The selection of x_{31} as the entering variable means that we want to ship through this route because it reduces the total shipping cost. What is the most that we can ship through the new route? Observe in Table 5.22 that if route (3, 1) ships θ units (i.e., $x_{31} = \theta$), then the maximum value of θ is determined based on two conditions.

1. Supply limits and demand requirements remain satisfied.
2. Shipments through all routes remain nonnegative.

These two conditions determine the maximum value of θ and the leaving variable in the following manner. First, construct a *closed loop* that starts and ends at the entering variable cell, (3, 1). The loop consists of *connected horizontal* and *vertical* segments only (no diagonals are allowed).[7] Except for the entering variable cell, each corner of the closed loop must coincide with a basic variable. Table 5.23 shows the loop for x_{31}. Exactly one loop exists for a given entering variable.

Next, we assign the amount θ to the entering variable cell (3, 1). For the supply and demand limits to remain satisfied, we must alternate between subtracting and adding the amount θ at the successive *corners* of the loop as shown in Table 5.23 (it is immaterial whether the loop is traced in a clockwise or counterclockwise direction). For $\theta \geq 0$, the new values of the variables then remain nonnegative if

$$x_{11} = 5 - \theta \geq 0$$
$$x_{22} = 5 - \theta \geq 0$$
$$x_{34} = 10 - \theta \geq 0$$

The corresponding maximum value of θ is 5, which occurs when both x_{11} and x_{22} reach zero level. Because only one current basic variable must leave the basic solution, we can choose either x_{11} or x_{22} as the leaving variable. We arbitrarily choose x_{11} to leave the solution.

The selection of x_{31} (= 5) as the entering variable and x_{11} as the leaving variable requires adjusting the values of the basic variables at the corners of the closed loop as Table 5.24 shows. Because each unit shipped through route (3, 1) reduces the shipping cost by \$9 ($= u_3 + v_1 - c_{31}$), the total cost associated with the new schedule is \$9 × 5 = \$45 less than in the previous schedule. Thus, the new cost is \$520 − \$45 = \$475.

TABLE 5.23 Determination of Closed Loop for x_{31}

	$v_1 = 10$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 = 0$	[10] $5 - \theta$ (−)	[2] $10 + \theta$ (+)	[20] −16	[11] 4	15
$u_2 = 5$	[12] 3	[7] $5 - \theta$ (−)	[9] 15	[20] $5 + \theta$ (+)	25
$u_3 = 3$	[4] θ (+) 9	[14] −9	[16] −9	[18] $10 - \theta$ (−)	10
Demand	5	15	15	15	

[7]TORA's tutorial module allows you to determine the cells of the *closed loop* interactively with immediate feedback regarding the validity of your selections. See TORA Moment on page 216.

TABLE 5.24 Iteration 2 Calculations

	$v_1 = 1$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 \equiv 0$	10 −9	2 **15 − Θ** −	20 −16	11 **Θ** + 4	**15**
$u_2 = 5$	12 −6	7 **0 + Θ** +	9 **15**	20 **10 − Θ** −	**25**
$u_3 = 3$	4 **5**	14 −9	16 −9	18 **5**	**10**
Demand	**5**	**15**	**15**	**15**	

TABLE 5.25 Iteration 3 Calculations (Optimal)

	$v_1 = -3$	$v_2 = 2$	$v_3 = 4$	$v_4 = 11$	Supply
$u_1 \equiv 0$	10 −13	2 **5**	20 −16	11 **10**	**15**
$u_2 = 5$	12 −10	7 **10**	9 **15**	20 −4	**25**
$u_3 = 7$	4 5	14 −5	16 −5	18 5	**10**
Demand	**5**	**15**	**15**	**15**	

Given the new basic solution, we repeat the computation of the multipliers u and v, as Table 5.24 shows. The entering variable is x_{14}. The closed loop shows that $x_{14} = 10$ and that the leaving variable is x_{24}.

The new solution, shown in Table 5.25, costs $4 \times 10 = \$40$ less than the preceding one, thus yielding the new cost $\$475 - \$40 = \$435$. The new $u_i + v_j - c_{ij}$ are now negative for all nonbasic x_{ij}. Thus, the solution in Table 5.25 is optimal.

The following table summarizes the optimum solution.

From silo	To mill	Number of truckloads
1	2	5
1	4	10
2	2	10
2	3	15
3	1	5
3	4	5
Optimal cost = $435		

TORA Moment.

From Solve/Modify Menu, select Solve ⇒ Iterations, and choose one of the three methods (northwest corner, least-cost, or Vogel) to start the transportation model iterations. The iterations module offers two useful interactive features:

1. You can set any u or v to zero before generating Iteration 2 (the default is $u_1 = 0$). Observe then that although the values of u_i and v_j change, the evaluation of the nonbasic cells ($= u_i + v_j - c_{ij}$) remains the same. This means that, initially, any u or v can be set to zero (in fact, any value) without affecting the optimality calculations.
2. You can test your understanding of the selection of the *closed loop* by clicking (in any order) the *corner* cells that comprise the path. If your selection is correct, the cell will change color (green for entering variable, red for leaving variable, and gray otherwise).

Solver Moment.

Entering the transportation model into Excel spreadsheet is straightforward. Figure 5.4 provides the Excel Solver template for Example 5.3-1 (file solverEx5.3-1.xls), together with all the formulas and the definition of range names.

In the input section, data include unit cost matrix (cells B4:E6), source names (cells A4:A6), destination names (cells B3:E3), supply (cells F4:F6), and demand (cells B7:E7). In the output section, cells B11:E13 provide the optimal solution in matrix form. The total cost formula is given in target cell A10.

AMPL Moment.

Figure 5.5 provides the AMPL model for the transportation model of Example 5.3-1 (file amplEx5.3-1a.txt). The names used in the model are self-explanatory. Both the constraints and the objective function follow the format of the LP model presented in Example 5.1-1.

The model uses the sets `sNodes` and `dNodes` to conveniently allow the use of the alphanumeric set members `{S1, S2, S3}` and `{D1, D2, D3, D4}` which are entered in the data section. All the input data are then entered in terms of these set members as shown in Figure 5.5.

Although the alphanumeric code for set members is more readable, generating them for large problems may not be convenient. File amplEx5.3-1b shows how the same sets can be defined as `{1..m}` and `{1..n}`, where `m` and `n` represent the number of sources and the number of destinations. By simply assigning numeric values for `m` and `n`, the sets are automatically defined for any size model.

The data of the transportation model can be retrieved from a spreadsheet (file TM.xls) using the AMPL `table` statement. File amplEx3.5-1c.txt provides the details. To study this model, you will need to review the material in Section A.5.5.

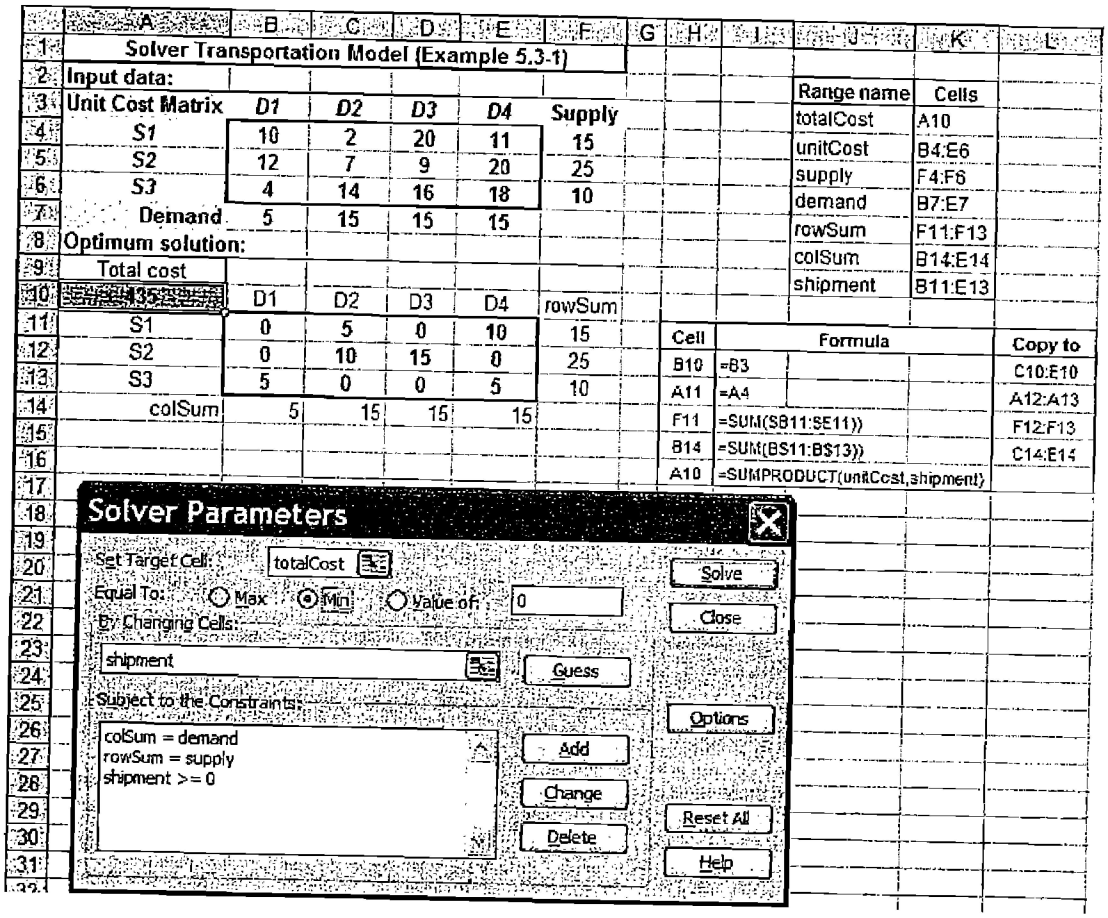

FIGURE 5.4

Excel Solver solution of the transportation model of Example 5.3-1 (File solverEx5.3-1.xls)

PROBLEM SET 5.3B

1. Consider the transportation models in Table 5.26.
   - **(a)** Use the northwest-corner method to find the starting solution.
   - **(b)** Develop the iterations that lead to the optimum solution.
   - **(c)** *TORA Experiment.* Use TORA's Iterations module to compare the effect of using the northwest-corner rule, least-cost method, and Vogel method on the number of iterations leading to the optimum solution.
   - **(d)** *Solver Experiment.* Solve the problem by modifying file solverEx5.3-1.xls.
   - **(e)** *AMPL Experiment.* Solve the problem by modifying file amplEx5.3-1b.txt.
2. In the transportation problem in Table 5.27, the total demand exceeds the total supply. Suppose that the penalty costs per unit of unsatisfied demand are \$5, \$3, and \$2 for destinations 1, 2, and 3, respectively. Use the least-cost starting solution and compute the iterations leading to the optimum solution.

```
#----- Transporation model (Example 5.3-1)-----
set sNodes;
set dNodes;
param c{sNodes,dNodes};
param supply{sNodes};
param demand{dNodes};
var x{sNodes,dNodes}>=0;
minimize z:sum {i in sNodes,j in dNodes}c[i,j]*x[i,j];
subject to
source {i in sNodes}:sum{j in dNodes}x[i,j]=supply[i];
dest {j in dNodes}:sum{i in sNodes}x[i,j]=demand[j];
data;
set sNodes:=S1 S2 S3;
set dNodes:=D1 D2 D3 D4;
param c:
   D1  D2  D3  D4 :=
S1 10  2   20  11
S2 12  7   9   20
S3 4   14  16  18;
param supply:= S1 15 S2 25 S3 10;
param demand:=D1 5 D2 15 D3 15 D4 15;
solve;display z, x;
```

FIGURE 5.5

AMPL model of the transportation model of Example 5.3-1 (File amplEx5.3-1a.txt)

TABLE 5.26 Transportation Models for Problem 1

(i)

\$0	\$2	\$1	**6**
\$2	\$1	\$5	**9**
\$2	\$4	\$3	**5**
5	**5**	**10**	

(ii)

\$10	\$4	\$2	**8**
\$2	\$3	\$4	**5**
\$1	\$2	\$0	**6**
7	**6**	**6**	

(iii)

—	\$3	\$5	**4**
\$7	\$4	\$9	**7**
\$1	\$8	\$6	**19**
5	**6**	**19**	

TABLE 5.27 Data for Problem 2

\$5	\$1	\$7	**10**
\$6	\$4	\$6	**80**
\$3	\$2	\$5	**15**
75	**20**	**50**	

3. In Problem 2, suppose that there are no penalty costs, but that the demand at destination 3 must be satisfied completely.

(a) Find the optimal solution.

(b) *Solver Experiment.* Solve the problem by modifying file solverEx5.3-1.xls.

(c) *AMPL Experiment.* Solve the problem by modifying file amplEx5.3b-1.txt.

TABLE 5.28 Data for Problem 4

\$1	\$2	\$1	**20**
\$3	\$4	\$5	**40**
\$2	\$3	\$3	**30**
30	**20**	**20**	

TABLE 5.29 Data for Problem 6

10			**10**
	20	**20**	**40**
10	**20**	**20**	

4. In the unbalanced transportation problem in Table 5.28, if a unit from a source is not shipped out (to any of the destinations), a storage cost is incurred at the rate of \$5, \$4, and \$3 per unit for sources 1, 2, and 3, respectively. Additionally, all the supply at source 2 must be shipped out completely to make room for a new product. Use Vogel's starting solution and determine all the iterations leading to the optimum shipping schedule.

***5.** In a 3×3 transportation problem, let x_{ij} be the amount shipped from source i to destination j and let c_{ij} be the corresponding transportation cost per unit. The amounts of supply at sources 1, 2, and 3 are 15, 30, and 85 units, respectively, and the demands at destinations 1, 2, and 3 are 20, 30, and 80 units, respectively. Assume that the starting northwest-corner solution is optimal and that the associated values of the multipliers are given as $u_1 = -2$, $u_2 = 3$, $u_3 = 5$, $v_1 = 2$, $v_2 = 5$, and $v_3 = 10$.

(a) Find the associated optimal cost.

(b) Determine the smallest value of c_{ij} for each nonbasic variable that will maintain the optimality of the northwest-corner solution.

6. The transportation problem in Table 5.29 gives the indicated *degenerate* basic solution (i.e., at least one of the basic variables is zero). Suppose that the multipliers associated with this solution are $u_1 = 1$, $u_2 = -1$, $v_1 = 2$, $v_2 = 2$, and $v_3 = 5$ and that the unit cost for all (basic and nonbasic) *zero* x_{ij} variables is given by

$$c_{ij} = i + j\theta, -\infty < \theta < \infty$$

(a) If the given solution is optimal, determine the associated optimal value of the objective function.

(b) Determine the value of θ that will guarantee the optimality of the given solution. (*Hint:* Locate the zero basic variable.)

7. Consider the problem

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

TABLE 5.30 Data for Problem 7

\$1	\$1	\$2	5
\$6	\$5	\$1	6
2	7	1	

subject to

$$\sum_{j=1}^{n} x_{ij} \geq a_i, i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0, \text{all } i \text{ and } j$$

It may appear logical to assume that the optimum solution will require the first (second) set of inequalities to be replaced with equations if $\sum a_i \geq \sum b_j$ ($\sum a_i \leq \sum b_j$). The counterexample in Table 5.30 shows that this assumption is not correct.

Show that the application of the suggested procedure yields the solution $x_{11} = 2$, $x_{12} = 3$, $x_{22} = 4$, and $x_{23} = 2$, with $z = \$27$, which is worse than the feasible solution $x_{11} = 2$, $x_{12} = 7$, and $x_{23} = 6$, with $z = \$15$.

5.3.3 Simplex Method Explanation of the Method of Multipliers

The relationship between the method of multipliers and the simplex method can be explained based on the primal-dual relationships (Section 4.2). From the special structure of the LP representing the transportation model (see Example 5.1-1 for an illustration), the associated dual problem can be written as

$$\text{Maximize } z = \sum_{i=1}^{m} a_i u_i + \sum_{j=1}^{n} b_j v_j$$

subject to

$$u_i + v_j \leq c_{ij}, \text{all } i \text{ and } j$$

$$u_i \text{ and } v_j \text{ unrestricted}$$

where

a_i = Supply amount at source i

b_j = Demand amount at destination j

c_{ij} = Unit transportation cost from source i to destination j

u_i = Dual variable of the constraint associated with source i

v_j = Dual variable of the constraint associated with destination j

5.4

From Formula 2, Section 4.2.4, the objective-function coefficients (reduced costs) of the variable x_{ij} equal the difference between the left- and right-hand sides of the corresponding dual constraint—that is, $u_i + v_j - c_{ij}$. However, we know that this quantity must equal zero for each *basic variable*, which then produces the following result:

$$u_i + v_j = c_{ij}, \text{ for each } \textit{basic} \text{ variable } x_{ij}.$$

There are $m + n - 1$ such equations whose solution (after assuming an arbitrary value $u_1 = 0$) yields the multipliers u_i and v_j. Once these multipliers are computed, the entering variable is determined from all the *nonbasic* variables as the one having the largest positive $u_i + v_j - c_{ij}$.

The assignment of an arbitrary value to one of the dual variables (i.e., $u_1 = 0$) may appear inconsistent with the way the dual variables are computed using Method 2 in Section 4.2.3. Namely, for a given basic solution (and, hence, inverse), the dual values must be unique. Problem 2, Set 5.3c, addresses this point.

PROBLEM SET 5.3C

1. Write the dual problem for the LP of the transportation problem in Example 5.3-5 (Table 5.21). Compute the associated optimum *dual* objective value using the optimal dual values given in Table 5.25, and show that it equals the optimal cost given in the example.
2. In the transportation model, one of the dual variables assumes an arbitrary value. This means that for the same basic solution, the values of the associated dual variables are not unique. The result appears to contradict the theory of linear programming, where the dual values are determined as the product of the vector of the objective coefficients for the basic variables and the associated inverse basic matrix (see Method 2, Section 4.2.3). Show that for the transportation model, although the inverse basis is unique, the vector of *basic* objective coefficients need not be so. Specifically, show that if c_{ij} is changed to $c_{ij} + k$ for all i and j, where k is a constant, then the optimal values of x_{ij} will remain the same. Hence, the use of an arbitrary value for a dual variable is implicitly equivalent to assuming that a specific constant k is added to all c_{ij}.

5.4 THE ASSIGNMENT MODEL

"The best person for the job" is an apt description of the assignment model. The situation can be illustrated by the assignment of workers with varying degrees of skill to jobs. A job that happens to match a worker's skill costs less than one in which the operator is not as skillful. The objective of the model is to determine the minimum-cost assignment of workers to jobs.

The general assignment model with n workers and n jobs is represented in Table 5.31.

The element c_{ij} represents the cost of assigning worker i to job j $(i, j = 1, 2, \ldots, n)$. There is no loss of generality in assuming that the number of workers always

TABLE 5.31 Assignment Model

		Jobs			
	1	2	...	n	
Worker 1	c_{11}	c_{12}	...	c_{1n}	1
2	c_{21}	c_{22}	...	c_{2n}	1
⋮	⋮	⋮	⋮	⋮	⋮
n	c_{n1}	c_{n2}	...	c_{nn}	1
	1	1	...	1	

equals the number of jobs, because we can always add fictitious workers or fictitious jobs to satisfy this assumption.

The assignment model is actually a special case of the transportation model in which the workers represent the sources, and the jobs represent the destinations. The supply (demand) amount at each source (destination) exactly equals 1. The cost of "transporting" worker i to job j is c_{ij}. In effect, the assignment model can be solved directly as a regular transportation model. Nevertheless, the fact that all the supply and demand amounts equal 1 has led to the development of a simple solution algorithm called the **Hungarian method.** Although the new solution method appears totally unrelated to the transportation model, the algorithm is actually rooted in the simplex method, just as the transportation model is.

5.4.1 The Hungarian Method[8]

We will use two examples to present the mechanics of the new algorithm. The next section provides a simplex-based explanation of the procedure.

Example 5.4-1

Joe Klyne's three children, John, Karen, and Terri, want to earn some money to take care of personal expenses during a school trip to the local zoo. Mr. Klyne has chosen three chores for his children: mowing the lawn, painting the garage door, and washing the family cars. To avoid anticipated sibling competition, he asks them to submit (secret) bids for what they feel is fair pay for each of the three chores. The understanding is that all three children will abide by their father's decision as to who gets which chore. Table 5.32 summarizes the bids received. Based on this information, how should Mr. Klyne assign the chores?

The assignment problem will be solved by the Hungarian method.

Step 1. For the original cost matrix, identify each row's minimum, and subtract it from all the entries of the row.

[8]As with the transportation model, the classical Hungarian method, designed primarily for *hand* computations, is something of the past and is presented here purely for historical reasons. Today, the need for such computational shortcuts is not warranted as the problem can be solved as a regular LP using highly efficient computer codes.

TABLE 5.32 Klyne's Assignment Problem

	Mow	Paint	Wash
John	\$15	\$10	\$9
Karen	\$9	\$15	\$10
Terri	\$10	\$12	\$8

Step 2. For the matrix resulting from step 1, identify each column's minimum, and subtract it from all the entries of the column.

Step 3. Identify the optimal solution as the feasible assignment associated with the zero elements of the matrix obtained in step 2.

Let p_i and q_j be the minimum costs associated with row i and column j as defined in steps 1 and 2, respectively. The row minimums of step 1 are computed from the original cost matrix as shown in Table 5.33.

Next, subtract the row minimum from each respective row to obtain the reduced matrix in Table 5.34.

The application of step 2 yields the column minimums in Table 5.34. Subtracting these values from the respective columns, we get the reduced matrix in Table 5.35.

TABLE 5.33 Step 1 of the Hungarian Method

	Mow	Paint	Wash	Row minimum
John	15	10	9	$p_1 = 9$
Karen	9	15	10	$p_2 = 9$
Terri	10	12	8	$p_3 = 8$

TABLE 5.34 Step 2 of the Hungarian Method

	Mow	Paint	Wash
John	6	1	0
Karen	0	6	1
Terri	2	4	0
Column minimum	$q_1 = 0$	$q_2 = 1$	$q_3 = 0$

TABLE 5.35 Step 3 of the Hungarian Method

	Mow	Paint	Wash
John	6	<u>0</u>	0
Karen	<u>0</u>	5	1
Terri	2	3	<u>0</u>

The cells with underscored zero entries provide the optimum solution. This means that John gets to paint the garage door, Karen gets to mow the lawn, and Terri gets to wash the family cars. The total cost to Mr. Klyne is $9 + 10 + 8 = \$27$. This amount also will always equal $(p_1 + p_2 + p_3) + (q_1 + q_2 + q_3) = (9 + 9 + 8) + (0 + 1 + 0) = \$27$. (A justification of this result is given in the next section.)

The given steps of the Hungarian method work well in the preceding example because the zero entries in the final matrix happen to produce a *feasible* assignment (in the sense that each child is assigned a distinct chore). In some cases, the zeros created by steps 1 and 2 may not yield a feasible solution directly, and further steps are needed to find the optimal (feasible) assignment. The following example demonstrates this situation.

Example 5.4-2

Suppose that the situation discussed in Example 5.4-1 is extended to four children and four chores. Table 5.36 summarizes the cost elements of the problem.

The application of steps 1 and 2 to the matrix in Table 5.36 (using $p_1 = 1, p_2 = 7, p_3 = 4, p_4 = 5, q_1 = 0, q_2 = 0, q_3 = 3$, and $q_4 = 0$) yields the reduced matrix in Table 5.37 (verify!).

The locations of the zero entries do not allow assigning unique chores to all the children. For example, if we assign child 1 to chore 1, then column 1 will be eliminated, and child 3 will not have a zero entry in the remaining three columns. This obstacle can be accounted for by adding the following step to the procedure outlined in Example 5.4-1:

Step 2a. If no feasible assignment (with all zero entries) can be secured from steps 1 and 2,

(i) Draw the *minimum* number of horizontal and vertical lines in the last reduced matrix that will cover *all* the zero entries.

TABLE 5.36 Assignment Model

		Chore 1	2	3	4
Child	1	\$1	\$4	\$6	\$3
	2	\$9	\$7	\$10	\$9
	3	\$4	\$5	\$11	\$7
	4	\$8	\$7	\$8	\$5

TABLE 5.37 Reduced Assignment Matrix

		Chore 1	2	3	4
Child	1	**0**	3	2	2
	2	2	**0**	**0**	2
	3	**0**	1	4	3
	4	3	2	**0**	**0**

TABLE 5.38 Application of Step 2a

		Chore 1	2	3	4
Child	1	0	3	2	2
	2	2	0	0	2
	3	0	*1*	4	3
	4	3	2	0	0

TABLE 5.39 Optimal Assignment

		Chore 1	2	3	4
Child	1	<u>0</u>	2	1	1
	2	3	0	<u>0</u>	2
	3	0	<u>0</u>	3	2
	4	4	2	0	<u>0</u>

(ii) Select the *smallest uncovered* entry, subtract it from every uncovered entry, then add it to every entry at the intersection of two lines.

(iii) If no feasible assignment can be found among the resulting zero entries, repeat step 2a. Otherwise, go to step 3 to determine the optimal assignment.

The application of step 2a to the last matrix produces the shaded cells in Table 5.38. The smallest unshaded entry (shown in italics) equals 1. This entry is added to the bold intersection cells and subtracted from the remaining shaded cells to produce the matrix in Table 5.39.

The optimum solution (shown by the underscored zeros) calls for assigning child 1 to chore 1, child 2 to chore 3, child 3 to chore 2, and child 4 to chore 4. The associated optimal cost is $1 + 10 + 5 + 5 = \$21$. The same cost is also determined by summing the p_i's, the q_j's, and the entry that was subtracted after the shaded cells were determined—that is, $(1 + 7 + 4 + 5) + (0 + 0 + 3 + 0) + (1) = \$21$.

AMPL Moment.

File amplEx5.4-2.txt provides the AMPL model for the assignment model. The model is very similar to that of the transportation model.

PROBLEM SET 5.4A

1. Solve the assignment models in Table 5.40.
   - **(a)** Solve by the Hungarian method.
   - **(b)** *TORA Experiment.* Express the problem as an LP and solve it with TORA.
   - **(c)** *TORA Experiment.* Use TORA to solve the problem as a transportation model.

TABLE 5.40 Data for Problem 1

(i)

$3	$8	$2	$10	$3
$8	$7	$2	$9	$7
$6	$4	$2	$7	$5
$8	$4	$2	$3	$5
$9	$10	$6	$9	$10

(ii)

$3	$9	$2	$3	$7
$6	$1	$5	$6	$6
$9	$4	$7	$10	$3
$2	$5	$4	$2	$1
$9	$6	$2	$4	$5

(d) *Solver Experiment.* Modify Excel file solverEx5.3-1.xls to solve the problem.

(e) *AMPL Experiment.* Modify amplEx5.3-1b.txt to solve the problem.

2. JoShop needs to assign 4 jobs to 4 workers. The cost of performing a job is a function of the skills of the workers. Table 5.41 summarizes the cost of the assignments. Worker 1 cannot do job 3 and worker 3 cannot do job 4. Determine the optimal assignment using the Hungarian method.

3. In the JoShop model of Problem 2, suppose that an additional (fifth) worker becomes available for performing the four jobs at the respective costs of $60, $45, $30, and $80. Is it economical to replace one of the current four workers with the new one?

4. In the model of Problem 2, suppose that JoShop has just received a fifth job and that the respective costs of performing it by the four current workers are $20, $10, $20, and $80. Should the new job take priority over any of the four jobs JoShop already has?

***5.** A business executive must make the four round trips listed in Table 5.42 between the head office in Dallas and a branch office in Atlanta.

The price of a round-trip ticket from Dallas is $400. A discount of 25% is granted if the dates of arrival and departure of a ticket span a weekend (Saturday and Sunday). If the stay in Atlanta lasts more than 21 days, the discount is increased to 30%. A one-way

TABLE 5.41 Data for Problem 2

		Job 1	Job 2	Job 3	Job 4
Worker	1	$50	$50	—	$20
	2	$70	$40	$20	$30
	3	$90	$30	$50	—
	4	$70	$20	$60	$70

TABLE 5.42 Data for Problem 5

Departure date from Dallas	Return date to Dallas
Monday, June 3	Friday, June 7
Monday, June 10	Wednesday, June 12
Monday, June 17	Friday, June 21
Tuesday, June 25	Friday, June 28