**Posterior Distribution**

The posterior distribution combines the prior information about the unknown parameter θ with the information contained in the observed data to give a composite picture of the final judgments about θ. In order to arrive at a single number as the estimate of θ, it may be necessary to bring in the notion of the loss suffered by the decision maker as a result of estimating the true value θ by the number θ. The crux of the Bayesian approach is the synthesis of the prior distribution and the current sample data into a posterior probability distribution from which all decisions and inferences are made.

**Measure of location of Posterior Distribution**

One can calculate any measure of location to know where the posterior distribution is located on the number line. There are three possible measures of location we will consider: posterior mode, posterior median and posterior mean.

**Posterior Mode**

This is the value of unknown(s) that maximizes the posterior distribution. If the posterior distribution is continuous, it is found by setting the first derivative w.r.t. unknown(s) of posterior density equal to zero, provided that second derivative is negative. We can follow maximum likelihood and use the mode of the distribution.



where  is the mode of the mode of the posterior distribution of .

The posterior mode has some potential disadvantages as measure of location.

* It may lie at or near one end of the distribution, and thus may not be representative of the distribution as a whole.
* There may be multiple local maximums. When we set the derivative function equal to zero and solve, we will find all of them and the local minimums as well.

**Posterior Median**

This is the value that has 50% of posterior distribution below it and 50% above it. It is an excellent measure of location.

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where  is the median of the posterior distribution of . The only disadvantage of the posterior median is that it has to be found numerically.

**Posterior Mean**

The posterior mean is a very frequently used measure of location. It is the expected value, or mean of the posterior distribution. For continuous distribution it is defined as:



Where  is the mean of the posterior distribution of . Similarly for discrete random variable it is defined as:



The posterior mean is strongly affected when the distribution has a heavy tail. For a skewed distribution with one heavy tail, the posterior mean may be quite a distance away from most of the probability (Bolstad, 2004).