Critical Thinking

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A Remark on Spelling

Observant readers are likely to notice that throughout this book I use the spelling 'premiss' rather than 'premise', despite the fact that the latter is the more common American spelling. In defense of the former spelling I offer the following, from Charles S. Peirce.

> Since somebody may think that I write *premiss* instead of *premise* from negligence, may I be permitted to say that desperately negligent as I am of non-logical matters, I endeavor to attend to all the minutiae of logic. The word *praemissa* as a substantive meaning a premiss came into Latin very late and was never very common. Consequently, the English word was for a long time little used; but when it was used, it was always spelled *premiss*. But when it became more common so as to be written by persons of insufficient learning, it was confused with another word, the legal word, generally used in the plural, *premises*. This word is simply a French legal adjective meaning "aforesaid," and commonly used in the phrase "les choses premises." It thus passed into English in its plural form; and this plural form masked its adjectival nature, so that the unlearned did not know what it meant.[1]

In English the word 'premises' has come to mean a piece of land, including the buildings and "appurtenances" that occupy it. The word is frequently used in such legalistic phrases as 'occupy the premises' and 'vacate the premises'. I imagine that the custom on legal documents—eviction notices and the like—was to begin with a description of a certain property, and end with a phrase such as 'The resident is hereby ordered to vacate the *aforesaid*'. With the French word used regularly in place of 'aforesaid', one would soon get the idea that 'premises' was a lawyers' term of art meaning 'real estate'. In any case, Peirce's point (and mine) is that the word 'premise' has nothing to do with logic; and it is only by an unfortunate coincidence that the logical term 'premiss' has the same pronunciation.

Certain technical terms—notably the technical names for the quantification levels, 'Particular', 'Common', 'Majority', 'Predominant', and 'Universal'—are also ordinary words in standard English. This fact occasionally creates problems. To avoid ambiguity I adopt the convention of capitalizing the first letter of these words when they are used in their technical sense. Hence, 'a Particular proposition' means a proposition that employs the quantifier 'some'; while 'a particular proposition' means a given proposition (of unspecified type).

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[1]Charles S. Peirce, MS 75, an unpublished manuscript dated 1902.

INTRODUCTION

The Grammar of Argumentation

> Few persons care to study logic, because everybody conceives himself to be proficient enough in the art of reasoning already. But I observe that this satisfaction is limited to one's own ratiocination, and does not extend to that of other men.[1] – Charles S. Peirce

As Peirce says, we all imagine that we are already sufficiently skilled in the art of reasoning, and therefore most of us see no reason to make a special study of the subject. And, to some extent, we are right. We begin learning how to think the moment we are born (perhaps even before), and by the time we have reached college age, we are quite accomplished at it. Indeed, anyone who can successfully find his shoes in the morning has achieved considerable expertise in the art of reasoning. Finding ones shoes requires all of the basic skills involved in critical thinking: generalizing from past experience, the formulating of reasonable hypotheses, the deduction of consequences that would follow from various hypotheses, and the testing of hypotheses through experimentation.

But, notice that there is a distinction between knowing how to reason and being able to explain how reasoning is done. In much the same way, there is a distinction between being able to digest food and being able to describe the chemical and biological processes involved in digestion. Understanding the chemistry of digestion will not make you better at digesting food. Likewise, knowing how to describe the structure of rational arguments will not necessarily make you a better thinker. A critical thinking class is unlikely to help you find your shoes. It will only help you to understand the process that you engage in whenever you set out to find your shoes.

On the other hand, it can't hurt. Knowing something about the chemistry of digestion may not improve the way in which enzymes interact with nutrients; but, it may make us better at selecting a healthy diet. Likewise, knowing how we reason may help us extrapolate our methods to more difficult and important problems than just the finding of shoes. It may be true that we begin learning how to think the moment we are born, but it does not follow that we have nothing further to learn. Reasoning, unlike digestion, is a skill, and it is always possible to improve. Indeed, as Peirce suggests, we may not be as good at reasoning as we think we are. The purpose of reasoning is to find the truth. If we were all as good at reasoning as we think we are, there would be a lot fewer differences of opinion in the world.

Learning how to describe the process of reasoning may be one way in which we can begin to improve our skill in reasoning, and so be better at finding the truth.

Terms, Propositions, and Arguments

I am sorry to have to tell you that the study of logic often feels a lot like the study of grammar; and, if few persons care to study logic, even fewer care to study grammar. However, it

cannot be helped. Thinking is done by means of what Peirce calls "signs," by which he means anything that *refers* to something. We are chiefly interested in signs that can take the form of utterances, specifically, utterances that have "semantic content," or meaning, such as words and groups of words. Grunts, moans, and disconnected syllables may also be considered utterances, but they do not *refer* in the way that words do, so they fall outside our study.

Utterances that have semantic content occur at three levels of complexity. Words, of course, have meaning in their own right, but we can also put words together to form more complex units of meaning called sentences. The rules by which words are put together to create sentences is what we mean by 'grammar'. Hence, logic inevitably involves a certain amount of grammar. But, just as words can be put together to form sentences, so sentences can be put together to form a still more complex level of meaning called the argument. The rules by which sentences are put together to form arguments is what we mean by 'logic'. In a sense, logic is the "grammar" of argumentation; although, since logic is the broader study, requiring consideration of all three levels of meaning, it is really more accurate to call grammar "the logic of sentences."

Rather than talking about 'words' and 'sentences', logicians prefer to talk about 'terms' and 'propositions', as well as arguments. The distinction is subtle, but important. Some objects that we talk about are not individual *things* at all but are, rather, **ideal objects**. When we talk about ideal objects we are not talking about a mere physical manifestation, but about an "idea" which may be represented physically. For example, the letter 'e' is an ideal object. The letter 'e' occurs eight times in the previous sentence, twice in the word 'example', once in the word 'the', twice in 'letter', once by itself in quotation marks, and once each in the two words 'ideal' and 'object'. But in another sense, there is only *one* letter 'e', namely the fifth letter of the alphabet. An ideal object is the unity behind all of its various manifestations. Here are some examples.

TERMS

(a)	'this particular piece of paper.'	*Although there are two different phrases here, nevertheless, only one idea is being expressed. Hence there is only one term.*
(b)	'the piece of paper on which this is written.'	

PROPOSITIONS

(a')	'This piece of paper is white.'	*Although there are two different sentences here, nevertheless, each expresses the same combination of ideas. Hence there is only one proposition.*
(b')	'The piece of paper on which this is written is white.'	

ARGUMENTS

(a")	'This particular piece of paper is white. Hence some paper is white.'	*Although these two passages are worded differently, each one offers the same evidence in support of the same claim. Hence there is only one argument.*
(b")	'Some paper is white, since this specific piece of paper is white.'	

While words and sentences, like letters, are ideal objects, they are more concrete than terms and propositions. That is, two words, such as "cats" and "felines," must be regarded as different individual words; yet, each expresses the same term, since the words are synonyms. Likewise, different individual sentences may be used to express identically the same proposition, and by extension, different individual sentences may be used to construct identically the same argument. Since there is often more than one way to express the same idea, you should avoid relying upon identical words and phrases in your analyses of arguments. The exercises in this book tend to make things easy, by using precisely the same wording every time the same term is expressed. Real authors of arguments do just the reverse. Since using different words and phrases to express the same idea tends to make an argument more interesting to read, real authors tend to use different wording even when they have the same idea in mind. That is one reason that arguments encountered in "real life" are usually more difficult to analyze than those encountered in logic textbooks. (In fact re-wording for the sake of variety is not always a good idea. A certain amount of repetition will often make your writing clearer, and give it a feeling of unity.)

Complex Terms and Degenerate Arguments

Terms, propositions, and arguments differ from each other both with respect to their structure and with respect to their function.

Definitions by *function*:	**Definitions by *basic structure*:**	EXAMPLES:
A **term** presents a topic for discussion, but says nothing about the topic so identified.	A **term** is a simple, or unanalyzed, utterance. (Being complex sequences of sounds, they are obviously open to *some* sort of analysis, but that analysis falls outside the scope of logic.)	*Simple Term:* 'cats'
A **proposition** urges some fact upon the listener, but offers no justification of the fact so urged.	A **proposition** is a pair of terms, called the **subject** and the **predicate**, related in such a way that the topic referred to by the predicate is also referred to by the subject, to a greater or lesser extent.	*Simple Proposition:* 'Cats are animals.'
An **argument** manifests the truth of one fact by connecting it to another fact that is presumably already taken to be true.	An **argument** is a pair of propositions, called the **premiss** and the **conclusion**, related in such a way that the fact urged by the conclusion is also urged by the premiss, to a greater or lesser extent.	*Simple Argument:* 'Cats are animals; so some animals are cats.'

Now the picture becomes complex. To some extent it is possible to mix and match structure with function, using an utterance that has the structure of a proposition to perform the function of a term, or using an utterance that has the structure of an argument to perform the function of a proposition. But this is possible only to some extent. Structure places some limits

on possible functions. Consider the following image. It is possible to break a window by throwing a rock through it, it is possible to break a window by throwing a typewriter through it, and it is even possible to break a window by throwing a laptop computer through it. Any of these three instruments will serve, since breaking a window is not a very complex activity. It is, of course, also possible to use a typewriter to write a letter, but it is not possible to use a rock to write a letter. Writing a letter is a more complex activity, which requires a more complex or highly structured instrument. You may use a typewriter to do a rock's job, but you cannot use a rock to do a typewriter's job. Likewise, you can use a laptop computer to type a letter (assuming you can send the document to a printer), but you can also use a laptop computer to check your email, make a spreadsheet, go online, etc. A typewriter can do none of those things. Again, you can use a computer to do a typewriter's job, but you cannot use a typewriter to do a computer's job. And, of course, you can use a computer to do a rock's job, but you cannot use a rock to do a computer's job.

Similarly, functioning as an argument is a very complex activity, and it can only be done by highly structured units of discourse. By contrast, functioning as a term is a very simple activity, and it can be done by pretty much any unit of discourse at all. Propositions fall somewhere in the middle.

The situation is described by the following matrix:

	TERM	PROPOSITION	ARGUMENT
	(as viewed according to function or use in context)		
TERM	*Simple term:* 'cats'		
PROPOSITION	*Complex term (Degenerate proposition):* 'black cats'	*Simple proposition:* 'Some cats are black.'	
ARGUMENT *(as viewed according to structure)*	*Complex term (Degenerate argument):* 'a cat's being bad luck because of its being black'	*Complex proposition (Degenerate argument):* 'If some cats are black, then they are bad luck.'	*Simple argument:* 'Black cats are bad luck. Some cats are black, so they are bad luck.'

A simple utterance, such as 'cats' or 'black', can present a topic for discussion. But such an utterance cannot be used to urge a fact upon a listener. It simply does not have a sufficiently

complex structure to permit it to *assert* something. Likewise, a pair of terms related by predication can urge a fact upon a listener, but it cannot appeal to the connectedness of one fact to another in order to defend a further fact. Again this is because it does not have a sufficiently complex structure.

But going the other direction, there is no reason why a related pair of terms related by predication should not be used merely to present a topic for discussion. For example, the utterance 'black cats' is a pair of terms related by predication, i.e. it predicates 'black' of 'cats'. But the utterance is not being used to *urge the fact* that some cats are black. It merely offers black cats to us as a topic for further discussion. An utterance that has the structure of a proposition, but is used as if it were a mere term, may be regarded as a 'degenerate proposition', although for our purposes, it is more useful to call such degenerate propositions 'complex terms'.

Similarly, a group of propositions related by an inference has a sufficiently complex structure that it could be used for all three purposes. Besides genuine arguments, there will naturally be two types of "degenerate" arguments. The first will be utterances having the structure of an argument that merely urge a fact upon the listener, and the second will be utterances having the structure of an argument that merely present a topic for discussion.

There are two important practical consequences to all of this:

(1) Terms (meaning utterances that have the function of terms) can be any size whatsoever. A term is not restricted to a single word, but may involve complex phrases. Moreover, notice that terms are not restricted to naming concrete objects, but may refer to highly abstract concepts as well.

(2) Some propositions have the structure of arguments, and may, therefore, be mistaken for arguments, if one is not careful.

Deduction, Induction, and Retroduction

The ancient Greeks were the first to examine the basic structures of rational argumentation. Aristotle laid out the basic structure of argumentation in a book called *Prior Analytics*. Aristotle's description of reasoning was accepted through the middle ages as pretty much the final word on the subject. However, Aristotle's description of argumentation really describes only one kind of argument: the kind that we call deduction. At the end of the middle ages, Francis Bacon realized that the reasoning used by the scientists of his time were not well described by Aristotle's deductive syllogisms. Bacon proposed, in a book called *Novum Organum*, that a new kind of logic, induction, was as important as deduction. At the end of the 19th Century, Charles S. Peirce further expanded our understanding of the basic patterns of reasoning by proposing that there are actually three patterns of rational thought: deduction, induction, and a third pattern that he (eventually) called retroduction.

Peirce's belief that there are three types of reasoning was based, in part at least, on the idea that, using syllogisms to represent forms of argumentation, there are precisely three logically possible forms that argumentation can take. To use Peirce's example, suppose we discover a room in which there are several bags of beans of various colors. Suppose we also notice a handful of beans lying on a table. We may draw various conclusions, depending upon what we know about the situation.

Deduction—In a deduction, we begin with a RULE, or general statement, and a CASE, which is a class or instance subsumed under this rule. The conclusion is an OBSERVATION of fact, which Peirce called the "result." (I prefer the term "observation," since it more accurately describes the characteristics of this statement in all three types of argumentation.)

RULE -	All the beans in this bag are white.
CASE -	The beans on the table are from this bag.
OBSERVATION -	The beans on the table are white.

In a deductive argument, the conclusion follows from the premisses because of the meanings of the connector words used to express the argument—words, such as 'if', 'and', 'not', etc. Connector words do not, by themselves, make any reference to objects; they merely structure propositions (and arguments). Thus, they have meaning only when they occur *with* content-bearing words. In the above example, the argument turns primarily on the meanings of the words 'all' and 'are', which express the concept of class inclusion. Given the agreed meaning of these words, the premisses in effect *stipulate* that the conclusion is true. Thus, in a deductive argument the conclusion cannot be false unless at least one of the premisses is false. This makes deduction the most powerful type of reasoning, but also the least applicable to our experiences.

Induction—In an induction, we begin with an OBSERVATION made upon a sample, or CASE. The conclusion is the generalization, or RULE.

OBSERVATION -	The beans on the table are white.
CASE -	The beans on the table are from this bag.
RULE -	All the beans in this bag are white.

The persuasive power of an inductive argument comes from the fact that the sample is taken as representative of the larger population from which it was drawn. Thus, some care must be taken to *cause* the sample to reflect the population, such as drawing the sample in a random manner, or otherwise making sure that distinct elements of the population are represented in the sample. Errors are, of course, possible, so the conclusion may be false, even if the premisses are both true. The premisses do not stipulate that the conclusion is true, but they do *indicate* that the conclusion is true. Induction is not as powerful as deduction, but it is, in many ways, more important, since it applies directly to our experiences with the world around us.

Retroduction—In a retroduction, we also begin with an OBSERVATION, usually a surprising fact that catches our attention and demands an explanation. The explanation is suggested by a RULE, or general statement, that has something in common with what we have observed. The conclusion is, then, the CASE which explains the surprising observation by fitting it into a general pattern.

RULE -	All the beans in this bag are white.
OBSERVATION -	The beans on the table are white.
CASE -	The beans on the table are from this bag.

A retroductive argument follows because of some sort of similarity, or "concomitance." Any type of concomitance may be used, including (but not limited to) location in space and time, similarity of size, color, material composition, etc. The concomitance could easily turn out to be just an irrelevant coincidence, so retroduction, like induction, may have true premisses and a false conclusion.[2] Indeed, retroduction is the weakest of the three types of reasoning. It cannot even indicate that its conclusion is likely to be true, it can only *suggest* that its conclusion *might* be true. Fortunately human beings have an uncanny instinct for being able to recognize the difference between relevant connections and irrelevant ones. Retroduction may be the weakest form of reasoning, but it is nevertheless important. Deduction and induction both merely draw out the consequences of given premisses, so no new knowledge emerges from them. Only retroduction has the ability to suggest *new* ideas, since—psychologically speaking—the RULE is decided upon at the same time that the conclusion is proposed. Retroduction is the kind of reasoning involved in discovery and invention. We could not get along without it.

Validity and Critique

I have offered a rather unromantic view of logic according to which logic is closely akin to grammar. The business of grammar is to distinguish between well-formed, or grammatical, sentences and ill-formed, or ungrammatical sentences. Similarly, the business of logic is to distinguish between well-formed and ill-formed *arguments*. A well-formed argument is called a **valid** argument; while an ill-formed argument—an argument that does not follow correct form—is called an **invalid** argument.

However, determining whether an argument is valid or invalid has little to do with the *critique* of the argument. That is, the purpose of validity is not to tell whether an argument is good or bad. Rather, the purpose of validity is to tell us which type of reasoning the argument employs. I have worked out criteria for the validity of syllogisms under which any deductively valid argument will also meet the criteria or inductive and retroductive validity. Any inductively valid argument will also meet the criteria for retroductive validity. Hence, an argument that meets the criteria for deductive validity may be said to be a deductive argument; an argument that fails the criteria for deductive validity, but meets the criteria for inductive validity, may be called an inductive argument; and, of course, an argument that fails to be either deductive or inductive may still be a valid retroductive argument.[3] An "argument" that fails to meet *any* criteria of validity is not, strictly speaking, an argument at all. Invalid arguments exist only in the sense in which ungrammatical sentences exist. There are, of course, ungrammatical sentences: student papers are often full of them. But such groupings of words are called "sentences" only by courtesy. In the same way, an invalid "argument" is an argument only by courtesy, in recognition of the author's (unsuccessful) intentions.

Once it has been determined what type of validity an argument possesses, it then remains to be decided whether an argument is good or bad, i.e. whether or not we should permit ourselves to be persuaded by it. In the case of deduction, this is relatively easy: we only need to consider whether or not the premisses are true. With regard to induction and retroduction the critique is more difficult. It is not enough merely to be convinced that the premisses are true; other criteria must also be considered. In any case, only in the case of deduction does the truth of the premises *force* or *compel* us to accept the truth of the conclusion. The true premises of an induction *indicate* that a conclusion is true, but indications—evidence—can be affected by extraneous forces. A weathervane might point east, leading us to infer that the wind is blowing out of the west, although in fact the wind is out of the north. Our conclusion is false. However, the

assertion, "The weathervane is pointing east," remains true, even after we discover that the weathervane is rusted in place and *always* point east. So it may be wise to trust the conclusions presented to us by inductive evidence, but we should never feel *compelled* to accept them.

Nearly all logic or critical thinking texts refer to what are called "formal fallacies," that is, to forms of reasoning that fail to be deductively valid. One form is called Affirming the Consequent, for example, "If it rains there will be clouds in the sky. Look! There are clouds in the sky, so it will rain." In fact, the so-called "formal fallacies" are *valid forms*, but they are valid forms of retroductive reasoning, not deductive or inductive reasoning. The error—or fallacy—in this example is not in its form, but in our failure to recognize the force with which the conclusion should be put forward. Mere clouds hardly *force* us to believe that it will rain. They even fall short of providing *evidence* that it will rain. But they do *suggest the possibility* of rain. This is a valid retroductive argument. When I go out, I may (reasonably) decide to take an umbrella with me, just in case.

Notes

1. Charles S. Peirce, "The Fixation of Belief," *Philosophical Writings*, ed. Justus Buchler, Dover Publications, New York, 1955.

2. This is not, however, enough to establish that retroduction *is* induction, unless one defines induction as any argument in which the premisses could be true while the conclusion is false. But, by this definition, any unrelated pair of statements designated "premiss" and "conclusion" would be an inductive argument. In short, this definition confuses "induction" with the concept of deductive "invalidity." In fact, just as retroduction shares certain characteristics with induction that neither shares with deduction, retroduction shares certain characteristics with deduction that neither shares with induction. Finally, of course, deduction and induction share certain characteristics with each other that neither shares with retroduction.

3. What it is that makes the form of an argument "correct" for its type is discussed in B. E. R. Thompson, "Deductively valid, inductively valid, and retroductively valid syllogisms," *Transactions of the Charles S. Peirce Society*, Vol 54, No 4 (2016). Pp. 611 – 632. However, my more recent work on the validity of non-deductive syllogisms is currently unpublished.

UNIT I: Terms and Propositions

Section 1: Categorical Propositions

In the Introduction I defined logic as the grammar of argumentation. Having defined logic in this way, it stands to reason that we will need to incorporate some of the grammar of sentences into our system. Fortunately we only need the most basic elements of sentence grammar. The purpose of this chapter is to explain the grammatical structure of the type of sentences used in syllogistic logic.

The Ideal Basic Structure

A proposition was defined as a unit of language in which two terms, the subject term and the predicate term, are combined. To analyze the structure of a proposition, we need to locate the two terms, as well as the connector words by which the connection between the two terms is expressed. Let us assume that all sentences—even extremely long and complicated sentences—can be put into the following form:

Quantifier + Subject Term + Copula + [Negative Particle] + Predicate Term

Sentences in this form express categorical propositions. A **categorical proposition** is a proposition that says something about the relation of two classes, or categories of objects, to each other. Specifically, a categorical proposition says whether, and to what extent, one class overlaps with another class.

Syllogistic logic uses *only* categorical propositions. This system of syllogistic logic uses only *some types* of categorical propositions. The following discussions explain which specific forms of categorical propositions may be used in this system.

The Subject Term and the Predicate Term

The subject term and the predicate term must be words or phrases representing categories of "things." Any categories whatsoever are acceptable. We may have simple physical objects, such as balls, rocks, and digital computers. We may also include actions, activities, places, regions, moments of time, and spans of time. We may include compound entities, such as clubs and organizations, football teams, and other entities made up of component entities. We may include abstractions, such as 'peace' and 'justice'. The essential feature of a category is that its members be discrete entities. It must make sense to ask "How many?" even if the answer is, "Infinitely many," or "Only one." (On Plato's view at least, abstract entities, such as 'peace' and 'justice' are categories having only a single member.)

Term Variables and Term Constants

Often it will be convenient to use single capital letters to represent the terms. (For one thing, this practice can prevent writer's cramp.) For example, 'C' could be used to represent the term 'cows', or 'A' could be used to represent 'astronauts who are afraid of heights'. Such letters are called **term constants**, and they are always defined by means of a **key**. Other times we may not have a specific term in mind, and will wish merely to represent the form of a categorical

proposition, independent of any specific term. In such cases we will use the letter 'S' to represent the subject term (whatever it may be) and 'P' to represent the predicate term (whatever it may be). Letters used in this way are called **term variables**. As in algebra, a variable may stand for anything whatsoever, provided that multiple occurrences of the same variable must always stand for the *same* thing (whatever that may be).

The Copula

Copulas are various forms of the verb 'to be'. Copulas may be singular or plural, may take various tenses, and may be in either indicative or subjunctive mood. This system uses only the plural, present tense, indicative copula: 'are'.[1]

The Negative Particle

In English, the negative particle is the word 'not'. The brackets in the above form indicate that the negative particle is optional. Hence, not every categorical proposition has a negative particle.

As explained below, some quantifiers also have negative meaning. Don't confuse negative quantifiers with the negative particle. A categorical proposition may have both. When this occurs, the result is a double negative.

Quantifiers

There are many quantifiers that may be included in a syllogistic logic. These quantifiers include cardinal numbers ('one', 'two', etc.), ordinal numbers ('the first', 'the second', etc.), and purposive quantifiers ('enough', 'too many', etc.), to give a few examples.[2] Certain quantifiers, such as '10.34% of', 'Almost 15% of', 'More than 27% of', etc., express proportions. These may be called **proportional quantifiers**. The extreme ends of the proportional spectrum (the quantifiers 'all', 'none', and 'some') formed the basis of Aristotle's original system of syllogistic logic. Hence the quantifiers, 'all', 'none', and 'some', may be called the **classical quantifiers**. Proportional quantifiers that fall between these extreme ends of the spectrum are known as the **intermediate quantifiers**. Hence the set of classical quantifiers and the set of intermediate quantifiers together make up the set of proportional quantifiers.

This system of logic incorporates only proportional quantifiers. However, we will not start by attempting to comprehend the entire system of proportional quantifiers; nor will we merely limit ourselves to the classical quantifiers. Rather, there is a sub-group of quantifiers that is interesting because the quantifiers in this sub-set can be expressed by single, concise words in English. I call these the **common speech quantifiers**. They form a significantly larger set than the classical sub-set, but one that is still relatively small and manageable. In this system, we will recognize five quantifiers: no, few, most, many, and some.[3]

Interpretation of Quantifiers

Quantifiers can have two possible interpretations:

> A quantifier, Q, takes minimal interpretation if Q represents the smallest intended quantity. 'Q' means 'at least Q (possibly more)'.
>
> A quantifier, Q, takes maximal interpretation if Q represents the greatest intended quantity. 'Q' means 'at most Q (possibly less)'.

Quantifiers that receive minimal interpretation are affirmative in meaning, while quantifiers that receive maximal interpretation are negative in meaning. Of the quantifiers in this system, 'no' and 'few' receive maximal interpretation and are therefore negative in meaning. The quantifiers 'most', 'many', and 'some' receive minimal interpretation, and are therefore affirmative in meaning.

The Ten Categorical Propositions of the Common Speech System

Five quantifiers, with or without the optional negative particle, yields ten logically possible forms that a categorical proposition may take (in this system):

No S are P.	No S are not P.
Few S are P.	Few S are not P.
Most S are P.	Most S are not P.
Many S are P.	Many S are not P.
Some S are P.	Some S are not P.

Some of the sentences on this list have both a negative particle (the word 'not') and a negative quantifier. Such double negatives can be confusing. Therefore, since double negatives cancel in any case, let us adopt the following abbreviations:

All S are P =df No S are not P.

Almost all S are P =df Few S are not P.

Labeling the Categorical Propositions

The **quality** of a categorical proposition refers to whether it is affirmative or negative. The **quantity** of a categorical proposition refers to how much of the subject class is being discussed. There are five levels of quantity (in this system), and two types of quality:

		Quality	
		Affirmative	*Negative*
Quantity	*Universal*	A: All S are P.	E: No S are P.
	Predominant	P: Almost all S are P.	B: Few S are P.
	Majority	T: Most S are P.	D: Most S are not P.
	Common	K: Many S are P.	G: Many S are not P.
	Particular	I: Some S are P.	O: Some S are not P.

The Universal and Particular categorical propositions are called the 'classical' categorical propositions, since they have been studied since the time of Aristotle. The letters A, E, I, and O, as names for the four classical categorical propositions, were taken from the four vowels of the Latin alphabet, which also happen to occur in the Latin words 'affirmo' and 'nego'. The affirmative propositions were named after the first two vowels of 'affirmo', while the negative propositions were named after the first two vowels of 'nego':

A - **a**ffirmo **E** - n**e**go

I - aff**i**rmo **O** - n**o**go

The remaining categorical propositions are named after the consonant stops, i.e. the consonants that are made by stopping the flow of air through the mouth. Consonant stops may be either 'voiced' or 'unvoiced', depending upon whether the vocal cords are used in forming the sound. Affirmative intermediate propositions are named after the unvoiced stops, while negative intermediate propositions are named after the voiced stops. The unvoiced stops also occur in the word that identifies the quantity:

Predominant	**P** - unvoiced labial	**B** - voiced labial
Majo**r**ity	**T** - unvoiced dental	**D** - voiced dental
Kommon	**K** - unvoiced palatal	**G** - unvoiced palatal

Replacing Terms with Constants

It is usually convenient to replace the terms in categorical propositions with term constants. This makes the form of the proposition easier to recognize, and makes the proposition shorter to write. Use a key to indicate which letter is being used to represent which term. Be careful never to use the same letter to represent two different terms.

Example:

Many [astronauts who are afraid of heights] are not [people who get dizzy in weightless conditions].

Many A are not D.	A - astronauts who are afraid of heights. D - people who get dizzy in weightless conditions.

Exercises:
A. In the following categorical propositions, mark off and label the quantifier, subject term, copula, predicate term, and (where one occurs) the negative particle. Replace terms with constants. Provide a key stating what each letter stands for.

1. All apples are fruit.
2. No apples are vegetables.
3. Most apples are red-colored fruit.
4. Some apples are not red-colored fruit.
5. Many red-colored fruit are not apples.
6. No red-colored fruit are green-colored fruit.
7. Some red-colored vegetables are not things that people eat.
8. Few people who cook are people who believe tomatoes to be fruit.
9. Some people who cook are botanists.
10. Almost all botanists are people who believe tomatoes to be fruit.
11. Most people who cook are not botanists.
12. Few people who believe that tomatoes are vegetables are people who believe that tomatoes are fruit.
13. All people who believe that tomatoes are vegetables are people who have never studied botany.
14. Many people who have never studied botany are people who believe that tomatoes are not fruit.
15. Most people who have never studied botany are people who don't really care whether tomatoes are vegetables or whether they are fruit.

B. For each of the above categorical propositions, state its letter name.

Parameters

Every categorical proposition has two terms, and the two terms must be *capable* of overlapping, even if they do not overlap *in fact*. For example, it may be true that there is no overlap between the class 'citizens of Venezuela' and the class 'citizens of Albania', but it is

possible that there might be, since both classes are composed of people. Some countries recognize dual citizenship; but, even if no countries recognized dual citizenship, there is still a common class into which both terms fall. This class is called the **parameter** of the proposition. By contrast, it would never do to use the terms 'citizens of Venezuela' and 'colors of the rainbow' in the same categorical proposition. These are terms of a completely different logical type. There is no common class into which both terms can fall. The parameter of a propositions about 'citizens of Venezuela' and 'citizens of Albania' might be 'people', while the parameter of a proposition about 'colors of the rainbow' might be 'colors', but there is apparently no type of "things" that include both colors and people.

Logicians sometimes use the phrase 'universe of discourse' to refer to the broad class of objects into which all the terms under discussion can fall. This phrase seems to suggest that there are many different alternate realities about which we could speak. Unfortunately the concept of a 'universe of discourse' is much less romantic than that. In fact the phrase is nothing more than a synonym for the word 'parameter'.

The Complement of a Class

Within a given universe of discourse any term has the effect of dividing that universe into two parts: objects that are members of the class to which the term refers, and objects that are not. For example, the term 'citizens of Albania' draws our attention toward people who are citizens of Albania, and away from people who are not. Within the "universe" of people, the term creates a partition. Notice, however, that the term has little or no effect on the universe of colors.

Thus, for any class of objects, there is an opposite class within the same universe of discourse. These two classes are said to be complementary, or it is said that each one is the complement of the other. The **complement** of a class is the class that includes nothing that is a member of that class, and everything that is not.

When using term constants, the complement of any class, C, may be expressed 'non-C'. The complement of a class may occur in a categorical proposition, but when it does it has no effect upon the type of categorical proposition involved. For example, 'Most R are non-D', is a T proposition. It has the form 'Most S are P', even though the predicate term is 'non-D'.

In ordinary language, using the particle 'non-' may not be the best way to express the complement of a class. Consider for example, a class such as 'students who are adept at logic'. What is the complement of this class? The phrase 'non-students who are adept at logic' sounds somewhat peculiar. It suggests that perhaps we are talking about milkmen and grocers (or perhaps even tables and chairs!) who are adept at logic. But presumably our universe of discourse is limited to students. Hence it would be better to express the complement of the class in one of these ways:

(a)	'students who are un-adept at logic'	(using a prefix such as 'un-' or 'in-')
(b)	'students not adept at logic'	(using the word 'not' in the predicate, rather than as a negative particle)
(c)	'students who are poor at logic'	(using an antonym)

Exercises:
A. For each of the following categorical propositions, state the letter name.

1. Few A are non-B.
2. Most non-C are not D.
3. All non-E are non-F.
4. Some G are not H.
5. Many non-I are not non-J.
6. Some K are non-L.
7. Many non-M are N.
8. No O are non-P.
9. Almost all non-Q are R.
10. Most non-S are non-T.

B. For each of the following categorical propositions, replace terms with term constants and state the letter name. Use 'non-' to express negative (or complementary) terms.

1. Some politicians are unpatriotic citizens.
2. Most politicians are not unpatriotic citizens.
3. No non-politicians are dishonest people.
4. Many honest people are non-politicians.
5. Almost all dishonest people are people who are not patriotic.

Correct Categorical Form

Any sentence in categorical form must begin with a quantifier, and contain two distinctly stated terms, one in the subject and one in the predicate. In a categorical proposition, *both* terms must refer to categories of objects, though there need not necessarily be any objects in the category referred to. For example, the sentence 'All unicorns are white animals', expresses a categorical proposition, even though there are no unicorns. The word 'unicorns' refers to a class of objects, even though there are no existing members in that class.

It should be obvious that many sentences are simply not in categorical form. For example, the sentence 'Some fences are red', does not express a categorical proposition, since 'red' is a quality, not a category of objects. Sentences with active verbs, such as 'John gave the ball to Mary', are even more problematic. Nevertheless, the assumption of syllogistic logic is that sentences that are not already in categorical form may be *forced* into categorical form, with a little effort.

To put sentences into categorical form we must (a) see that the predicate is expressed as a noun phrase, and (b) see that the subject is preceded by a recognized quantifier. In this system of syllogistic logic we must also see to it that both terms are plural, since only the plural copula 'are' is permitted. However, this problem usually takes care of itself provided the other two are handled correctly.

Locating Subjects and Predicates

All declarative sentences can be divided into a subject portion and a predicate portion. The first step in forcing sentences with irregular predicates into correct categorical form is to find the subject/predicate break, i.e. the point in the sentence that separates the subject portion of the sentence from the predicate portion of the sentence. The subject portion of a sentence is (usually) the first part of the sentence up to, but not including, the main verb. The predicate portion of the sentence is the main verb plus (usually) everything after the main verb. There is no magic formula for finding the main verb. The only way to find the main verb is to read the sentence with at least some degree of comprehension.

Remember that terms may be of any size. It follows that the subject portion of the sentence and the predicate portion need not necessarily be the same size.

Examples:

All of the astronauts aboard the first flight of the Atlantis / were scientists.
The Queen of Hearts / baked some tarts, all on a summer's day.

While most normal, well-written sentences keep the subject portion of the sentence all in one place, some sentences divide the subject into two sections, and place part of the subject at the end of the sentence. This can occur when the subject portion of the sentence includes a relative clause (i.e. a clause beginning with 'who', 'which', or 'that'). It can also occur when the sentence has an exceptive quantifier (see Section II) that has been split apart (e.g. 'No...but' and 'No...unless'). In such cases, the concluding portion of the sentence is a clause that modifies the subject. Hence that clause is part of the subject for logical purposes, despite its location in the sentence. By the way, subject splitting is generally a poor idea. While you may find instances of it in other people's writing, I recommend that you avoid it in your own writing.

Examples:

All children / are delightful [that are well behaved].
No sentences / are difficult to understand [except those that have relative clauses].

Exercises:
In each of the following propositions, draw a line to indicate where the break between subject and predicate occurs. Indicate if any subject splitting occurs in the sentence.

1. Little Red Riding Hood took a great big basket of goodies through the dark lonely woods to Grandmother's house.

2. Goldilocks, who should have known better than to be breaking into strange houses in the first place, was frightened.

3. Most little girls who carry baskets of goodies to their grandmothers are not persons who deserve to be eaten by wolves.

4. All little girls deserve to be frightened who break into strange houses owned by bears.

5. No little girls are burglars unless they have been badly brought up.

6. Many little girls are delicious, at least to big bad wolves.

7. Big bad wolves think that little girls are delicious.

8. Bears who are hungry and expect to eat a hearty breakfast shouldn't leave their food unattended while they go for a walk.

Putting the Predicate into Categorical Form

Once the subject/predicate break has been located, the second step in forcing sentences into categorical form is to build a proper predicate term by adding three simple words:

1. At the subject/predicate break, insert the copula 'are'.

2. Follow the copula with a dummy word. By a dummy word I mean an extremely general word, such as 'persons', 'beings', 'things', 'objects', 'creatures', 'animals', 'activities', 'events', etc. When the addition of a dummy word is required to force a proposition into categorical form, the dummy word added will always name the parameter of the proposition, so it must name the parameter to which the *subject* term belongs.

3, Follow the dummy word with an appropriate relative pronoun, such as 'who', 'which', or 'that'.

The newly added copula is a connector word, so it does not fall into either the subject term or the predicate term. The predicate term begins with the dummy word. Assuming there are no other complications, the rest of the sentence, from the dummy word to the period, will be included in the predicate term.

Examples:

'No cows eat French fries'.

becomes...

'No [cows] *are* [*animals that* eat french fries]'.

No C are F.	C - cows. F - animals that eat French fries.

'Most cows are larger than most dogs'.

becomes...

'Most [cows] *are* [*animals that* are larger than most dogs]'.

Most C are D.	C - cows. D - animals that are larger than most dogs.

While this method will work on virtually all problematic sentences, it sometimes creates sentences that are more convoluted than necessary. For example, 'Most night students are willing to do extra work', could be made into 'Most [night students] are [students who are willing to do extra work]', but dropping the words 'who are' doesn't change the meaning, and actually creates a smoother sentence. Why not simply say 'Most [night students] are [students willing to do extra work]'? There is nothing wrong with such alternative solutions. Indeed, where they exist they are undoubtedly preferable. The method of inserting 'are things that' has the virtue that it is highly general. It solves a large class of problems with a single rule. But it should not be treated as the *only* solution, or even as the best solution in all cases. All that really matters is that problematic sentences be forced into categorical form without significantly altering their meaning.

Example:

'Most cows are brown'.

becomes...

'Most [cows] are [brown *animals*]'.

Most C are B.	C - cows. B - brown animals.

Exercises:
A. The following are phrases that might appear in the predicate of an English sentence. Identify which of them are already proper categorical terms and which are not. If they are not, add language that will make them into categorical terms.

1. larger than a bread box.
2. ran as fast as he could.
3. home runs scored in the second inning.
4. unimportant, given the present situation.
5. places we visited while we were in Paris.

B. Force each of the following sentences into categorical form. Then replace the terms with appropriate letters, and provide a key explaining what phrase each letter represents.

1. No chess players are impatient.
2. Most baseball players are not patient.
3. Few baseball players hit home runs.
4. Some hockey players are not very good at tennis.
5. Many hockey players are sufficiently proficient at tennis to beat many chess players.
6. Some scientists are unfriendly.
7. Many scientists can't even balance their own checkbooks.
8. Most non-scientists don't understand the theory of relativity.
9. Few non-scientists eat cold French fries for breakfast.
10. No scientists don't occasionally eat cold French fries for breakfast.
11. All sentences can be forced into categorical form.
12. Some sentences cannot be forced into categorical form.
13. All sentences that are not in categorical form should be forced into categorical form.
14. Some sentences that are extremely difficult to force into categorical form are not coherent.
15. Most American businessmen who've hit it rich playing the stock market are greedy.
16. Many greedy American businessmen have not hit it rich playing the stock market.
17. Many American businessmen are not greedy.
18. No American businessmen should be greedy.
19. All human beings have certain rights, including among others the right not to be killed.
20. Almost all fetuses will eventually turn into human beings, provided that no one interferes with their development.

Locating the Quantifier

Sentences in English usually don't begin with a quantifier, and nearly all of them do not begin with one of the standard quantifiers permitted within this system of categorical propositions. Indeed, there are so many ways to express quantification that we will set most of those problems aside for now. However, one quantifier problem is so common and important that it must be dealt with right away. That is the case in which the quantifier is missing altogether. Often this is because the subject of the sentence is an individual. For example, the sentence, "Plato is a philosopher," seems to express a categorical proposition. It asserts that the entity 'Plato' is a member of the class 'philosophers'. But, is 'Plato' a categorical term? That is, can we consider 'Plato' to be a countable category? The answer is yes. 'Plato' is a category of objects that includes only a single member.

Unfortunately, our form permits only the plural copula. So grammatical correctness requires that the subject term must be made into 'Platos', or 'persons identical to Plato', or 'persons who are Plato'. We can then make the copula into 'are', and make the predicate plural as well. An alternative solution would be to ignore the rules of grammar and allow "Plato are philosophers" to be good enough for our purposes. The result will be ugly no matter what we do.

But what about the quantifier? What proportion of the class 'Plato' are we talking about? Well, there is only one member in the class, so to talk about that one member is to talk about *every* member. Hence the categorical proposition is Universal. We can reword "Plato is a philosopher" as "All persons who are Plato are philosophers," or as "All Plato are philosophers." It is important to understand that this does not mean all persons who are *named* Plato. It means all persons who are identical to the one and only Plato that we are talking about. Even such a sentence as "Smith is a philosopher" can be reworded as "All persons who are Smith are philosophers," since in this case we mean all persons who are identical to the particular Smith referred to in the proposition (of which, of course, there is only one).

Propositions Not Expressed as Declarative Sentences

All categorical propositions take the form of what grammarians call a "declarative sentence." However, there are other types of grammatical constructions: imperative sentences (including commands, instructions, and requests), interrogative utterances (questions), and exclamatory utterances (e.g. 'Wow!' and 'Oh, my!'). Of these, imperatives also express a proposition and interrogatives *can* be used to express a proposition. (Exclamations only express attitude, and, for our purposes, may be ignored.)

1. Imperatives – Any command may be restated as a declarative sentence, usually by adding the words 'You should...' or 'Everyone should...' to the front of the sentence.

Example:

'Listen to me!' *becomes...* 'All [you] are [people who should listen to me]'.

All Y are L.

Y – you
L – people who should listen to me

(Bear in mind that 'you' are an individual, as in the previous section.)

2. Rhetorical Questions – Some questions are just questions: they honestly request information that the questioner needs and does not have. Sometimes, however, a question is asked, not to obtain information, but to elicit a specific reply. That is, the "questioner" wants a specific answer, and knows in advance what he expects to hear. This is known as a 'rhetorical question'. Like imperatives, all rhetorical questions are really propositions in disguise. They can be restated as declarative sentences, and then forced into categorical form in the usual way.

To restate a rhetorical question as a declarative sentence, it is important to know what reply the rhetorical question expects. Typically the expected reply to a rhetorical question is negative when the question is affirmative, and affirmative when the question is negative.

Example:

'Should we listen to him?' expects the reply 'No!'

'Shouldn't we listen to him?' expects the reply 'Yes!'

Hence, when restating these questions as declarative sentences,

'Should we listen to him?' *becomes...* 'We shouldn't listen to him.'

'Shouldn't we listen to him?' *becomes...* 'We should listen to him.'

Rhetorical questions may be used to make your writing style more dramatic and flamboyant. While it is naturally desirable to make your writing more interesting, students tend to overuse rhetorical questions. Frequently rhetorical questions strike a reader as manipulative and offensive. Rhetorical questions become especially offensive when they are used in place of reasoning. Sometimes an author tries to escape from the need to back up his position by stating it as a rhetorical question—as if his position were too obvious to need explicit support! Most writing teachers recommend avoiding rhetorical questions altogether.

Exercises:
Force each of the following imperative and interrogative sentences into categorical form.

1. Vote to re-elect Senator Slugfest!

2. How can you expect those poor little children to understand calculus?

3. Wouldn't you rather be in Hawaii?

4. Attach the faceplate to the frame using ¼ inch machine screws, as shown in the diagram above.

5. Act only on that maxim whereby thou canst at the same time will it to be a universal law.
—Immanuel Kant

Times and Places

'Times' and 'places' are frequently used parameters. It may be especially helpful to use these parameters in rendering propositions that include adverbs of time and adverbs of place.

COMMON ADVERBS OF TIME		COMMON ADVERBS OF PLACE
when	today	where
whenever	yesterday	wherever
now	frequently	here
then	occasionally	there
always	sometimes	everywhere
never		nowhere

Examples:

'It don't rain in Indianapolis in the summertime'.

becomes...

'No times that are summertime are times that it rains in Indianapolis'.

No S are R.

S - times that are summertime.
R - times that it rains in Indianapolis.

'Wherever there's smoke there's fire'.

becomes...

'All places where there's smoke are places where there's fire'.

All S are F.

S - places where there's smoke.
F - places where there's fire.

In Section 4 we will see that adverbs of time can also be used as quantifiers. There is no particular rule for distinguishing how an adverb of time is being used. You must use your own common sense to tell when a word is being used as an adverb of time and when it is being used as an adverbial quantifier.

Exercises:
Force the following sentences into categorical form. In each case use either 'times' or 'places' as the parameter.

1. When the going gets tough, the tough get going.

2. Today is the first day of the rest of your life.

3. Sometimes you just have to make the best of a bad situation.

4. Whenever you need me, I'll be there.

5. Home is where the heart is.

6. Frequently life just isn't fair.

7. Never say never.

8. When in Rome, do as the Romans do.

9. The grass is always greener in someone else's yard.

10. Whenever it rains, it pours.

Notes

[1]Some logicians prefer to use the singular copula 'is'. There is not, so far as I know, any very good reason for preferring one over the other; but I notice that logicians who accept the Boolean position on existence claims (see the discussion of the problem of existence in Section 3) tend to prefer the singular copula. For some reason the singular formulation makes the Boolean interpretation of categorical propositions seem more natural. My use of the plural copula indicates my preference (explained and defended in Section 3) for the Aristotelian interpretation of categorical propositions, instead of the Boolean interpretation.

[2]See Mark Brown, "Generalized quantifiers and the square of opposition," *The Notre Dame Journal of Formal Logic*, Vol. 25, No. 4 (Oct, 1984), pp. 303-322.

[3]Another (non-pedagogical) reason for introducing the five-tiered system first is that these quantifiers were the first to be incorporated into syllogistic logic. See Philip L. Peterson, "On the logic of 'few', 'many', and 'most'," *The Notre Dame Journal of Formal Logic*, Vol. 20, No. 1 (Jan, 1979), pp. 155-179, and Bruce E. R. Thompson, "Syllogisms using 'few', 'many', and 'most'," *The Notre Dame Journal of Formal Logic*, Vol. 27, No. 1 (Jan, 1982), pp. 75-84.