35% more space than Grano, which amounts to allocating about 57% to Wheatie and 43% to Grano. What do you think?

11. Jack is an aspiring freshman at Ulern University. He realizes that "all work and no play make Jack a dull boy." As a result, Jack wants to apportion his available time of about 10 hours a day between work and play. He estimates that play is twice as much fun as work. He also wants to study at least as much as he plays. However, Jack realizes that if he is going to get all his homework assignments done, he cannot play more than 4 hours a day. How should Jack allocate his time to maximize his pleasure from both work and play?

12. Wild West produces two types of cowboy hats. A type 1 hat requires twice as much labor time as a type 2. If the all available labor time is dedicated to Type 2 alone, the company can produce a total of 400 Type 2 hats a day. The respective market limits for the two types are 150 and 200 hats per day. The profit is $8 per Type 1 hat and $5 per Type 2 hat. Determine the number of hats of each type that would maximize profit.

13. Show & Sell can advertise its products on local radio and television (TV). The advertising budget is limited to $10,000 a month. Each minute of radio advertising costs $15 and each minute of TV commercials $300. Show & Sell likes to advertise on radio at least twice as much as on TV. In the meantime, it is not practical to use more than 400 minutes of radio advertising a month. From past experience, advertising on TV is estimated to be 25 times as effective as on radio. Determine the optimum allocation of the budget to radio and TV advertising.

***14.** Wyoming Electric Coop owns a steam-turbine power-generating plant. Because Wyoming is rich in coal deposits, the plant generates its steam from coal. This, however, may result in emission that does not meet the Environmental Protection Agency standards. EPA regulations limit sulfur dioxide discharge to 2000 parts per million per ton of coal burned and smoke discharge from the plant stacks to 20 lb per hour. The Coop receives two grades of pulverized coal, $C1$ and $C2$, for use in the steam plant. The two grades are usually mixed together before burning. For simplicity, it can be assumed that the amount of sulfur pollutant discharged (in parts per million) is a weighted average of the proportion of each grade used in the mixture. The following data are based on consumption of 1 ton per hour of each of the two coal grades.

Coal grade	Sulfur discharge in parts per million	Smoke discharge in lb per hour	Steam generated in lb per hour
$C1$	1800	2.1	12,000
$C2$	2100	.9	9,000

(a) Determine the optimal ratio for mixing the two coal grades.

(b) Determine the effect of relaxing the smoke discharge limit by 1 lb on the amount of generated steam per hour.

15. Top Toys is planning a new radio and TV advertising campaign. A radio commercial costs $300 and a TV ad costs $2000. A total budget of $20,000 is allocated to the campaign. However, to ensure that each medium will have at least one radio commercial and one TV ad, the most that can be allocated to either medium cannot exceed 80% of the total budget. It is estimated that the first radio commercial will reach 5000 people, with each additional commercial reaching only 2000 new ones. For TV, the first ad will reach 4500 people and each additional ad an additional 3000. How should the budgeted amount be allocated between radio and TV?

16. The Burroughs Garment Company manufactures men's shirts and women's blouses for Walmark Discount Stores. Walmark will accept all the production supplied by Burroughs. The production process includes cutting, sewing, and packaging. Burroughs employs 25 workers in the cutting department, 35 in the sewing department, and 5 in the packaging department. The factory works one 8-hour shift, 5 days a week. The following table gives the time requirements and profits per unit for the two garments:

	Minutes per unit			
Garment	*Cutting*	*Sewing*	*Packaging*	Unit profit ($)
Shirts	20	70	12	8
Blouses	60	60	4	12

Determine the optimal weekly production schedule for Burroughs.

17. A furniture company manufactures desks and chairs. The sawing department cuts the lumber for both products, which is then sent to separate assembly departments. Assembled items are sent for finishing to the painting department. The daily capacity of the sawing department is 200 chairs or 80 desks. The chair assembly department can produce 120 chairs daily and the desk assembly department 60 desks daily. The paint department has a daily capacity of either 150 chairs or 110 desks. Given that the profit per chair is $50 and that of a desk is $100, determine the optimal production mix for the company.

***18.** An assembly line consisting of three consecutive stations produces two radio models: HiFi-1 and HiFi-2. The following table provides the assembly times for the three workstations.

	Minutes per unit	
Workstation	*HiFi-1*	*HiFi-2*
1	6	4
2	5	5
3	4	6

The daily maintenance for stations 1, 2, and 3 consumes 10%, 14%, and 12%, respectively, of the maximum 480 minutes available for each station each day. Determine the optimal product mix that will minimize the idle (or unused) times in the three workstations.

19. *TORA Experiment.* Enter the following LP into TORA and select the graphic solution mode to reveal the LP graphic screen.

$$\text{Minimize } z = 3x_1 + 8x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\geq 8 \\ 2x_1 - 3x_2 &\leq 0 \\ x_1 + 2x_2 &\leq 30 \\ 3x_1 - x_2 &\geq 0 \\ x_1 &\leq 10 \\ x_2 &\geq 9 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Next, on a sheet of paper, graph and scale the x_1- and x_2-axes for the problem (you may also click Print Graph on the top of the right window to obtain a ready-to-use scaled sheet). Now, graph a constraint manually on the prepared sheet, then click it on the left window of the screen to check your answer. Repeat the same for each constraint and then terminate the procedure with a graph of the objective function. The suggested process is designed to test and reinforce your understanding of the graphical LP solution through immediate feedback from TORA.

20. *TORA Experiment.* Consider the following LP model:

$$\text{Maximize } z = 5x_1 + 4x_2$$

subject to

$$\begin{aligned} 6x_1 + 4x_2 &\le 24 \\ 6x_1 + 3x_2 &\le 22.5 \\ x_1 + x_2 &\le 5 \\ x_1 + 2x_2 &\le 6 \\ -x_1 + x_2 &\le 1 \\ x_2 &\le 2 \\ x_1, x_2 &\ge 0 \end{aligned}$$

In LP, a constraint is said to be *redundant* if its removal from the model leaves the feasible solution space unchanged. Use the graphical facility of TORA to identify the redundant constraints, then show that their removal (simply by not graphing them) does not affect the solution space or the optimal solution.

21. *TORA Experiment.* In the Reddy Mikks model, use TORA to show that the removal of the raw material constraints (constraints 1 and 2) would result in an *unbounded solution space.* What can be said in this case about the optimal solution of the model?

22. *TORA Experiment.* In the Reddy Mikks model, suppose that the following constraint is added to the problem.

$$x_2 \ge 3$$

Use TORA to show that the resulting model has conflicting constraints that cannot be satisfied simultaneously and hence it has *no feasible solution.*

2.2.2 Solution of a Minimization Model

Example 2.2-2 (Diet Problem)

Ozark Farms uses at least 800 lb of special feed daily. The special feed is a mixture of corn and soybean meal with the following compositions:

	lb per lb of feedstuff		
Feedstuff	*Protein*	*Fiber*	Cost ($/lb)
Corn	.09	.02	.30
Soybean meal	.60	.06	.90

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. Ozark Farms wishes to determine the daily minimum-cost feed mix.

Because the feed mix consists of corn and soybean meal, the decision variables of the model are defined as

x_1 = lb of corn in the daily mix

x_2 = lb of soybean meal in the daily mix

The objective function seeks to minimize the total daily cost (in dollars) of the feed mix and is thus expressed as

$$\text{Minimize } z = .3x_1 + .9x_2$$

The constraints of the model reflect the daily amount needed and the dietary requirements. Because Ozark Farms needs at least 800 lb of feed a day, the associated constraint can be expressed as

$$x_1 + x_2 \geq 800$$

As for the protein dietary requirement constraint, the amount of protein included in x_1 lb of corn and x_2 lb of soybean meal is $(.09x_1 + .6x_2)$ lb. This quantity should equal at least 30% of the total feed mix $(x_1 + x_2)$ lb—that is,

$$.09x_1 + .6x_2 \geq .3(x_1 + x_2)$$

In a similar manner, the fiber requirement of at most 5% is constructed as

$$.02x_1 + .06x_2 \leq .05(x_1 + x_2)$$

The constraints are simplified by moving the terms in x_1 and x_2 to the left-hand side of each inequality, leaving only a constant on the right-hand side. The complete model thus becomes

$$\text{minimize } z = .3x_1 + .9x_2$$

subject to

$$\begin{aligned} x_1 + \quad x_2 &\geq 800 \\ .21x_1 - .30x_2 &\leq 0 \\ .03x_1 - .01x_2 &\geq 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Figure 2.3 provides the graphical solution of the model. Unlike those of the Reddy Mikks model (Example 2.2-1), the second and third constraints pass through the origin. To plot the associated straight lines, we need one additional point, which can be obtained by assigning a value to one of the variables and then solving for the other variable. For example, in the second constraint, $x_1 = 200$ will yield $.21 \times 200 - .3x_2 = 0$, or $x_2 = 140$. This means that the straight line $.21x_1 - .3x_2 = 0$ passes through (0, 0) and (200, 140). Note also that (0, 0) cannot be used as a reference point for constraints 2 and 3, because both lines pass through the origin. Instead, any other point [e.g., (100, 0) or (0, 100)] can be used for that purpose.

Solution:

Because the present model seeks the minimization of the objective function, we need to reduce the value of z as much as possible in the direction shown in Figure 2.3. The optimum solution is the intersection of the two lines $x_1 + x_2 = 800$ and $.21x_1 - .3x_2 = 0$, which yields $x_1 = 470.59$ lb and $x_2 = 329.41$ lb. The associated minimum cost of the feed mix is $z = .3 \times 470.59 + .9 \times 329.42 =$ \$437.65 per day.

Remarks. We need to take note of the way the constraints of the problem are constructed. Because the model is minimizing the total cost, one may argue that the solution will seek exactly 800 tons of feed. Indeed, this is what the optimum solution given above does. Does this mean then that the first constraint can be deleted altogether simply by including the amount 800 tons

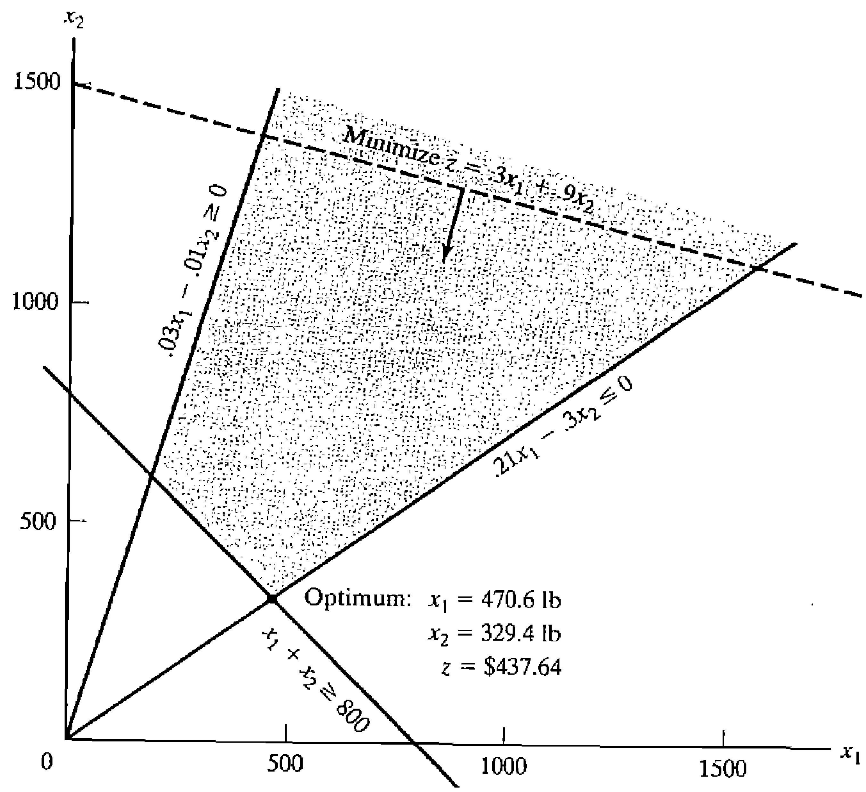

FIGURE 2.3
Graphical solution of the diet model

in the remaining constraints? To find the answer, we state the new protein and fiber constraints as

$$.09x_1 + .6x_2 \geq .3 \times 800$$
$$.02x_1 + .06x_2 \leq .05 \times 800$$

or

$$.09x_1 + .6x_2 \geq 240$$
$$.02x_1 + .06x_2 \leq 40$$

The new formulation yields the solution $x_1 = 0$, and $x_2 = 400$ lb (verify with TORA!), which does not satisfy the *implied* requirement for 800 lb of feed. This means that the constraint $x_1 + x_2 \geq 800$ must be used explicitly and that the protein and fiber constraints must remain exactly as given originally.

Along the same line of reasoning, one may be tempted to replace $x_1 + x_2 \geq 800$ with $x_1 + x_2 = 800$. In the present example, the two constraints yield the same answer. But in general this may not be the case. For example, suppose that the daily mix must include at least 500 lb of corn. In this case, the optimum solution will call for using 500 lb of corn and 350 lb of soybean (verify with TORA!), which is equivalent to a daily feed mix of $500 + 350 = 850$ lb. Imposing the equality constraint a priori will lead to the conclusion that the problem has no feasible solution (verify with TORA!). On the other hand, the use of the inequality is inclusive of the equality case, and hence its use does not prevent the model from producing exactly 800 lb of feed mix, should the remaining constraints allow it. The conclusion is that we should not "pre-guess" the solution by imposing the additional equality restriction, and we should always use inequalities unless the situation explicitly stipulates the use of equalities.

PROBLEM SET 2.2B

1. Identify the direction of decrease in z in each of the following cases:
   *(a) Minimize $z = 4x_1 - 2x_2$.
   (b) Minimize $z = -3x_1 + x_2$.
   (c) Minimize $z = -x_1 - 2x_2$.
2. For the diet model, suppose that the daily availability of corn is limited to 450 lb. Identify the new solution space, and determine the new optimum solution. 2.
3. For the diet model, what type of optimum solution would the model yield if the feed mix should not exceed 800 lb a day? Does the solution make sense?
4. John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores. In store 1, he can work between 5 and 12 hours a week, and in store 2 he is allowed between 6 and 10 hours. Both stores pay the same hourly wage. In deciding how many hours to work in each store, John wants to base his decision on work stress. Based on interviews with present employees, John estimates that, on an ascending scale of 1 to 10, the stress factors are 8 and 6 at stores 1 and 2, respectively. Because stress mounts by the hour, he assumes that the total stress for each store at the end of the week is proportional to the number of hours he works in the store. How many hours should John work in each store?

*5. OilCo is building a refinery to produce four products: diesel, gasoline, lubricants, and jet fuel. The minimum demand (in bbl/day) for each of these products is 14,000, 30,000, 10,000, and 8,000, respectively. Iran and Dubai are under contract to ship crude to OilCo. Because of the production quotas specified by OPEC (Organization of Petroleum Exporting Countries) the new refinery can receive at least 40% of its crude from Iran and the remaining amount from Dubai. OilCo predicts that the demand and crude oil quotas will remain steady over the next ten years. 2.

   The specifications of the two crude oils lead to different product mixes: One barrel of Iran crude yields .2 bbl of diesel, .25 bbl of gasoline, .1 bbl of lubricant, and .15 bbl of jet fuel. The corresponding yields from Dubai crude are .1, .6, .15, and .1, respectively. OilCo needs to determine the minimum capacity of the refinery (in bbl/ day).

6. Day Trader wants to invest a sum of money that would generate an annual yield of at least \$10,000. Two stock groups are available: blue chips and high tech, with average annual yields of 10% and 25%, respectively. Though high-tech stocks provide higher yield, they are more risky, and Trader wants to limit the amount invested in these stocks to no more than 60% of the total investment. What is the minimum amount Trader should invest in each stock group to accomplish the investment goal?

*7. An industrial recycling center uses two scrap aluminum metals, A and B, to produce a special alloy. Scrap A contains 6% aluminum, 3% silicon, and 4% carbon. Scrap B has 3% aluminum, 6% silicon, and 3% carbon. The costs per ton for scraps A and B are \$100 and \$80, respectively. The specifications of the special alloy require that (1) the aluminum content must be at least 3% and at most 6%, (2) the silicon content must lie between 3%

and 5%, and (3) the carbon content must be between 3% and 7%. Determine the optimum mix of the scraps that should be used in producing 1000 tons of the alloy.

8. *TORA Experiment.* Consider the Diet Model and let the objective function be given as

$$\text{Minimize } z = .8x_1 + .8x_2$$

Use TORA to show that the optimum solution is associated with *two* distinct corner points and that both points yield the same objective value. In this case, the problem is said to have *alternative optima.* Explain the conditions leading to this situation and show that, in effect, the problem has an infinite number of alternative optima, then provide a formula for determining all such solutions.

2.3 SELECTED LP APPLICATIONS

This section presents realistic LP models in which the definition of the variables and the construction of the objective function and constraints are not as straightforward as in the case of the two-variable model. The areas covered by these applications include the following:

1. Urban planning.
2. Currency arbitrage.
3. Investment.
4. Production planning and inventory control.
5. Blending and oil refining.
6. Manpower planning.

Each model is fully developed and its optimum solution is analyzed and interpreted.

2.3.1 Urban Planning[1]

Urban planning deals with three general areas: (1) building new housing developments, (2) upgrading inner-city deteriorating housing and recreational areas, and (3) planning public facilities (such as schools and airports). The constraints associated with these projects are both economic (land, construction, financing) and social (schools, parks, income level). The objectives in urban planning vary. In new housing developments, profit is usually the motive for undertaking the project. In the remaining two categories, the goals involve social, political, economic, and cultural considerations. Indeed, in a publicized case in 2004, the mayor of a city in Ohio wanted to condemn an old area of the city to make way for a luxury housing development. The motive was to increase tax collection to help alleviate budget shortages. The example presented in this section is fashioned after the Ohio case.

[1]This section is based on Laidlaw (1972).