Although site 1 property is more expensive (on a per ft^2 basis), the construction cost is less than at site 2, because the infrastructure at site 1 is in a much better shape. Construction cost is $25 million at site 1 and $27 million at site 2. Which site should be selected, and what properties should be acquired?

*3. A city will undertake five urban renewal housing projects over the next five years. Each project has a different starting year and a different duration. The following table provides the basic data of the situation:

	Year 1	Year 2	Year 3	Year 4	Year 5	Cost (million $)	Annual income (million $)
Project 1	Start		End			5.0	.05
Project 2		Start			End	8.0	.07
Project 3	Start				End	15.0	.15
Project 4			Start	End		1.2	.02
Budget (million $)	3.0	6.0	7.0	7.0	7.0		

Projects 1 and 4 must be finished completely within their durations. The remaining two projects can be finished partially within budget limitations, if necessary. However, each project must be at least 25% completed within its duration. At the end of each year, the completed section of a project is immediately occupied by tenants and a proportional amount of income is realized. For example, if 40% of project 1 is completed in year 1 and 60% in year 3, the associated income over the five-year planning horizon is $.4 \times \$50{,}000$ (for year 2) $+ .4 \times \$50{,}000$ (for year 3) $+ (.4 + .6) \times \$50{,}000$ (for year 4) $+ (.4 + .6) \times \$50{,}000$ (for year 5) $= (4 \times .4 + 2 \times .6) \times \$50{,}000$. Determine the optimal schedule for the projects that will maximize the total income over the five-year horizon. For simplicity, disregard the time value of money.

4. The city of Fayetteville is embarking on an urban renewal project that will include lower- and middle-income row housing, upper-income luxury apartments, and public housing. The project also includes a public elementary school and retail facilities. The size of the elementary school (number of classrooms) is proportional to the number of pupils, and the retail space is proportional to the number of housing units. The following table provides the pertinent data of the situation:

	Lower income	Middle income	Upper income	Public housing	School room	Retail unit
Minimum number of units	100	125	75	300		0
Maximum number of units	200	190	260	600		25
Lot size per unit (acre)	.05	.07	.03	.025	.045	.1
Average number of pupils per unit	1.3	1.2	.5	1.4		
Retail demand per unit (acre)	.023	.034	.046	.023	.034	
Annual income per unit($)	7000	12,000	20,000	5000	—	15,000

The new school can occupy a maximum space of 2 acres at the rate of at most 25 pupils per room. The operating annual cost per school room is $10,000. The project will be located on a 50-acre vacant property owned by the city. Additionally, the project can make use of an adjacent property occupied by 200 condemned slum homes. Each condemned home occupies .25 acre. The cost of buying and demolishing a slum unit is $7000. Open space, streets, and parking lots consume 15% of total available land.

Develop a linear program to determine the optimum plan for the project.

5. Realco owns 800 acres of undeveloped land on a scenic lake in the heart of the Ozark Mountains. In the past, little or no regulation was imposed upon new developments around the lake. The lake shores are now dotted with vacation homes, and septic tanks, most of them improperly installed, are in extensive use. Over the years, seepage from the septic tanks led to severe water pollution. To curb further degradation of the lake, county officials have approved stringent ordinances applicable to all future developments: (1) Only single-, double-, and triple-family homes can be constructed, with single-family homes accounting for at least 50% of the total. (2) To limit the number of septic tanks, minimum lot sizes of 2, 3, and 4 acres are required for single-, double-, and triple-family homes, respectively. (3) Recreation areas of 1 acre each must be established at the rate of one area per 200 families. (4) To preserve the ecology of the lake, underground water may not be pumped out for house or garden use. The president of Realco is studying the possibility of developing the 800-acre property. The new development will include single-, double-, and triple-family homes. It is estimated that 15% of the acreage will be allocated to streets and utility easements. Realco estimates the returns from the different housing units as follows:

Housing unit	Single	Double	Triple
Net return per unit ($)	10,000	12,000	15,000

The cost of connecting water service to the area is proportionate to the number of units constructed. However, the county charges a minimum of $100,000 for the project. Additionally, the expansion of the water system beyond its present capacity is limited to 200,000 gallons per day during peak periods. The following data summarize the water service connection cost as well as the water consumption, assuming an average size family:

Housing unit	Single	Double	Triple	Recreation
Water service connection cost per unit ($)	1000	1200	1400	800
Water consumption per unit (gal/day)	400	600	840	450

Develop an optimal plan for Realco.

6. Consider the Realco model of Problem 5. Suppose that an additional 100 acres of land can be purchased for $450,000, which will increase the total acreage to 900 acres. Is this a profitable deal for Realco?

2.3.2 Currency Arbitrage[2]

In today's global economy, a multinational company must deal with currencies of the countries in which it operates. Currency arbitrage, or simultaneous purchase and sale of currencies in different markets, offers opportunities for advantageous movement of money from one currency to another. For example, converting £1000 to U.S. dollars in 2001 with an exchange rate of $1.60 to £1 will yield $1600. Another way of making the conversion is to first change the British pound to Japanese yen and then convert the yen to U.S. dollars using the 2001 exchange rates of £1 = ¥175 and $1 = ¥105. The

[2]This section is based on J. Kornbluth and G. Salkin (1987, Chapter 6).

resulting dollar amount is $\frac{(£1{,}000 \times ¥175)}{¥105} = \$1{,}666.67$. This example demonstrates the advantage of converting the British money first to Japanese yen and then to dollars. This section shows how the arbitrage problem involving many currencies can be formulated and solved as a linear program.

Example 2.3-2 (Currency Arbitrage Model)

Suppose that a company has a total of 5 million dollars that can be exchanged for euros (€), British pounds (£), yen (¥), and Kuwaiti dinars (KD). Currency dealers set the following limits on the amount of any single transaction: 5 million dollars, 3 million euros, 3.5 million pounds, 100 million yen, and 2.8 million KDs. The table below provides typical spot exchange rates. The bottom diagonal rates are the reciprocal of the top diagonal rates. For example, rate(€ → $) = 1/rate($ → €) = 1/.769 = 1.30.

	$	€	£	¥	KD
$	1	.769	.625	105	.342
€	$\frac{1}{.769}$	1	.813	137	.445
£	$\frac{1}{.625}$	$\frac{1}{.813}$	1	169	.543
¥	$\frac{1}{105}$	$\frac{1}{137}$	$\frac{1}{169}$	1	.0032
KD	$\frac{1}{.342}$	$\frac{1}{.445}$	$\frac{1}{.543}$	$\frac{1}{.0032}$	1

Is it possible to increase the dollar holdings (above the initial $5 million) by circulating currencies through the currency market?

Mathematical Model: The situation starts with $5 million. This amount goes through a number of conversions to other currencies before ultimately being reconverted to dollars. The problem thus seeks determining the amount of each conversion that will maximize the total dollar holdings.

For the purpose of developing the model and simplifying the notation, the following numeric code is used to represent the currencies.

Currency	$	€	£	¥	KD
Code	1	2	3	4	5

Define

$$x_{ij} = \text{Amount in currency } i \text{ converted to currency } j,\ i \text{ and } j = 1, 2, \ldots, 5$$

For example, x_{12} is the dollar amount converted to euros and x_{51} is the KD amount converted to dollars. We further define two additional variables representing the input and the output of the arbitrage problem:

$$I = \text{Initial dollar amount } (= \$5 \text{ million})$$

$$y = \text{Final dollar holdings (to be determined from the solution)}$$

Our goal is to determine the maximum final dollar holdings, y, subject to the currency flow restrictions and the maximum limits allowed for the different transactions.

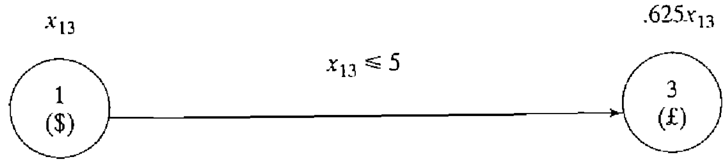

FIGURE 2.4

Definition of the input/output variable, x_{13}, between \$ and £

We start by developing the constraints of the model. Figure 2.4 demonstrates the idea of converting dollars to pounds. The dollar amount x_{13} at originating currency 1 is converted to $.625x_{13}$ pounds at end currency 3. At the same time, the transacted dollar amount cannot exceed the limit set by the dealer, $x_{13} \leq 5$.

To conserve the flow of money from one currency to another, each currency must satisfy the following input-output equation:

$$\begin{pmatrix}\text{Total sum available}\\ \text{of a currency (input)}\end{pmatrix} = \begin{pmatrix}\text{Total sum converted to}\\ \text{other currencies (output)}\end{pmatrix}$$

1. *Dollar* ($i = 1$):

$$\begin{aligned}\text{Total available dollars} &= \text{Initial dollar amount } + \\ &\quad\ \text{dollar amount from other currencies}\\ &= I + (€ \rightarrow \$) + (£ \rightarrow \$) + (¥ \rightarrow \$) + (\text{KD} \rightarrow \$)\\ &= I + \tfrac{1}{.769}x_{21} + \tfrac{1}{.625}x_{31} + \tfrac{1}{105}x_{41} + \tfrac{1}{.342}x_{51}\end{aligned}$$

$$\begin{aligned}\text{Total distributed dollars} &= \text{Final dollar holdings } + \\ &\quad\ \text{dollar amount to other currencies}\\ &= y + (\$ \rightarrow €) + (\$ \rightarrow £) + (\$ \rightarrow ¥) + (\$ \rightarrow \text{KD})\\ &= y + x_{12} + x_{13} + x_{14} + x_{15}\end{aligned}$$

Given $I = 5$, the dollar constraint thus becomes

$$y + x_{12} + x_{13} + x_{14} + x_{15} - \left(\tfrac{1}{.769}x_{21} + \tfrac{1}{.625}x_{31} + \tfrac{1}{105}x_{41} + \tfrac{1}{.342}x_{51}\right) = 5$$

2. *Euro* ($i = 2$):

$$\begin{aligned}\text{Total available euros} &= (\$ \rightarrow €) + (£ \rightarrow €) + (¥ \rightarrow €) + (\text{KD} \rightarrow €)\\ &= .769x_{12} + \tfrac{1}{.813}x_{32} + \tfrac{1}{137}x_{42} + \tfrac{1}{.445}x_{52}\end{aligned}$$

$$\begin{aligned}\text{Total distributed euros} &= (€ \rightarrow \$) + (€ \rightarrow £) + (€ \rightarrow ¥) + (€ \rightarrow \text{KD})\\ &= x_{21} + x_{23} + x_{24} + x_{25}\end{aligned}$$

Thus, the constraint is

$$x_{21} + x_{23} + x_{24} + x_{25} - \left(.769x_{12} + \tfrac{1}{.813}x_{32} + \tfrac{1}{137}x_{42} + \tfrac{1}{.445}x_{52}\right) = 0$$