THE CONSTRUCTION OF NORMAL-YIELD AND STAND TABLES FOR EVEN-AGED TIMBER STANDS [1]

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INTRODUCTION

A normal-yield table is a tabulated statement of yields to be expected from well-stocked, even-aged stands of a given timber type, at successive ages, for the range of site qualities characteristic of the type. Normal-stand tables are frequency distributions of tree stems by diameter classes within stands of given age and site quality.

All of the timber-yield tables published in the United States during the last 10 years have been based, either entirely or partially, upon the Bruce method (*1*) [2] or Reineke's modification of it (*7*). The accompanying stand tables have usually been based upon one of three principles of construction: (1) Correlation of percentiles of frequency with average diameter of stand (*2*, *8*); (2) graphic fitting of either the normal curve of error or some logarithmic transformation of the normal curve to actual frequency distributions (*1*, *7*); or (3) mathematical fitting of the Gram-Charlier series to the actual distributions after harmonizing calculated parameters with average diameter (*4*, *5*, *9*, *10*).

It is the purpose of this paper to describe (1) a general method of constructing yield tables for the entire stand that is free from the inherent rigidity of the Bruce-Reineke method, and (2) a method of constructing stand tables that is more objective than the purely graphic and less laborious in calculation than the Gram-Charlier series.

YIELD TABLES

The data upon which yield and stand tables are based consist of measurements of sample plots in the desired timber type. The sample plots selected should be in even-aged stands and large enough to contain 100 to 300 trees. The objective is to enclose a comparatively complete crown canopy by excluding the larger openings which follow accident or failure of reproduction and at the same time to include the ground area equivalent to that used by the enclosed timber. After surveying plot boundaries in order to determine plot area, the following measurements are taken: (1) Diameter breast high [3] of every tree by species and crown class; (2) heights of enough trees (usually 15 to 25) to permit the reliable estimate of average height of each diameter class; and (3) age counts, taken with an increment borer on several trees, to establish plot age.

From these data, the yield and the stem distribution of each plot are calculated on an acre basis. Yield is expressed in several ways, such as number of trees, basal area, average diameter, and volume in cubic and board feet. The relation of yield to age and site quality

[1] Received for publication Apr. 30, 1935; issued December 1935.
[2] Reference is made by number (italic) to Literature Cited, p. 564.
[3] Four and a half feet above the ground; abbreviation, d. b. h.

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is then worked out. For the western white pine type, as an example, Haig (*3*) constructed tables for the following expressions of yield:

A. For the entire stand:
   - Number of trees per acre.
   - Average diameter breast high.
   - Basal area per acre.
   - Cubic-foot volume inside bark per acre.

B. For the trees 6.6 inches d. b. h. and larger:
   - Number of trees per acre.
   - Average diameter breast high.
   - Basal area per acre.
   - Board-foot volume per acre, according to the International log rule.

C. For the trees 7.6 inches d. b. h. and larger:
   - Board-foot volume per acre, according to the Scribner log rule.

D. For the trees 12.6 inches d. b. h. and larger:
   - Number of trees per acre.
   - Average diameter breast high.
   - Basal area per acre.
   - Board-foot volume per acre according to the International log rule.
   - Board-foot volume per acre according to the Scribner log rule.

E. For the dominant stand:
   - Average diameter breast high.
   - Cubic-foot volume per acre.
   - Height of average dominant (dominant and codominant) western white pine.

In Haig's investigation, as in practically all such studies, the direct relation of yield to age and site quality was worked out for the basic variables of the entire stand only, yields of the partial stand (with the exception of the height of the average dominant western white pine) being derived therefrom.

These derived curves of yield have proved workable and satisfactory. But the curves of yield of the entire stand in terms of age and site quality are forced into too rigid a mold. The nature of this rigidity will be shown and a method of analysis that entirely overcomes it described. The data consist of measurements of 99 fully stocked, even-aged sample plots of red gum (*Liquidambar styraciflua* L.) measured by R. K. Winters of the Southern Forest Experiment Station.

THE DETERMINATION OF SITE INDEX

The quantitative measure of site quality, known as site index, is the height of the average dominant (usually including the average codominant) tree at a given age, called reference age, for which 50 years is commonly taken, as in the present discussion. A necessary condition for the correct determination of site index is that it be intrinsically independent of plot age; for then the proportion of the sample plots whose site indexes are better—or poorer—than any given site index is the same, in the long run, at one age as at another, and, conversely, site index may be identified by associating it with the probability of occurrence. For example, the site qualities that are the most productive 10 percent of the area in a timber type should be represented by 10 percent of the sample plots in each age class whose height of average dominant—or dominant and codominant—tree is greatest.

The condition stated above may be fulfilled in field sampling if, while trying for approximately equal numbers of plots in each age class, conscious effort is made to secure random distribution of the sample plots with respect to site quality. It then follows that the curve of height on age (shown in fig. 1, *A*, for the red gum data) is

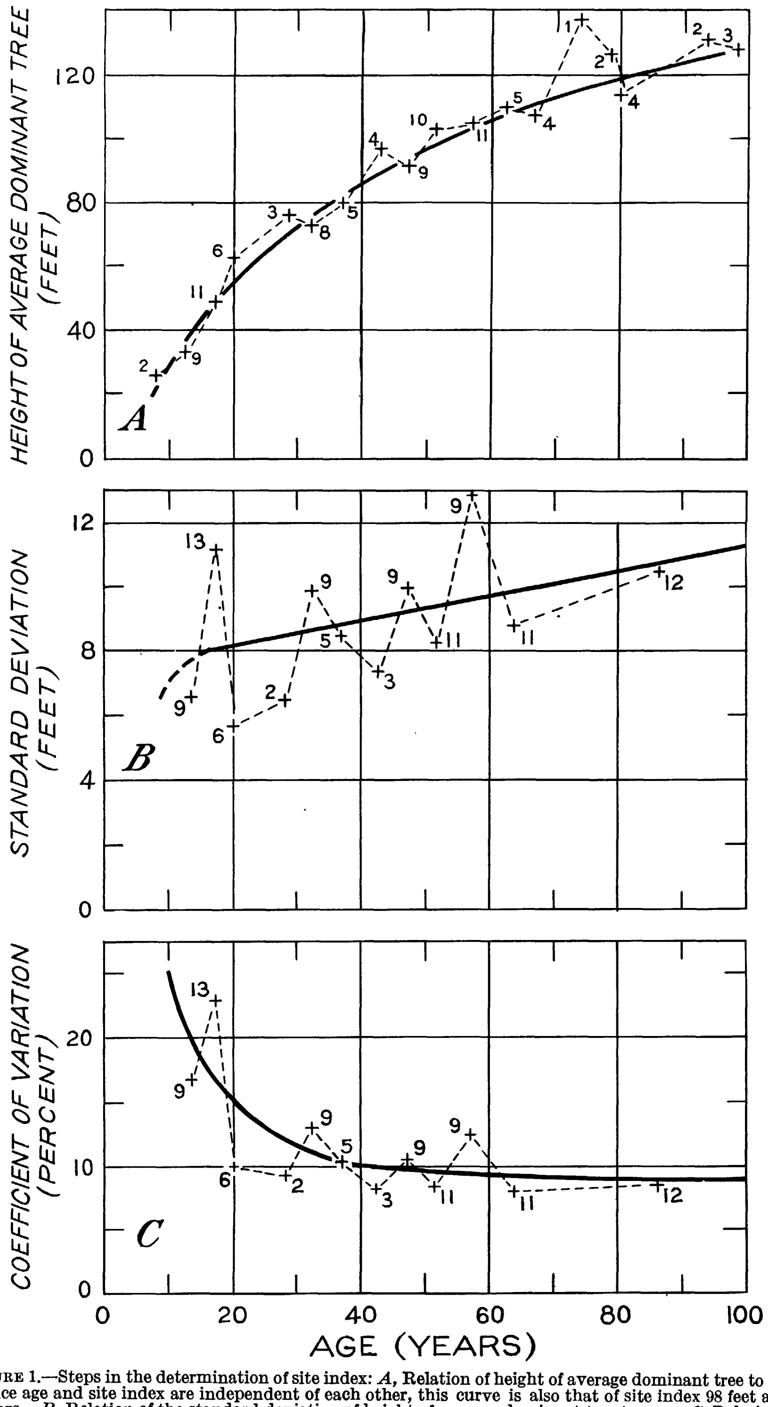

FIGURE 1.—Steps in the determination of site index: *A*, Relation of height of average dominant tree to age; since age and site index are independent of each other, this curve is also that of site index 98 feet at 50 years. *B*, Relation of the standard deviation of height of average dominant tree to age. *C*, Relation of the coefficient of variation to age.

not only the gross regression but also a partial regression, that is, it represents the relation of height to age for a constant site index as well as for the average of the site indexes, in this case 98 feet at 50 years.

The curves of height on age for other site indexes are next needed. In all of the recent yield studies in this country these have been defined as curves of the same form as that of average site index but differing therefrom by a constant ratio at all ages; for example, the curve of height of dominant stand on age for the 80-foot site index of red gum would be defined as $\left(\frac{80-98}{98}\right)$ or -0.184; that is, 18.4 percent less than the curve values of figure 1, *A*, at any age.

Now if site index thus defined holds true in nature, it follows that the coefficient of variation of height is independent of age. Otherwise there will not be the same proportion of the sample plots of a

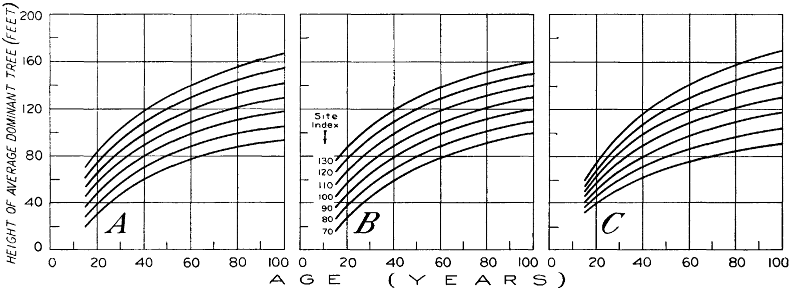

FIGURE 2.—The relation of height of average dominant tree to age and site index: *A*, As determined from the relation of the standard deviation or of the coefficient of variation to age; *B*, on the assumption that the standard deviation is independent of age; *C*, on the assumption that the coefficient of variation is independent of age.

given site class in all ages. This is an important hypothesis, and one that may be tested in each investigation. A method of making the test will be illustrated.

The distributions of residuals from the curve of the average site index of figure 1, *A*, are collected in table 1 by selected age classes. At the bottom of the table are given the standard deviations in feet, and the coefficients of variation; the latter are the standard deviations expressed as percentages of the curve value of the mean. The relation of these two constants of distribution to age is shown in figure 1, *B* and *C*. The three curves of figure 1 permit of ready cross checking, because any value of the standard-deviation curve divided by the corresponding average must equal the coefficient of variation. Since, according to the figure, the coefficient of variation is not independent of age in the early life of the stand, the definition upon which site index has generally been determined in yield studies during the past decade does not hold for the red gum data; that is, the ratio of height of dominant stand of a given site index to that of the average site index is not constant, but depends, rather, upon the age of the stand.[4]

[4] Roy A. Chapman, of the Southern Forest Experiment Station staff, tested this hypothesis for 3 of the southern pines for which yield tables had been published; for 2 of them it did not hold.

TABLE 1.—*Frequency distribution of the plot residuals from figure 1, A, by selected age classes*

Deviation of height of average dominant and codominant tree from curved height for average site index (feet)	Plots in age class indicated											
	8–14 years	15–19 years	20–24 years	25–29 years	30–34 years	35–39 years	40–44 years	45–49 years	50–54 years	55–59 years	60–69 years	70+ years
	Number	*Number*	*Number*	*Number*	*Number*	*Number*	*Number*	*Number*	*Number*	*Number*	*Number*	*Number*
+19.50 to +22.49		1										1
+16.50 to +19.49										1		1
+13.50 to +16.49									1			
+10.50 to +13.49	1						1		2	1	1	
+7.50 to +10.49		1		1					1	1	1	
+4.50 to +7.49	1	2	6		3	1	1	2	1	1		
+1.50 to +4.49	2	2		1	1			1	1	2	2	3
−1.50 to +1.49		1			1	1		2	2		3	4
−4.50 to −1.49	1	1				2	1		1	1	1	2
−7.50 to −4.49	3				2							
−10.50 to −7.49		1							1		2	
−13.50 to −10.49	1				1			2	1			
−16.50 to −13.49		3						1				
−19.50 to −16.49		1				1		1				
−22.50 to −19.49										2	1	
−25.50 to −22.49					1							1
Total	9	13	6	2	9	5	3	9	11	9	11	12
Standard deviation[1] feet	6.6	11.2	5.7	6.5	9.9	8.5	7.4	10.0	8.3	12.9	8.8	10.5
Curved height at average age of class feet	39.5	49.2	56.1	70.0	76.2	82.6	89.8	95.2	99.1	104.4	109.3	123.1
Coefficient of variation percent	16.7	22.8	10.2	9.3	13.0	10.3	8.2	10.5	8.4	12.4	8.1	8.5

[1] Computed from deviations measured to the nearest foot. The data are grouped into 3-foot classes for conciseness.

Fortunately, however, the curves of height of dominant stand which result from such varying ratios may be easily deduced from figure 1. Again taking the 80-foot site index as an illustration, the curve for the height of dominant stand, as determined above, is 18.4 percent less than the curve values of figure 1, *A*, but this is at the reference age of 50 only. It should, however, be this same number of units of the coefficient of variation at all ages, so that the probability of occurrence of a given site index would be the same at all ages. Since the coefficient of variation (as read from figure 1, *C*) is 9.5 percent at 50 years, site index 80 becomes $\frac{-18.4}{9.5}$ or -1.94 units measured from the average curve. And 1.94 times the curve value of the coefficient of variation at any given age is the percentage reduction to be applied to the average curve to give the curve height for site index 80 feet at that age. The curves for other site indexes are calculated in a similar manner. Several are shown in figure 2, *A*.

Site index, then, may be assigned to each plot by plotting the height of its average dominant tree over its age on this graph and interpolating between the nearest curves.

In practice it is perhaps simpler to base the site-index curves upon the standard deviation rather than upon the coefficient of variation. As site index is independent of age, though the standard deviation of dominant height may or may not be independent of age, site index may be defined as a deviation in standard units from the curve of dominant height on age. The absolute value of a standard unit at any age for red gum is given in figure 1, *B*. For the reference age of 50 years it is 9.3 feet. Hence the 80-foot site index curve is $\left(\frac{80-98}{9.3}\right) = -1.94$ standard units measured from the curve of figure 1, *A*. At 100 years of age, for instance, the height of dominant stand for site index 80 feet is $130+(-1.94\times11.3)=108$ feet. This process suggests a general expression of height of dominant stand in terms of age and site index. We have (1) the expression

$$H=f_1(A)$$

where H is height of dominant stand for average site index, expressed as a function f_1 of age A (fig. 1, *A*); (2) the standard deviation σ_H of the height of dominant stand as a function of age A, that is,

$$\sigma_H=f_2(A)$$

where f_2 is such a function as defined in figure 1, *B*. From these two expressions and from the definition of site index as given above,

$$H=f_1(A)+f_2(A)\left\{\frac{S-f_1(A_R)}{f_2(A_R)}\right\}$$

in which H is height of dominant stand at age A for site index S, and A_R is the reference age which determines the numerical expression of site index. This is a general expression of the joint effect of two independent variables upon a dependent variable when the correlation between the independents is zero.

If the standard deviation had been found to be independent of age, a series of site-index curves, such as those of figure 2, *B*, would have resulted, in which each is a constant difference from any other. On the other hand, had the coefficient of variation been independent of age, the result would have been a series of site-index curves, each a constant ratio of any other (fig. 2, *C*). Since, however, both the standard deviation and the coefficient of variation are dependent upon age, the best estimate, as shown in figure 2, *A*, is intermediate between the others.

THE CONSTRUCTION OF YIELD TABLES FOR THE ENTIRE STAND

The same general method may be used for the expression of entire stand yield in terms of age and site index. Since age and site index are independent of each other, the gross regression of yield on age is also the partial regression for the average of all site indexes present. For a particular site index, however, the regression of yield on age is (in the general case) at a level which differs from the gross regression by a certain number of units of the standard deviation of yield around the gross regression. How the number of such standard units corresponding to each site index is defined may be illustrated through the analysis of yield of red gum in cubic feet and in number of trees.

In figure 3 is shown the relation to age of the gross regression of volume, the standard deviation of volume, and the coefficient of variation of volume. The last-named is primarily for cross checking with the other two. The plotted points were computed in the same way as and the curves are analogous to those of figure 1.

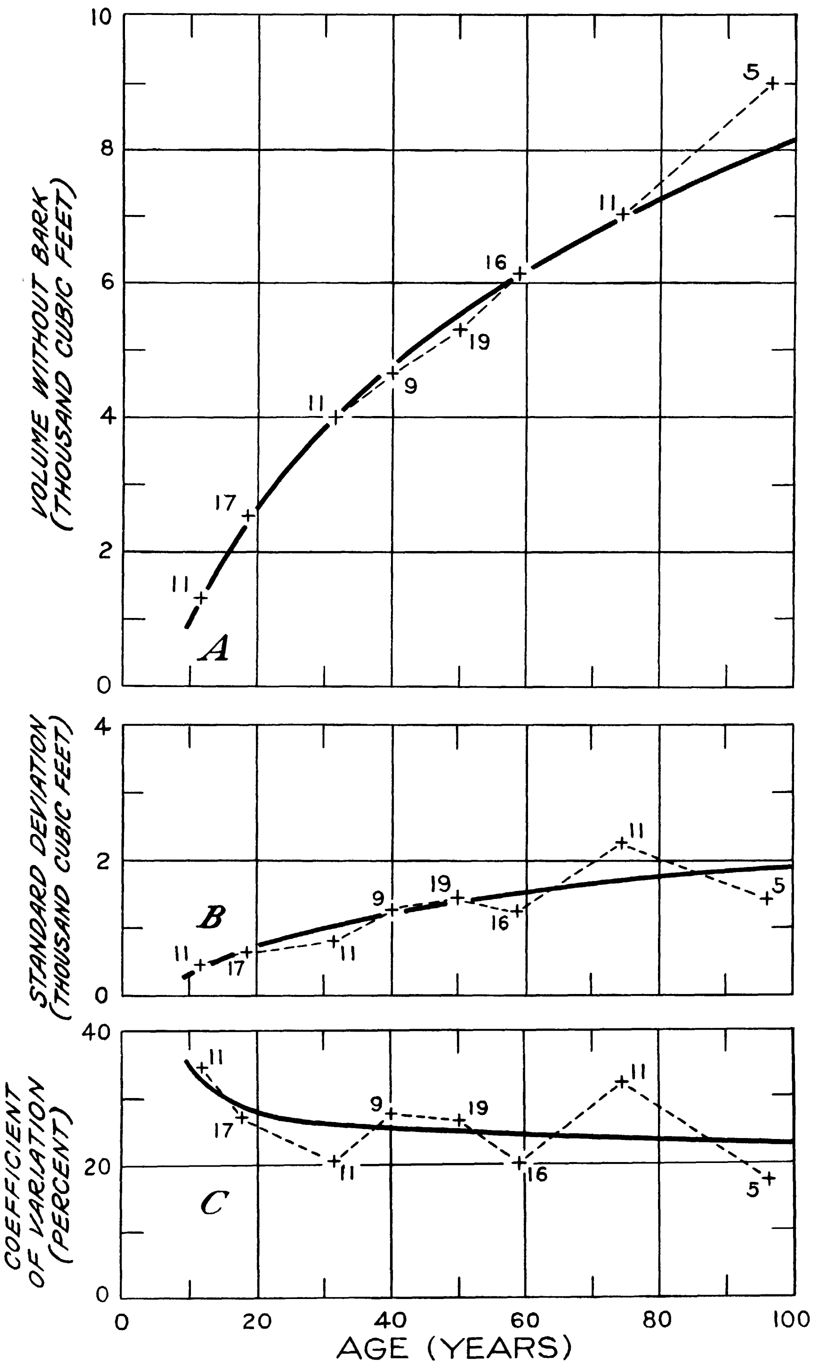

FIGURE 3.—Steps in the correlation of volume with age and site index: *A*, Relation of volume per acre to age for the average of the site indexes; *B*, relation of the standard deviation of volume to age; C, relation of the coefficient of variation of volume to age.

The sample-plot data are next sorted according to site-index classes and entered on a form such as table 2, which shows a partial listing of the data for red gum, site index class 70–74 feet. In column 5 are given the estimated volumes of the plots according to plot age, taken from figure 3, *A*. The residuals are given in column 6. In column 7 are the estimated standard deviations according to age as read from figure 3, *B*, and in column 8 the ratio of the residual to the standard deviation. For each site-index class the average site index and the average of the corresponding standard units are then computed. These standard units are then plotted on site index, and a freehand curve fitted to the points (fig. 4, *A*), the curve defining the deviation of the volume curve of any site index from the

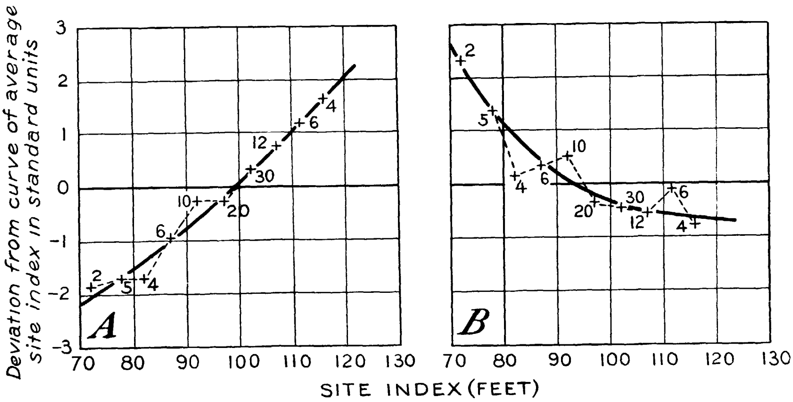

FIGURE 4.—The partial regression of yield (in units of the standard deviation at any age) on site index for *A*, volume in cubic feet, and *B*, number of trees.

volume curve for the average of the site indexes, in standard units of volume at a particular age. It is converted to units of volume by multiplying by the standard deviation for the age.

TABLE 2.—*Form of compilation and calculations preliminary to the determination of the net regression curves of volume in cubic feet on age*

Site-index class (feet)	Site index	Age	Volume per acre: Actual	Volume per acre: Curved (for the average site index)	Volume per acre: Actual minus curved	Standard deviation at indicated age	Actual minus curved ÷ standard deviation
	Feet	*Years*	*M cu. ft.*	*M cu. ft.*	*M cu. ft.*	*M cu. ft.*	*Standard units*
70–74	70	31	1.18	3.95	−2.77	1.02	−2.71
	74	56	3.37	5.92	−2.55	1.46	−1.75
Sum	144						−4.46
Average	72						−2.23

To summarize, three relationships have been established: (1) The expression of volume for the average of the site indexes, in terms of age (fig. 3, *A*) represented by

$$V = f_1(A)$$

in which V is volume, f_1 is its expression in terms of age (A) for the average of site indexes; (2) the standard deviation of volume in terms of age (fig. 3, B), or

$$\sigma_V = f_2(A)$$

in which σ_V is the standard deviation of volume, and f_2 is its expression in terms of age (A); and (3) from figure 4, A—

$$\frac{v}{\sigma_V} = f_3(S)$$

in which v is the deviation of the volume for a given site index from f_1 (A), σ_V has the same meaning as above, and f_3 is the expression of the ratio in terms of site index (S). The volume for any age and site index may therefore be expressed as follows:

$$V = f_1(A) + \{f_2(A) . f_3(S)\}$$

In figures 5 and 4, B, are presented the set of curves needed to express number of trees per acre in terms of age and site index. It is preferable, in the case of this variable, to plot both the average number of trees and the standard deviation on logarithmic scale, as the same relative accuracy is thus assured at all ages.

As stated above, the methods now in use for arriving at the yields of the partial stands have proven workable and satisfactory. These methods are based upon the relation of one of the following values of the partial stand to the average diameter breast high of the entire stand: (1) The ratio of the partial stand yield to the entire stand yield, such as the ratio of the basal area of the trees 6.6 inches d. b. h. and larger to the basal area of the entire stand, or the ratio of board-foot volume in the trees 12.6 inches d. b. h. and larger to the cubic-foot volume of the entire stand; (2) the difference between the partial stand yield and the corresponding yield of the entire stand, such as the difference between the average diameter breast high of the trees more than 12.5 inches and the average diameter breast high of the entire stand.

The yields for the partial stand are calculated by applying the proper ratio or difference to the particular yield of the entire stand which had served at the base of the ratio or difference, by reference to the average diameter breast high of the entire stand.

While this method is subject to a theoretical imperfection in that the ratios or differences as related to the average diameter breast high of the entire stand have not been proven to be completely independent of age and site quality, it has two important practical advantages: (1) Proper weights may be readily assigned to ratios which are correlated with only a single variable; (2) the correlation of ratios, or differences, with average diameter breast high serves to prevent absurdities which may follow the fitting of yield curves independent of one another to age and site quality.[5]

[5] It is obvious, for instance, that the average diameter breast high of the trees 6.6 inches and larger cannot be less than the average diameter breast high of the entire stand; that the ratio of the basal area of a partial stand to the basal area of the entire stand cannot exceed 1.

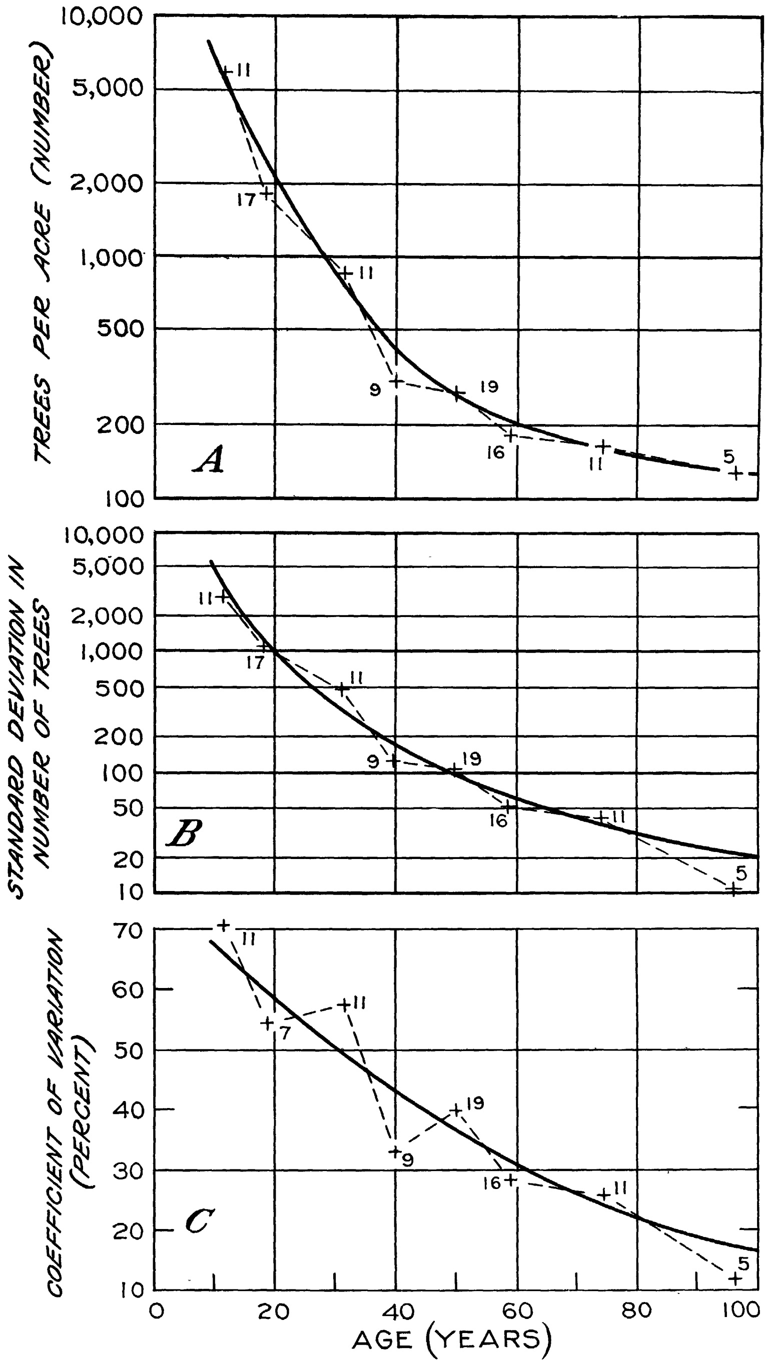

FIGURE 5.—Steps in the correlation of number of trees with age and site index: *A*, Relation of number of trees per acre to age for the average of the site indexes; *B*, relation of the standard deviation of the number of trees to age; *C*, relation of the coefficient of variation to age.

STAND TABLES DEFINED BY THE PEARL-REED POPULATION GROWTH CURVE

The value of an objective method of stand-table construction is apparent. It should be flexible enough to maintain the inherent variation of timber stands, but it should not necessitate laborious computations in application. The search for such a method led to the Pearl-Reed population growth curve (*6*). This curve is of the form

$$y=c+\frac{k}{1+me^{f(x)}} \tag{1}$$

where y is the population in a restricted geographical area, c is the lower asymptote or the population at the beginning of a cultural epoch, $(k+c)$ is the upper limiting asymptote of population, m is an

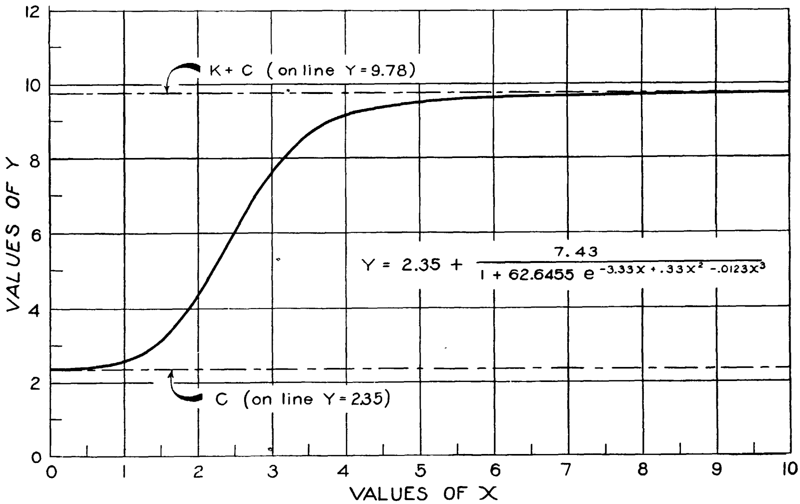

FIGURE 6.—An illustration of the Pearl-Reed population growth curve.

arbitrary constant, e is the base of the Naperian logarithms, x is the date, and $f(x)$ is of the form $b_1x+b_2x^2+---+b_nx^n$. Figure 6 is an example of this type of curve.

A special case occurs when $c=o$ and $k=100$. If, under these conditions, y is defined as the number of trees in an even-aged timber stand whose diameters are less than a given limit x, expressed in percentage of the total number of trees in the stand; then the curve of equation (1) closely resembles the cumulative frequency curve of a typical forest-stand diameter distribution (fig. 7).

In an analysis of a curve of this type it is convenient to use the calculus method. A reader not interested in the theory may pass over the subsequent discussion as far as equation (2) without loss of continuity.

The curves of figure 7 approach the ordinates $y=0$ and $y=100$ very slowly; hence these ordinates may be considered asymptotes to the curves. This suggests the condition—

$$\frac{dy}{dx}=(100-y)y\ f(x)$$

for which $\frac{dy}{dx}$ vanishes at $y=0$ and at $y=100$. To facilitate the integration, the right-hand member may be divided by -100. This operation does not change the values of y when $\frac{dy}{dx}=0$. Accordingly,

$$\frac{dy}{dx}=\frac{(100-y)y}{-100}f(x).$$

Separating variables, we have—

$$\frac{-100dy}{(100-y)y}=f(x)dx.$$

Adding and subtracting $\frac{ydy}{(100-y)y}$, this is reduced to—

$$\frac{-dy}{100-y}-\frac{dy}{y}=f(x)dx.$$

Integrating—

$$\int\frac{-dy}{100-y}-\int\frac{dy}{y}=\int f(x)dx+a$$

or

$$\log\left\{\frac{100-y}{y}\right\}=f'(x)+a.$$

If we define

$$f'(x)=b_1x+b_2x^2+b_3x^3+\text{ - - - - }+b_nx^n$$

then

$$\log\left\{\frac{100-y}{y}\right\}=a+b_1x+b_2x^2+\text{ - - - - }+b_nx^n. \tag{2}$$

The curve is most easily fitted to data in the form of equation (2). Its exponential form is—

$$\frac{100-y}{y}=e^{a+b_1x+b_2x^2+\text{ - - - }+b_nx^n}$$

or

$$y=\frac{100}{1+e^{a+b_1x+b_2x^2+\text{ - - - }+b_nx^n}} \tag{3}$$

and this is identical with the Pearl-Reed curve when $c=0$, $k=100$, and $m=e^a$.

In order to determine the number of terms in x necessary to fit equation (2) satisfactorily to even-aged stands as encountered, coordinates were read from a Pearson type III curve, the β_1 and β_2 coefficients of which were 1.73 and 4.96 respectively; and the following equations fitted successively:

$$\log\left\{\frac{100-y}{y}\right\}=a+bx$$

$$\log\left\{\frac{100-y}{y}\right\}=a+b_1x+b_2x^2$$

$$\log\left\{\frac{100-y}{y}\right\}=a+b_1x+b_2x^2+b_3x^3$$

The first two proved inadequate but the third followed the data closely. This is deemed sufficient reason to apply the last form to the red gum distributions.

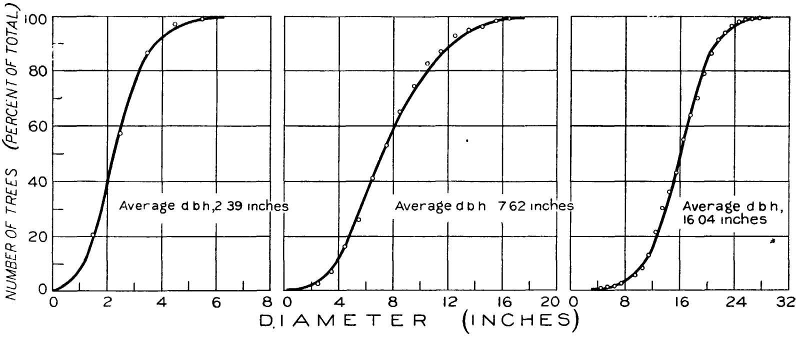

FIGURE 7.—Comparison of actual with calculated cumulative frequencies in stands of small (*A*), medium (*B*), and large (*C*) timber.

APPLICATION TO INDIVIDUAL STANDS

The plots were therefore sorted according to average diameter (by basal area) into groups with ½-inch class intervals. A combined stand tally for each group was then computed, showing the frequency of trees in each 1-inch diameter class from which the arithmetic average of the diameters was calculated. The frequencies of successive diameters, starting with the lowest, were then accumulated, and the cumulative values expressed as percentages of the total frequency. Table 3 shows the form of this summary, a line for each group.

TABLE 3.—*Form of summarizing field data on forest stands*

Average stand diameter class by basal area (inches)	Arithmetic average diameter breast high	Plots	Cumulative frequency percentages (y) for diameter class limits (upper), inches (x)							
			1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5
	Inches	*Number*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
2.5–2.9	2.39	5	20.6	57.4	86.8	97.0	99.0	99.6	100	--------
3.0–3.4	2.93	3	10.8	46.0	72.0	87.1	95.0	98.1	99.5	100

Defining y as the cumulative frequency to the upper limit of any diameter class, and x as the corresponding upper limit itself, a curve of the form of equation (2) in which—

$$\log\left\{\frac{100-y}{y}\right\}=a+b_1x+b_2x^2+b_3x^3$$

was fitted to the data of each group by the method of least squares, the constants a, b_1, b_2, and b_3 being derived from the solution. In this work the y values were given unit weight and the x values were assumed to be without error.

The actual data of each group and the derived curve were then plotted on the same graph. Three of these comparisons are shown in figure 7. After completing this work for each of the 27 groups, the constants a, b_1, b_2, and b_3 were plotted over the arithmetic average diameter breast high of the group as in figure 8.

ADJUSTMENT OF THE CONSTANTS

The distribution constants plotted in figure 8 should be adjusted in such a way that they progress smoothly with increase in average diameter breast high and also that they be consistent with one another. To effect the required harmonization a method of successive approximation was adopted which permits of the adjustment of two of the constants at a time, through the following relationships:

$$\left.\begin{aligned} &na+b_1\Sigma x+b_2\Sigma x^2+b_3\Sigma x^3=\Sigma\log\left(\frac{100-y}{y}\right) \\ &a+b_1M_d+b_2M_d^{\,2}+b_3M_d^{\,3}=0 \end{aligned}\right\} \qquad (4)$$

in which n is the number of diameter classes in an actual distribution and M_d is the median diameter breast high of the computed curve. The first equation is simply one of summation over all diameters of an actual distribution. The second equation, based upon the definition of the median, is simply that $y=50$ when x=median diameter breast high; that is—

$$\log\left(\frac{100-y}{y}\right)=\log\left(\frac{100-50}{50}\right)=\log 1=0$$

at median diameter breast high. Accordingly, the median diameter was calculated from each curve. The relation of the median diameter to the arithmetic average diameter itself is shown in figure 9. By means of this relation the median diameter breast high for the second of equations (4) is determined.

Since the data of figure 8 define trends of the constants b_2 and b_3 that are obviously more definite than those of a and b_1, freehand curves representing second estimates of the former two constants were drawn as shown. The curve values of each group were then substituted for b_2 and b_3 in equations (4), and the corresponding medians taken from figure 9, as stated above. The simultaneous solution of equations (4) for each group thus affords second estimates of the constants a and b_1 which have well-defined trends and considerably less variation than the first estimates; these were used in drawing the trend curves shown in the upper half of figure 8.

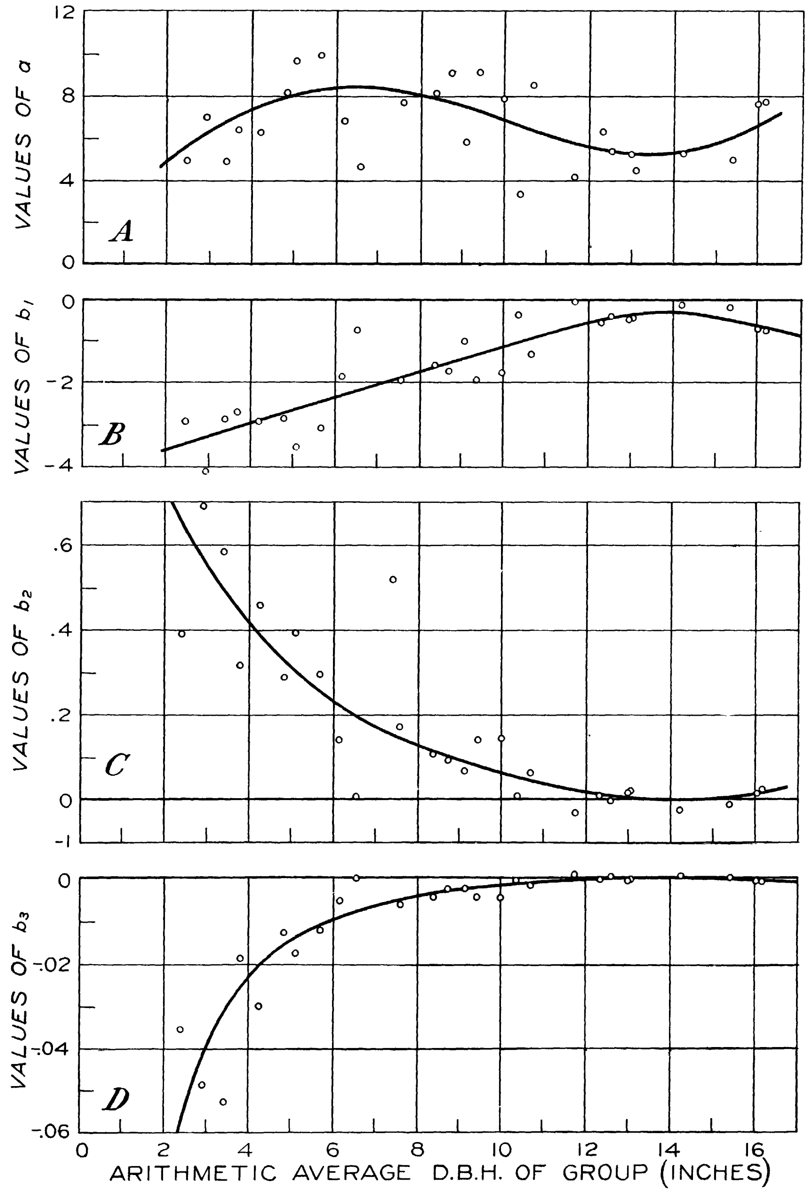

FIGURE 8.—Steps in the adjustment of cumulative frequency constants. The plotted points, called first estimates, were calculated for each group. Freehand curves through b_2 and b_3 (*C*, *D*), called second estimates of these constants, were used to calculate second estimates of a and b_1 (*A*, *B*); through the latter freehand curves were drawn, and these are compared with the first estimates of a and b_1 (*A*, *B*).

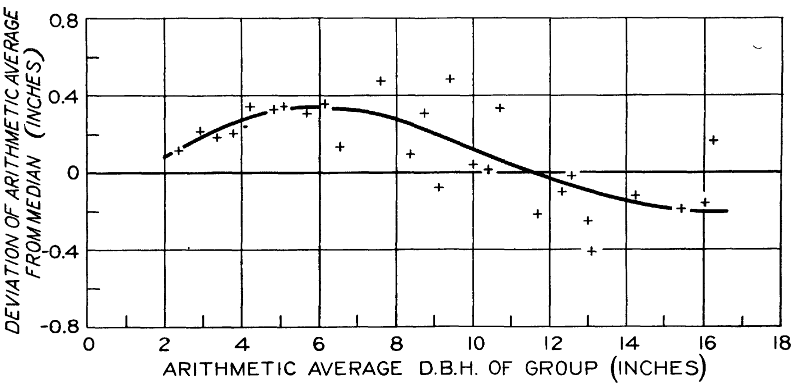

Figure 9.—The deviation of the arithmetic average diameter breast high from median diameter breast high as related to the arithmetic average diameter of each group.

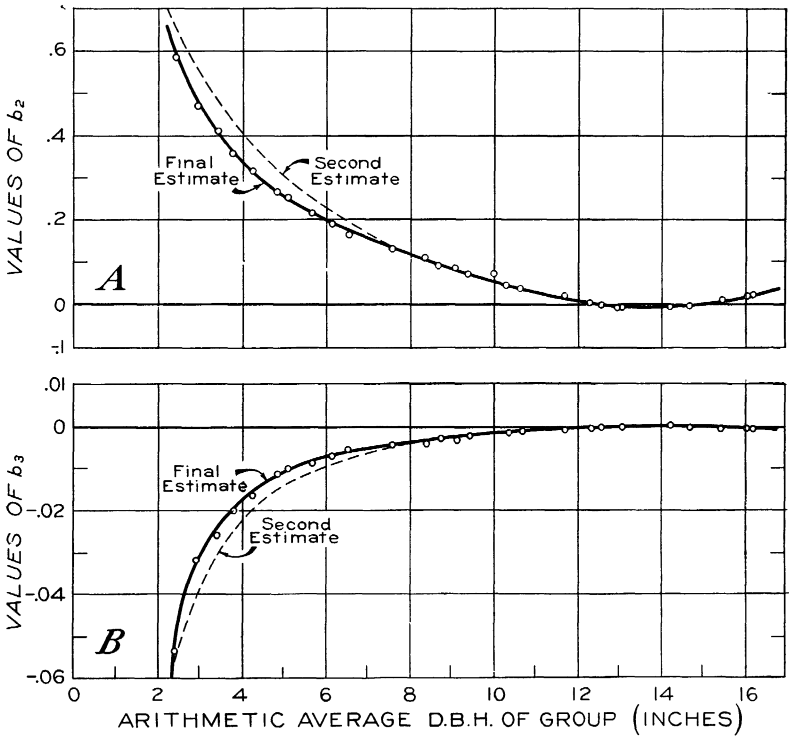

Figure 10.—Final adjustment of cumulative frequency constants b_2 and b_3, at *A* and *B* respectively.

As a check, the new curve estimates of a and b_1 were substituted in equations (4), and new values of b_2 and b_3 calculated. In figure 10 these are compared graphically with the previous estimates. Slight changes were made as indicated.

In order to retain three figures for the numerical values of b_2 and b_3, the curves of figure 10 were redrawn on semilogarithmic paper, with breaks, of course, as b_2 becomes negative and b_3 approaches zero. These final adjusted constants permit the calculation of such cumulative stand tables as may be desired, by substituting them in equation (2) or (3). One for each inch of average diameter breast high by basal area is presented in figure 11. Since the constants are expressed in terms of the arithmetic average diameter of the 27 original groups (figs. 8 and 10), a subsidiary curve of the latter on average diameter by basal area was first constructed, and from this, the arithmetic average diameter corresponding to each whole inch of average diameter by basal area was read, and the associated constants taken from figures 9 and 10 as redrawn on logarithmic scale.

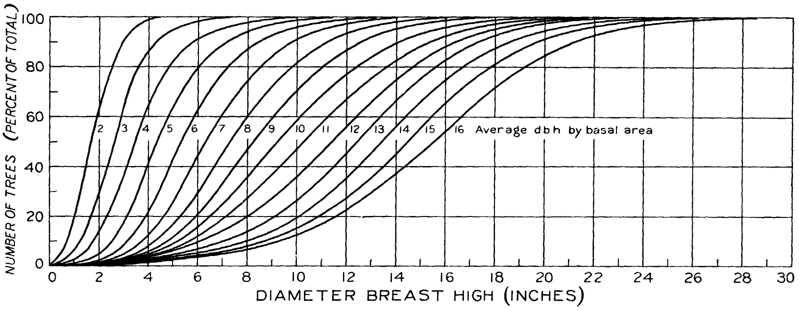

FIGURE 11.—Cumulative frequency percentages in number of trees by diameter-breast-high classes for red gum.

SUMMARY

A method of constructing normal-yield tables is described which does not presuppose, as the Bruce-Reineke method does, that the coefficient of variation of yield of the entire stand is the same at all ages. It is shown, using normal stands of red gum as examples, that the coefficient of variation for height of dominant stand, for volume, and for number of trees is dependent upon age of stand.

The basis of the method is that the relation of the standard deviation, or the coefficient of variation of yield, to stand age determines the form of the growth curve of any site index from the growth curve of the average site index.

Stand tables are constructed by application of the Pearl-Reed population growth curve to cumulative frequency distributions of red gum by diameter-breast-high classes.

A method of harmonizing the curves through adjustment of the descriptive constants two at a time, by successive approximation, is described. The work is less laborious than the use of the Gram-Charlier series and is more objective than the strictly graphic methods of constructing stand tables.

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