemical that is a blend of three chemical ingredients. The beginning-year cost per pound, the ending-year cost per pound, and the blend proportions follow.

	Cost per Pound ($)		Quantity (pounds)
Ingredient	Beginning	Ending	per 100 Pounds of Product
A	2.50	3.95	25
B	8.75	9.90	15
C	.99	.95	60

a. Compute the price relatives for the three ingredients.
b. Compute a weighted average of the price relatives to develop a one-year cost index for raw materials used in the product. What is your interpretation of this index value?

8. An investment portfolio consists of four stocks. The purchase price, current price, and number of shares are reported in the following table.

Stock	Purchase Price/Share ($)	Current Price/Share ($)	Number of Shares
Holiday Trans	15.50	17.00	500
NY Electric	18.50	20.25	200
KY Gas	26.75	26.00	500
PQ Soaps	42.25	45.50	300

Construct a weighted average of price relatives as an index of the performance of the portfolio to date. Interpret this price index.

9. Compute the price relatives for the R&B Beverages products in exercise 4. Use a weighted average of price relatives to show that this method provides the same index as the weighted aggregate method.

20.4 Some Important Price Indexes

We identified the procedures used to compute price indexes for single items or groups of items. Now let us consider some price indexes that are important measures of business and economic conditions. Specifically, we consider the Consumer Price Index, the Producer Price Index, and the Dow Jones averages.

Consumer Price Index

The CPI includes charges for services (e.g., doctor and dentist bills) and all taxes directly associated with the purchase and use of an item.

The **Consumer Price Index (CPI)**, published monthly by the U.S. Bureau of Labor Statistics, is the primary measure of the cost of living in the United States. The group of items used to develop the index consists of a *market basket* of 400 items including food, housing, clothing, transportation, and medical items. The CPI is a weighted aggregate price index with fixed weights.[1] The weight applied to each item in the market basket derives from a usage survey of urban families throughout the United States.

The February 2012 CPI, computed with a 1982–1984 base index of 100, was 227.7. This figure means that the cost of purchasing the market basket of goods and services increased 127.7% since the base period 1982–1984. The 50-year time series of the CPI from 1960–2010 is shown in Figure 20.1. Note how the CPI measure reflects the sharp inflationary behavior of the economy in the late 1970s and early 1980s.

Producer Price Index

The PPI is designed as a measure of price changes for domestic goods; imports are not included.

The **Producer Price Index (PPI)**, also published monthly by the U.S. Bureau of Labor Statistics, measures the monthly changes in prices in primary markets in the United States. The PPI is based on prices for the first transaction of each product in nonretail markets. All

[1]The Bureau of Labor Statistics actually publishes two Consumer Price Indexes: one for all urban consumers (CPI-U) and a revised Consumer Price Index for urban wage earners and clerical workers (CPI-W). The CPI-U is the one most widely quoted, and it is published regularly in *The Wall Street Journal*.