demand limits represented by the third and fourth constraints. The model determines the amounts (in tons/day) of interior and exterior paints that maximize the daily revenue (expressed in thousands of dollars).

The optimal dual solution shows that the dual price (worth per unit) of raw material $M1$ (resource 1) is $y_1 = .75$ (or \$750 per ton), and that of raw material $M2$ (resource 2) is $y_2 = .5$ (or \$500 per ton). These results hold true for specific *feasibility ranges* as we showed in Section 3.6. For resources 3 and 4, representing the market and demand limits, the dual prices are both zero, which indicates that their associated resources are abundant. Hence, their worth per unit is zero.

PROBLEM SET 4.3A

1. In Example 4.3-1, compute the change in the optimal revenue in each of the following cases (use TORA output to obtain the *feasibility ranges*):

(a) The constraint for raw material $M1$ (resource 1) is $6x_1 + 4x_2 \leq 22$.

(b) The constraint for raw material $M2$ (resource 2) is $x_1 + 2x_2 \leq 4.5$.

(c) The market condition represented by resource 4 is $x_2 \leq 10$.

***2.** NWAC Electronics manufactures four types of simple cables for a defense contractor. Each cable must go through four sequential operations: splicing, soldering, sleeving, and inspection. The following table gives the pertinent data of the situation.

	Minutes per unit				
Cable	*Splicing*	*Soldering*	*Sleeving*	*Inspection*	Unit revenue (\$)
SC320	10.5	20.4	3.2	5.0	9.40
SC325	9.3	24.6	2.5	5.0	10.80
SC340	11.6	17.7	3.6	5.0	8.75
SC370	8.2	26.5	5.5	5.0	7.80
Daily capacity (minutes)	4800.0	9600.0	4700.0	4500.0	

The contractor guarantees a minimum production level of 100 units for each of the four cables.

(a) Formulate the problem as a linear programming model, and determine the optimum production schedule.

(b) Based on the dual prices, do you recommend making increases in the daily capacities of any of the four operations? Explain.

(c) Does the minimum production requirements for the four cables represent an advantage or a disadvantage for NWAC Electronics? Provide an explanation based on the dual prices.

(d) Can the present unit contribution to revenue as specified by the dual price be guaranteed if we increase the capacity of soldering by 10%?

3. BagCo produces leather jackets and handbags. A jacket requires 8 m^2 of leather, and a handbag only 2 m^2. The labor requirements for the two products are 12 and 5 hours, respectively. The current weekly supplies of leather and labor are limited to 1200 m^2 and 1850 hours. The company sells the jackets and handbags at \$350 and \$120, respectively. The objective is to determine the production schedule that maximizes the net revenue. BagCo is considering an expansion of production. What is the maximum purchase price the company should pay for additional leather? For additional labor?

4.3.2 Economic Interpretation of Dual Constraints

The dual constraints can be interpreted by using Formula 2 in Section 4.2.4, which states that at any primal iteration,

$$\text{Objective coefficient of } x_j = \begin{pmatrix}\text{Left-hand side of}\\ \text{dual constraint } j\end{pmatrix} - \begin{pmatrix}\text{Right-hand side of}\\ \text{dual constraint } j\end{pmatrix}$$

$$= \sum_{i=1}^{m} a_{ij}y_i - c_j$$

We use dimensional analysis once again to interpret this equation. The revenue per unit, c_j, of activity j is in dollars per unit. Hence, for consistency, the quantity $\sum_{i=1}^{m} a_{ij}y_i$ must also be in dollars per unit. Next, because c_j represents revenue, the quantity $\sum_{i=1}^{m} a_{ij}y_i$, which appears in the equation with an opposite sign, must represent cost. Thus we have

$$\$\text{ cost} = \sum_{i=1}^{m} a_{ij}y_i = \sum_{i=1}^{m} \begin{pmatrix}\text{usage of resource } i\\ \text{per unit of activity } j\end{pmatrix} \times \begin{pmatrix}\text{cost per unit}\\ \text{of resource } i\end{pmatrix}$$

The conclusion here is that the dual variable y_i represents the **imputed cost** per unit of resource i, and we can think of the quantity $\sum_{i=1}^{m} a_{ij}y_i$ as the imputed cost of all the resources needed to produce one unit of activity j.

In Section 3.6, we referred to the quantity $\left(\sum_{i=1}^{m} a_{ij}y_i - c_j\right)$ as the **reduced cost** of activity j. The maximization optimality condition of the simplex method says that an increase in the level of an unused (nonbasic) activity j can improve revenue only if its reduced cost is negative. In terms of the preceding interpretation, this condition states that

$$\begin{pmatrix}\text{Imputed cost of}\\ \text{resources used by}\\ \text{one unit of activity } j\end{pmatrix} < \begin{pmatrix}\text{Revenue per unit}\\ \text{of activity } j\end{pmatrix}$$

The maximization optimality condition thus says that it is economically advantageous to increase an activity to a positive level if its unit revenue exceeds its unit imputed cost.

We will use the TOYCO model of Section 3.6 to demonstrate the computation. The details of the model are restated here for convenience.

Example 4.3-2

TOYCO assembles three types of toys: trains, trucks, and cars using three operations. Available assembly times for the three operations are 430, 460, and 420 minutes per day, respectively, and the revenues per toy train, truck, and car are \$3, \$2, and \$5, respectively. The assembly times per train for the three operations are 1, 3, and 1 minutes, respectively. The corresponding times per truck and per car are (2, 0, 4) and (1, 2, 0) minutes (a zero time indicates that the operation is not used).

Letting x_1, x_2, and x_3 represent the daily number of units assembled of trains, trucks and cars, the associated LP model and its dual are given as:

TOYCO primal	TOYCO dual
Maximize $z = 3x_1 + 2x_2 + 5x_3$	Minimize $w = 430y_1 + 460y_2 + 420y_3$
subject to	subject to
$x_1 + 2x_2 + x_3 \leq 430$ (Operation 1)	$y_1 + 3y_2 + y_3 \geq 3$
$3x_1 + 2x_3 \leq 460$ (Operation 2)	$2y_1 + 4y_3 \geq 2$
$x_1 + 4x_2 \leq 420$ (Operation 3)	$y_1 + 2y_2 \geq 5$
$x_1, x_2, x_3 \geq 0$	$y_1, y_2, y_3 \geq 0$
Optimal solution:	Optimal solution:
$x_1 = 0, x_2 = 100, x_3 = 230, z = \$1350$	$y_1 = 1, y_2 = 2, y_3 = 0, w = \$1350$

The optimal primal solution calls for producing no toy trains, 100 toy trucks, and 230 toy cars. Suppose that TOYCO is interested in producing toy trains as well. How can this be achieved? Looking at the problem from the standpoint of the interpretation of the *reduced cost* for x_1, toy trains will become attractive economically only if the imputed cost of the resources used to produce one toy train is strictly less than its unit revenue. TOYCO thus can either increase the unit revenue per unit by raising the unit price, or it can decrease the imputed cost of the used resources ($= y_1 + 3y_2 + y_3$). An increase in unit price may not be possible because of market competition. A decrease in the unit imputed cost is more plausible because it entails making improvements in the assembly operations. Letting r_1, r_2, and r_3 represent the proportions by which the unit times of the three operations are reduced, the problem requires determining r_1, r_2, and r_3 such that the new imputed cost per per toy train is less than its unit revenue—that is,

$$1(1 - r_1)y_1 + 3(1 - r_2)y_2 + 1(1 - r_3)y_3 < 3$$

For the given optimal values of $y_1 = 1$, $y_2 = 2$, and $y_3 = 0$, this inequality reduces to (verify!)

$$r_1 + 6r_2 > 4$$

Thus, any values of r_1 and r_2 between 0 and 1 that satisfy $r_1 + 6r_2 > 4$ should make toy trains profitable. However, this goal may not be achievable because it requires practically impossible reductions in the times of operations 1 and 2. For example, even reductions as high as 50% in these times (that is, $r_1 = r_2 = .5$) fail to satisfy the given condition. Thus, TOYCO should not produce toy trains unless an increase in its unit price is possible.

PROBLEM SET 4.3B

1. In Example 4.3-2, suppose that for toy trains the per-unit time of operation 2 can be reduced from 3 minutes to at most 1.25 minutes. By how much must the per-unit time of operation 1 be reduced to make toy trains just profitable?

*2. In Example 4.3-2, suppose that TOYCO is studying the possibility of introducing a fourth toy: fire trucks. The assembly does not make use of operation 1. Its unit assembly times on operations 2 and 3 are 1 and 3 minutes, respectively. The revenue per unit is $4. Would you advise TOYCO to introduce the new product?

*3. JoShop uses lathes and drill presses to produce four types of machine parts, $PP1$, $PP2$, $PP3$, and $PP4$. The table below summarizes the pertinent data.

	Machining time in minutes per unit of				
Machine	*PP1*	*PP2*	*PP3*	*PP4*	Capacity (minutes)
Lathes	2	5	3	4	5300
Drill presses	3	4	6	4	5300
Unit revenue ($)	3	6	5	4	

For the parts that are not produced by the present optimum solution, determine the rate of deterioration in the optimum revenue per unit increase of each of these products.

4. Consider the optimal solution of JoShop in Problem 3. The company estimates that for each part that is not produced (per the optimum solution), an across-the-board 20% reduction in machining time can be realized through process improvements. Would these improvements make these parts profitable? If not, what is the minimum percentage reduction needed to realize revenueability?

4.4 ADDITIONAL SIMPLEX ALGORITHMS

In the simplex algorithm presented in Chapter 3 the problem starts at a (basic) feasible solution. Successive iterations continue to be feasible until the optimal is reached at the last iteration. The algorithm is sometimes referred to as the **primal simplex** method.

This section presents two additional algorithms: The **dual simplex** and the **generalized simplex**. In the dual simplex, the LP starts at a better than optimal *infeasible* (basic) solution. Successive iterations remain infeasible and (better than) optimal until feasibility is restored at the last iteration. The generalized simplex combines both the primal and dual simplex methods in one algorithm. It deals with problems that start both nonoptimal and infeasible. In this algorithm, successive iterations are associated with basic feasible or infeasible (basic) solutions. At the final iteration, the solution becomes optimal and feasible (assuming that one exists).

All three algorithms, the primal, the dual, and the generalized, are used in the course of post-optimal analysis calculations, as will be shown in Section 4.5.

4.4.1 Dual Simplex Algorithm

The crux of the dual simplex method is to start with a better than optimal and infeasible basic solution. The optimality and feasibility conditions are designed to preserve the optimality of the basic solutions while moving the solution iterations toward feasibility.

Dual feasibility condition. The leaving variable, x_r, is the basic variable having the most negative value (ties are broken arbitrarily). If all the basic variables are nonnegative, the algorithm ends.

Dual optimality condition. Given that x_r is the leaving variable, let $\bar{c}_j$ be the reduced cost of nonbasic variable x_j and α_{rj} the constraint coefficient in the x_r-row and x_j-column of the tableau. The entering variable is the nonbasic variable with $\alpha_{rj} < 0$ that corresponds to

$$\min_{\text{Nonbasic } x_j} \left\{ \left| \frac{\bar{c}_j}{\alpha_{rj}} \right|, \alpha_{rj} < 0 \right\}$$

(Ties are broken arbitrarily.) If $\alpha_{rj} \geq 0$ for all nonbasic x_j, the problem has no feasible solution.

To start the LP optimal and infeasible, two requirements must be met:

1. The objective function must satisfy the optimality condition of the regular simplex method (Chapter 3).

2. All the constraints must be of the type $(\leq)$.

The second condition requires converting any $(\geq)$ to $(\leq)$ simply by multiplying both sides of the inequality $(\geq)$ by -1. If the LP includes $(=)$ constraints, the equation can be replaced by two inequalities. For example,

$$x_1 + x_2 = 1$$

is equivalent to

$$x_1 + x_2 \leq 1, x_1 + x_2 \geq 1$$

or

$$x_1 + x_2 \leq 1, -x_1 - x_2 \leq -1$$

After converting all the constraints to $(\leq)$, the starting solution is infeasible if at least one of the right-hand sides of the inequalities is strictly negative.

Example 4.4-1

$$\text{Minimize } z = 3x_1 + 2x_2 + x_3$$

subject to

$$\begin{aligned} 3x_1 + x_2 + x_3 &\geq 3 \\ -3x_1 + 3x_2 + x_3 &\geq 6 \\ x_1 + x_2 + x_3 &\leq 3 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

In the present example, the first two inequalities are multiplied by -1 to convert them to $(\leq)$ constraints. The starting tableau is thus given as:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	-3	-2	-1	0	0	0	0
x_4	-3	-1	-1	1	0	0	-3
x_5	3	-3	-1	0	1	0	-6
x_6	1	1	1	0	0	1	3

The tableau is optimal because all the reduced costs in the z-row are ≤ 0 $(\bar{c}_1 = -3, \bar{c}_2 = -2, \bar{c}_3 = -1, \bar{c}_4 = 0, \bar{c}_5 = 0, \bar{c}_6 = 0)$. It is also infeasible because at least one of the basic variables is negative $(x_4 = -3, x_5 = -6, x_6 = 3)$.

According to the dual feasibility condition, $x_5\ (= -6)$ is the leaving variable. The next table shows how the dual optimality condition is used to determine the entering variable.

	$j = 1$	$j = 2$	$j = 3$
Nonbasic variable	x_1	x_2	x_3
z-row $(\bar{c}_j)$	-3	-2	-1
x_5-row, α_{4j}	3	-3	-1
Ratio, $\lvert\frac{\bar{c}_j}{\alpha_{5j}}\rvert$, $\alpha_{5j} < 0$	—	$\frac{2}{3}$	1

The ratios show that x_2 is the entering variable. Notice that a nonbasic variable x_j is a candidate for entering the basic solution only if its α_{rj} is strictly negative. This is the reason x_1 is excluded in the table above.

The next tableau is obtained by using the familiar row operations, which give

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	-5	0	$-\frac{1}{3}$	0	$-\frac{2}{3}$	0	4
x_4	-4	0	$-\frac{2}{3}$	1	$-\frac{1}{3}$	0	-1
x_2	-1	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	2
x_6	2	0	$\frac{2}{3}$	0	$\frac{1}{3}$	1	1
Ratio	$\frac{5}{4}$	—	$\frac{1}{2}$	—	2	—	

The preceding tableau shows that x_4 leaves and x_3 enters, thus yielding the following tableau, which is both optimal and feasible:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	-3	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{9}{2}$
x_3	6	0	1	$-\frac{3}{2}$	$\frac{1}{2}$	0	$\frac{3}{2}$
x_2	-3	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{3}{2}$
x_6	-2	0	0	1	0	1	0

Notice how the dual simplex works. In all the iterations, optimality is maintained (all reduced costs are ≤ 0). At the same time, each new iteration moves the solution toward feasibility. At iteration 3, feasibility is restored for the first time and the process ends with the optimal feasible solution given as $x_1 = 0, x_2 = \frac{3}{2}, x_2 = \frac{3}{2}$, and $z = \frac{9}{2}$.

TORA Moment.

TORA provides a tutorial module for the dual simplex method. From the SOLVE/MODIFY menu select Solve ⇒ Algebraic ⇒ Iterations ⇒ Dual Simplex. Remember that you need to convert (=) constraints to inequalities. You do not need to convert (≥) constraints because TORA will do the conversion internally. If the LP does not satisfy the initial requirements of the dual simplex, a message will appear on the screen.

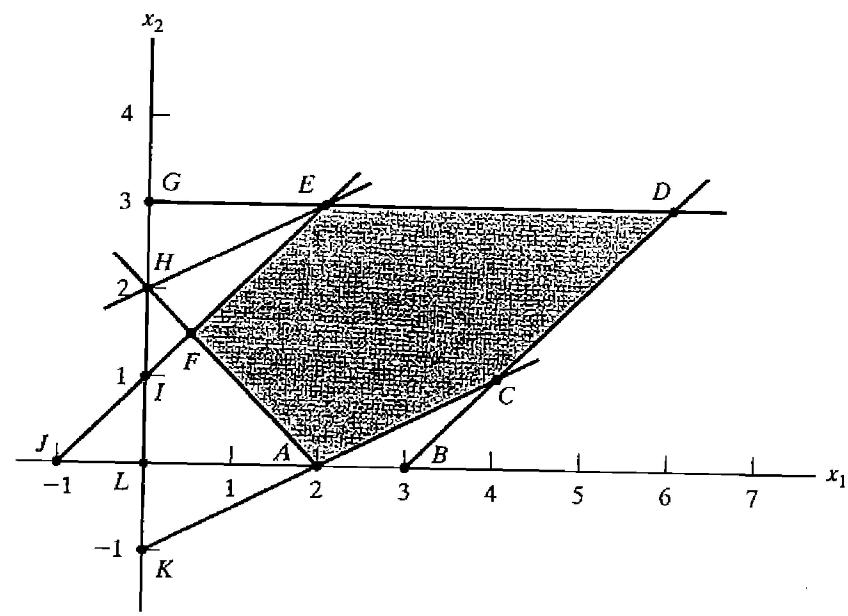
As in the regular simplex method, the tutorial module allows you to select the entering and the leaving variables beforehand. An appropriate feedback then tells you if your selection is correct.

PROBLEM SET 4.4A[2]

1. Consider the solution space in Figure 4.3, where it is desired to find the optimum extreme point that uses the *dual* simplex method to minimize $z = 2x_1 + x_2$. The optimal solution occurs at point $F = (0.5, 1.5)$ on the graph.
   (a) Can the dual simplex start at point A?
   *(b) If the starting basic (infeasible but better than optimum) solution is given by point G, would it be possible for the iterations of the dual simplex method to follow the path $G \rightarrow E \rightarrow F$? Explain.
   (c) If the starting basic (infeasible) solution starts at point L, identify a possible path of the dual simplex method that leads to the optimum feasible point at point F.
2. Generate the dual simplex iterations for the following problems (using TORA for convenience), and trace the path of the algorithm on the graphical solution space.
   (a) Minimize $z = 2x_1 + 3x_2$

FIGURE 4.3

Solution space for Problem 1, Set 4.4a



[2]You are encouraged to use TORA's tutorial mode where possible to avoid the tedious task of carrying out the Gauss-Jordan row operations. In this manner, you can concentrate on understanding the main ideas of the method.

subject to

$$2x_1 + 2x_2 \le 30$$
$$x_1 + 2x_2 \ge 10$$
$$x_1, x_2 \ge 0$$

(b) Minimize $z = 5x_1 + 6x_2$
subject to

$$x_1 + x_2 \ge 2$$
$$4x_1 + x_2 \ge 4$$
$$x_1, x_2 \ge 0$$

(c) Minimize $z = 4x_1 + 2x_2$
subject to

$$x_1 + x_2 = 1$$
$$3x_1 - x_2 \ge 2$$
$$x_1, x_2 \ge 0$$

(d) Minimize $z = 2x_1 + 3x_2$
subject to

$$2x_1 + x_2 \ge 3$$
$$x_1 + x_2 = 2$$
$$x_1, x_2 \ge 0$$

3. *Dual Simplex with Artificial Constraints.* Consider the following problem:

$$\text{Maximize } z = 2x_1 - x_2 + x_3$$

subject to

$$2x_1 + 3x_2 - 5x_3 \ge 4$$
$$-x_1 + 9x_2 - x_3 \ge 3$$
$$4x_1 + 6x_2 + 3x_3 \le 8$$
$$x_1, x_2, x_3 \ge 0$$

The starting basic solution consisting of surplus variables x_4 and x_5 and slack variable x_6 is infeasible because $x_4 = -4$ and $x_5 = -3$. However, the dual simplex is not applicable directly, because x_1 and x_3 do not satisfy the maximization optimality condition. Show that by adding the artificial constraint $x_1 + x_3 \le M$ (where M is sufficiently large not to eliminate any feasible points in the original solution space), and then using the new constraint as a pivot row, the selection of x_1 as the entering variable (because it has the most negative objective coefficient) will render an all-optimal objective row. Next, carry out the regular dual simplex method on the modified problem.

4. Using the artificial constraint procedure introduced in Problem 3, solve the following problems by the dual simplex method. In each case, indicate whether the resulting solution is feasible, infeasible, or unbounded.

(a) Maximize $z = 2x_3$
subject to

$$-x_1 + 2x_2 - 2x_3 \geq 8$$
$$-x_1 + x_2 + x_3 \leq 4$$
$$2x_1 - x_2 + 4x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

(b) Maximize $z = x_1 - 3x_2$
subject to

$$x_1 - x_2 \leq 2$$
$$x_1 + x_2 \geq 4$$
$$2x_1 - 2x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

***(c)** Minimize $z = -x_1 + x_2$
subject to

$$x_1 - 4x_2 \geq 5$$
$$x_1 - 3x_2 \leq 1$$
$$2x_1 - 5x_2 \geq 1$$
$$x_1, x_2 \geq 0$$

(d) Maximize $z = 2x_3$
subject to

$$-x_1 + 3x_2 - 7x_3 \geq 5$$
$$-x_1 + x_2 - x_3 \leq 1$$
$$3x_1 + x_2 - 10x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0$$

5. Solve the following LP in three different ways (use TORA for convenience). Which method appears to be the most efficient computationally?

$$\text{Minimize } z = 6x_1 + 7x_2 + 3x_3 + 5x_4$$

subject to

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$
$$x_2 - 5x_3 - 6x_4 \geq 10$$
$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

4.4.2 Generalized Simplex Algorithm

The (primal) simplex algorithm in Chapter 3 starts feasible but nonoptimal. The dual simplex in Section 4.4.1 starts (better than) optimal but infeasible. What if an LP model starts both nonoptimal and infeasible? We have seen that the primal simplex accounts for the infeasibility of the starting solution by using artificial variables. Similarly, the dual simplex accounts for the nonoptimality by using an artificial constraint (see Problem 3, Set 4.4a). Although these procedures are designed to enhance *automatic* computations, such details may cause one to lose sight of what the simplex algorithm truly entails—namely, the optimum solution of an LP is associated with a corner point (or basic) solution. Based on this observation, you should be able to "tailor" your own simplex algorithm for LP models that start both nonoptimal and infeasible. The following example illustrates what we call the generalized simplex algorithm.

Example 4.4-2

Consider the LP model of Problem 4(a), Set 4.4a. The model can be put in the following tableau form in which the starting basic solution (x_3, x_4, x_5) is both nonoptimal (because x_3 has a negative reduced cost) and infeasible (because $x_4 = -8$). (The first equation has been multiplied by -1 to reveal the infeasibility directly in the *Solution* column.)

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	0	0	-2	0	0	0	0
x_4	1	-2	2	1	0	0	-8
x_5	-1	1	1	0	1	0	4
x_6	2	-1	4	0	0	1	10

We can solve the problem without the use of any artificial variables or artificial constraints as follows: Remove infeasibility first by applying a version of the dual simplex feasibility condition that selects x_4 as the leaving variable. To determine the entering variable, all we need is a nonbasic variable whose constraint coefficient in the x_4-row is strictly negative. The selection can be done without regard to optimality, because it is nonexistent at this point anyway (compare with the dual optimality condition). In the present example, x_2 has a negative coefficient in the x_4-row and is selected as the entering variable. The result is the following tableau:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	0	0	-2	0	0	0	0
x_2	$-\frac{1}{2}$	1	-1	$-\frac{1}{2}$	0	0	4
x_5	$-\frac{1}{2}$	0	2	$\frac{1}{2}$	1	0	0
x_6	$\frac{3}{2}$	0	3	$-\frac{1}{2}$	0	1	14

The solution in the preceding tableau is now feasible but nonoptimal, and we can use the primal simplex to determine the optimal solution. In general, had we not restored feasibility in the preceding tableau, we would repeat the procedure as necessary until feasibility is satisfied or there is evidence that the problem has no feasible solution (which happens if a basic variable is negative and all its constraint coefficients are nonnegative). Once feasibility is established, the next step is to pay attention to optimality by applying the proper optimality condition of the primal simplex method.

Remarks. The essence of Example 4.4-2 is that the simplex method is not rigid. The literature abounds with variations of the simplex method (e.g., the primal-dual method, the symmetrical method, the criss-cross method, and the multiplex method) that give the impression that each procedure is different, when, in effect, they all seek a corner point solution, with a slant toward automated computations and, perhaps, computational efficiency.

PROBLEM SET 4.4B

1. The LP model of Problem 4(c), Set 4.4a, has no feasible solution. Show how this condition is detected by the *generalized simplex procedure.*
2. The LP model of Problem 4(d), Set 4.4a, has no bounded solution. Show how this condition is detected by the *generalized simplex procedure.*

4.5 POST-OPTIMAL ANALYSIS

In Section 3.6, we dealt with the sensitivity of the optimum solution by determining the ranges for the different parameters that would keep the optimum basic solution unchanged. In this section, we deal with making changes in the parameters of the model and finding the new optimum solution. Take, for example, a case in the poultry industry where an LP model is commonly used to determine the optimal feed mix per broiler (see Example 2.2-2). The weekly consumption per broiler varies from .26 lb (120 grams) for a one-week-old bird to 2.1 lb (950 grams) for an eight-week-old bird. Additionally, the cost of the ingredients in the mix may change periodically. These changes require periodic recalculation of the optimum solution. *Post-optimal analysis* determines the new solution in an efficient way. The new computations are rooted in the use duality and the primal-dual relationships given in Section 4.2.

The following table lists the cases that can arise in post-optimal analysis and the actions needed to obtain the new solution (assuming one exists):

Condition after parameters change	Recommended action
Current solution remains optimal and feasible.	No further action is necessary.
Current solution becomes infeasible.	Use dual simplex to recover feasibility.
Current solution becomes nonoptimal.	Use primal simplex to recover optimality.
Current solution becomes both nonoptimal and infeasible.	Use the generalized simplex method to obtain new solution.

The first three cases are investigated in this section. The fourth case, being a combination of cases 2 and 3, is treated in Problem 6, Set 4.5a.

The TOYCO model of Example 4.3-2 will be used to explain the different procedures. Recall that the TOYCO model deals with the assembly of three types of toys: trains, trucks, and cars. Three operations are involved in the assembly. We wish to