The slope of an isoquant is referred to by some economists as the *marginal rate of substitution* (*MRS*).[1] Other authors refer to it as the rate of technical substitution (RTS) or the marginal rate of technical substitution (*MRTS*). This text uses the terminology *MRS*.

The *MRS* is a measurement of how well one input substitutes for another as one moves along a given isoquant. Suppose that the horizontal axis is labeled x_1, and the vertical axis is labeled x_2. The terminology $MRS_{x_1x_2}$ is used to describe the slope of the isoquant assuming that input x_1 is increasing and x_2 is decreasing. In this example, x_1 is the replacing input and x_2 is the input being replaced, moving down and to the right along the isoquant.

Figure 5.3 illustrates an isoquant exhibiting a diminishing marginal rate of substitution. As one moves farther and farther downward and to the right along the isoquant representing constant output, each incremental unit of x_1 (Δx_1) replaces less and less x_2 (Δx_2). The diminishing marginal rate of substitution between inputs accounts for the usual shape of an isoquant bowed inward, or convex to the origin. The shape is also linked to the synergistic effect of inputs used in combination with each other. An input is normally more productive when used with ample quantities of other inputs.

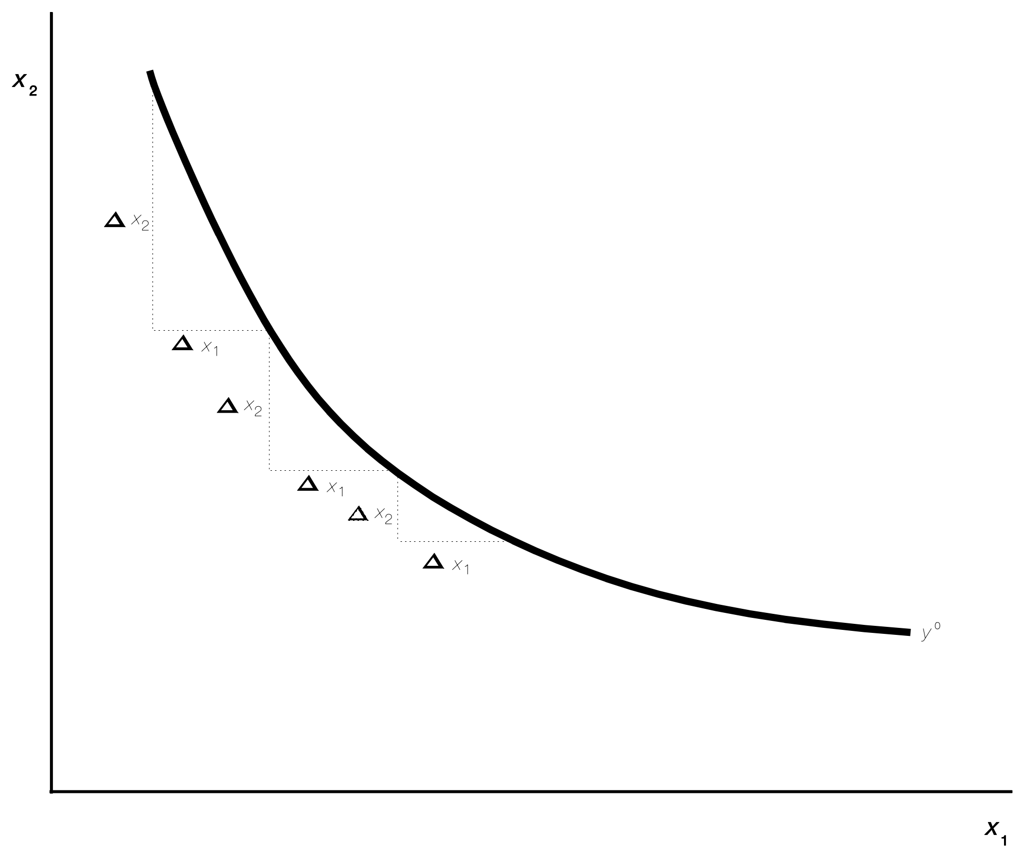


Figure 5.3 Illustration of Diminishing $MRS_{x_1x_2}$

The *MRS* might also measure the inverse slope of the isoquant. Suppose that the use of x_2 is being increased, while the use of x_1 is decreased. The terminology $MRS_{x_2x_1}$ is used to describe the inverse slope of the isoquant. In this example, x_2 is the replacing input, and x_1 is the input being replaced, as one moves up and to the left along the isoquant. The $MRS_{x_2x_1}$ is equal to $1/MRS_{x_1x_2}$.

The slope of an isoquant can also be defined as $\Delta x_2/\Delta x_1$.

Then[2]

(5.5) $$MRS_{x_1x_2} = \Delta x_2/\Delta x_1$$

and

(5.6) $$MRS_{x_2x_1} = \Delta x_1/\Delta x_2 = 1/MRS_{x_1x_2}$$

Isoquants are usually downward sloping, but not always. If the marginal product of both inputs is positive, isoquants will be downward sloping. It is possible for isoquants to slope upward if the marginal product of one of the inputs is negative.

Isoquants are usually bowed inward, convex to the origin, or exhibit diminishing marginal rates of substitution, but not always. The diminishing marginal rate of substitution is normally a direct result of the diminishing marginal product of each input. There are some instances, however, in which the *MPP* for both inputs can be increasing and yet the isoquant remains convex to the origin (see specific cases in Chapter 10).

Figure 5.4 illustrates the isoquants for a three-dimensional production surface derived from a polynomial production function that produces a three-dimensional surface illustrating all three stages of production, the two-input analog to the neoclassical production function employed in Chapter 2. To illustrate, horizontal cuts are made at varying output levels. In panel A, the entire three-dimensional production surface is illustrated. Panels C, D, E and F represent cuts at successively lower output levels. Note that in panel E, the isoquant is concave, rather than convex to the origin. Panel F illustrates an example isoquant beneath the production surface.

Figure 5.5 illustrates some possible patterns for isoquant maps and their corresponding production surfaces. Diagrams A and B illustrate isoquants as a series of concentric rings. The center of the series of rings corresponds to the input combination that results in maximum output or product. In Table 5.1, this would correspond with an input combination of 70 pounds of phosphate and 70 pounds of potash, for a yield of 136 bushels per acre. This pattern results when output is actually reduced because too much of both inputs have been used.

Diagrams C and D illustrate another common isoquant map and its corresponding production surface. The isoquants are not rings; rather they approach both axes but never reach them. These isoquants are called asymptotic to the x_1 and x_2 axes, since they approach but do not reach the axes. A diminishing marginal rate of substitution exists everywhere on these isoquants. These isoquants appear to be very similar to the average fixed-cost curve discussed in Chapter 4. However, depending on the relative productivity of the two inputs, these isoquants might be positioned nearer to or farther from one of the two axes. In this example, more of either input, or both inputs taken in combination, will always increase output. There are no maxima for the underlying production functions.

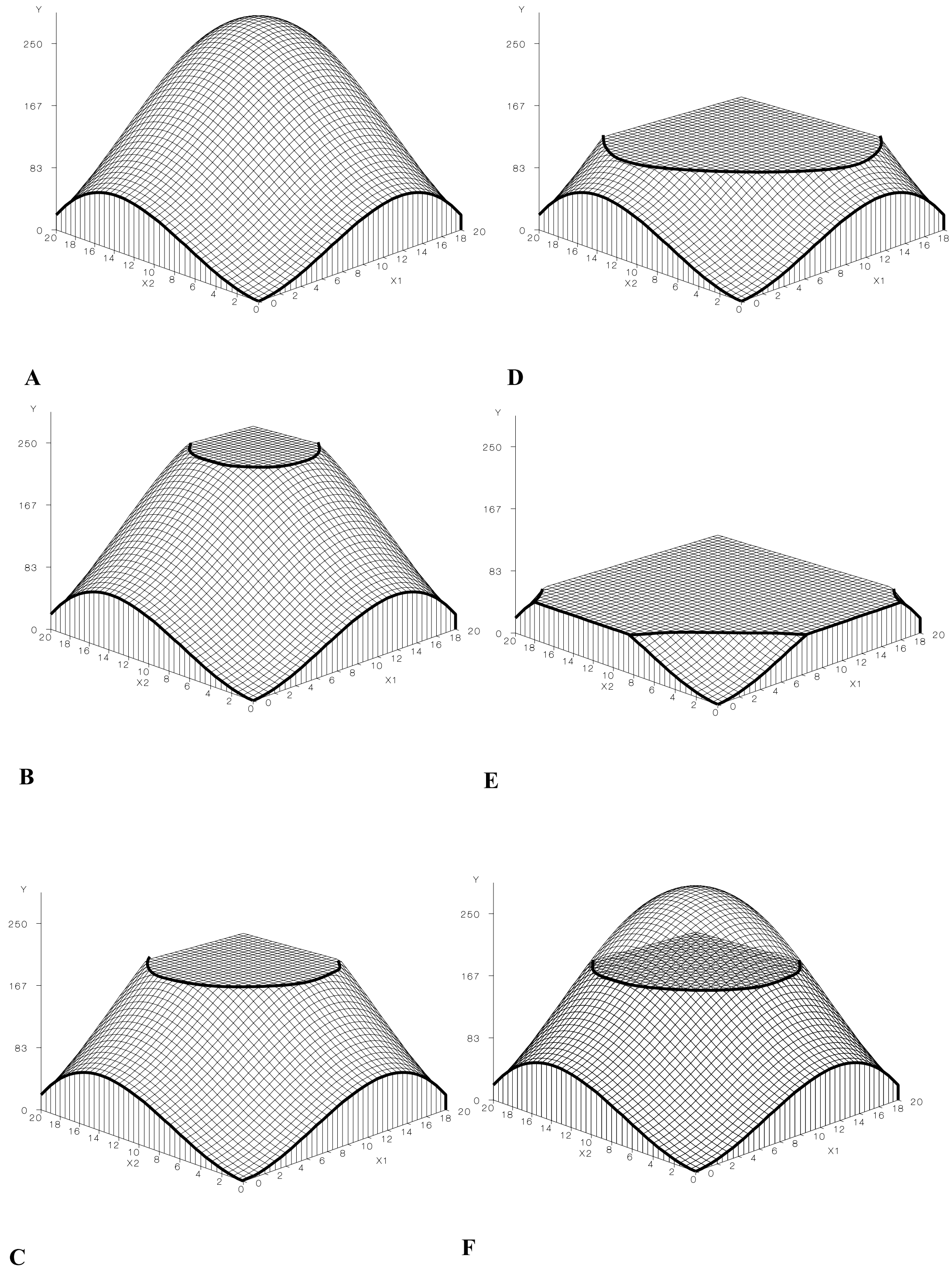
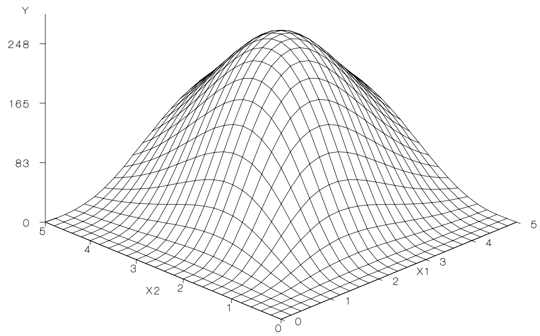
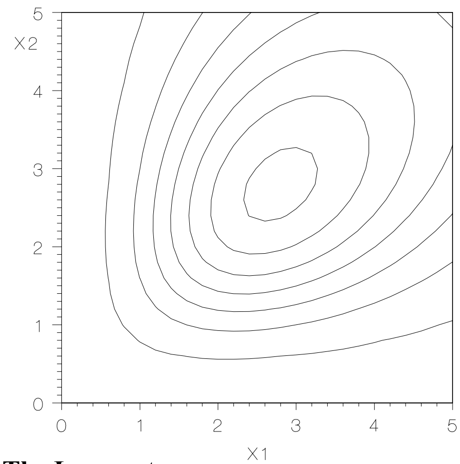


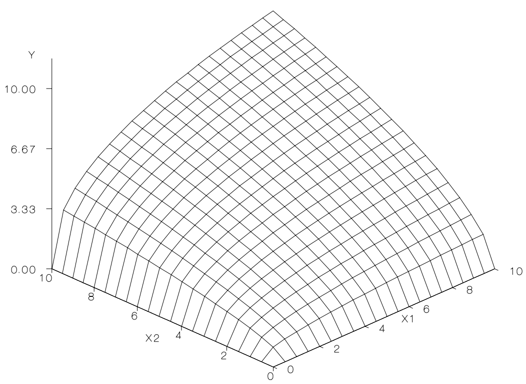
Figure 5.4 Isoquants and a Production Surface



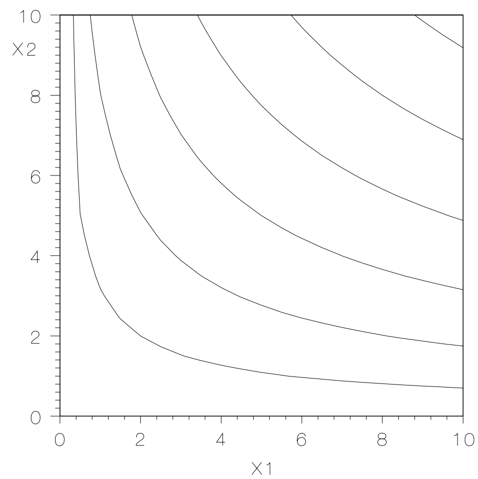
A The Production Surface



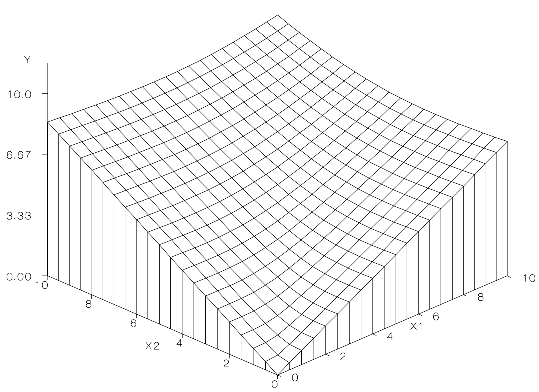
B The Isoquants



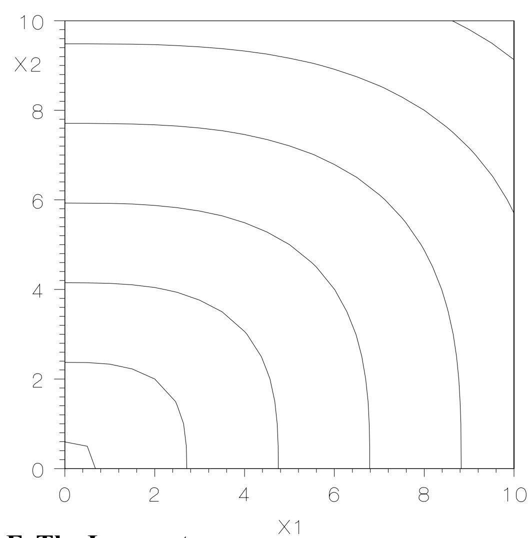
C The Production Surface



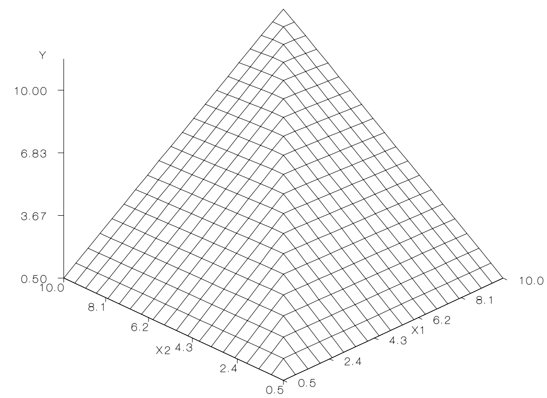
D The Isoquants



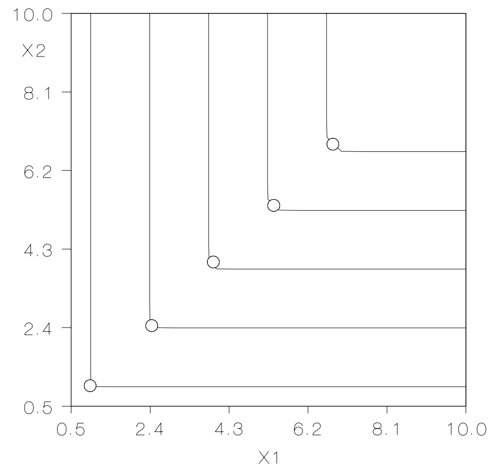
E The Production Surface



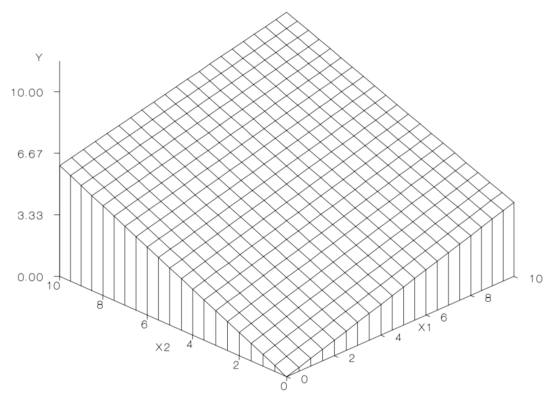
F The Isoquants



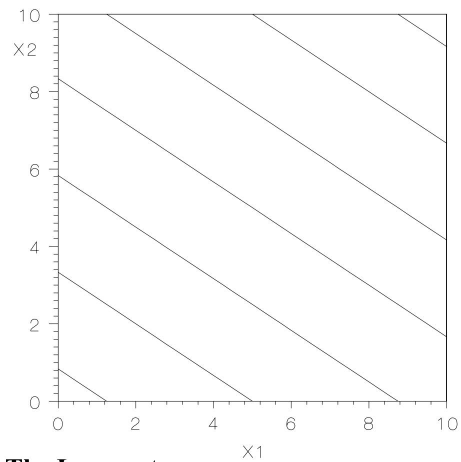
K The Production Surface



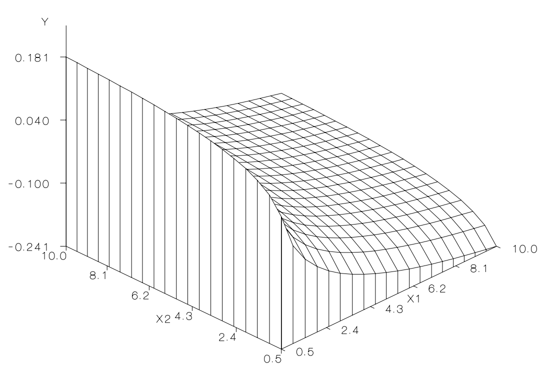
L The Isoquants



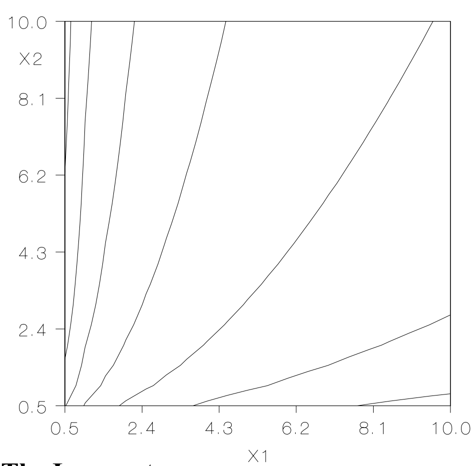
G The Production Surface



H The Isoquants



I The Production Surface



J The Isoquants

Figure 5.5 Some Possible Production Surfaces and Isoquant Maps

Another possibility is for a concave surface with isoquants bowed outward (concave to the origin (diagrams E and F). This pattern represents an increasing rather than diminishing marginal rate of substitution between input pairs. As the use of x_1 increases and the use of x_2 decreases along the isoquant, less and less additional x_1 is required to replace units of x_2 and maintain output. This shape is not very likely, because the pattern would suggest that the two inputs used in combination results in a decrease in relative productivity rather than the synergistic increase that was discussed earlier.

It is possible for isoquants to have a constant slope (diagrams G and H). The corresponding production surface is a hyperplane. In this instance, one input or factor of production substitutes for the other in a fixed proportion. Here, there is a constant, not a diminishing marginal rate of substitution. For example, if inputs substituted for each other in a fixed proportion of 1 unit x_1 to 2 units of x_2, the following input combinations would all result in exactly the same output−$4x_1$, $0\ x_2$; $3x_1$, $2x_2$; $2x_1$, $4x_2$; $1x_1$, $6x_2$; $0x_1$, $8x_2$.

It is also possible for isoquants to have a positive slope (Diagrams I and J). This can occur in a situation where additional amounts of one of the inputs (in this instance, input x_1) *reduces* output. Diagram B also includes some points where the isoquants have a positive slope.

Finally, isoquants might be right angles, and the corresponding production surface is shaped like a pyramid (diagrams K and L). This can occur when two inputs must be used in fixed proportion with each other. The classic example here is tractors and tractor drivers. A tractor without a driver produces no output. A driver without a tractor produces no output. These inputs must be used in a constant fixed proportion to each other one tractor driver to one tractor.

5.3 Isoquants and Ridge Lines

Two families of production functions underlie every isoquant map. Figure 5.6 illustrates this relationship. Assume x_2 to be fixed at some predetermined level x_2^*. A horizontal line is drawn from x_2^* across the diagram. A production function for x_1 holding x_2 constant at x_2^* can then be drawn by putting x_1 on the horizontal axis, and noting the output obtained from the intersection of the line drawn at x_2^* with each isoquant. Now choose another level of x_2. Call this level x_2^*. The process can be repeated over and over again for any level of x_2. Each alternative fixed level for x_2 generates a new production function for x_1 assuming that x_2 is held constant at the predetermined level.

Moreover, the same process can be repeated by holding x_1 constant and tracing out the production functions for x_2. Every time x_1 changes, a new production function is obtained for x_2. As one moves from one production function for x_2 to another, different quantities of output from x_2 are produced, despite the fact that neither the quality or quantity of x_2 has changed. This is because the varying assumptions about the quantity of x_1 either enhance or reduce the productivity of x_2. Another way of saying this is that the marginal productivity (or *MPP*) of x_2 is not independent of the assumption that was made about the availability of x_1, and the *MPP* of x_1 is not independent of the assumption that is made about the availability of x_2.

Now suppose that a level for x_2 is chosen of x_2^* that is just tangent to one of the isoquants. The point of tangency between the line drawn at x_2^* and the isoquant will represent the maximum possible output that can be produced from x_1 holding x_2 constant at x_2^*. The production function derived by holding x_2 constant at x_2^* will achieve its maximum at the point of tangency between the isoquant and the horizontal line drawn at x_2^*. The point of tangency is the point of zero slope on the isoquant and marks the dividing point between stages II and III for the production function

$$y = f(x_1 \mid x_2 = x_2^*) \tag{5.7}$$

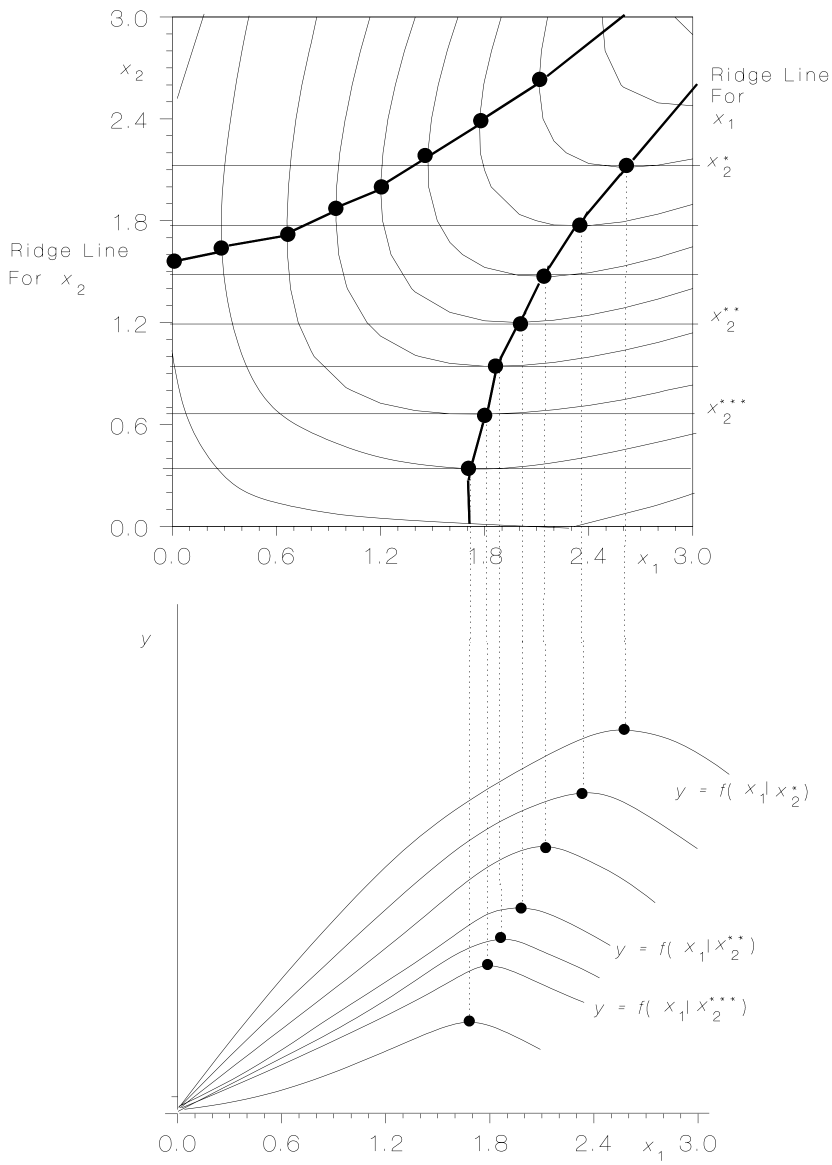


Figure 5.6 Ridge Lines and a Family of Production Functions for Input x_1

This process could be repeated over and over again by selecting alternative values for x_2 and drawing a horizontal line at the selected level for x_2. Each isoquant represents a different output level, just as each horizontal line represents a different assumption about the magnitude of x_2. An infinite number of isoquants could be drawn, each representing a slightly different output level. An infinite number of horizontal lines could be drawn across the isoquant map, each representing a slightly different assumption about the value for x_2. For each horizontal line, there would be a point of tangency on one (and only one!) of the isoquants. This point of tangency is a point of zero slope on the isoquant. Each isoquant would have a corresponding horizontal line tangent to it. The point of tangency represents the maximum for the underlying production function for x_1 under the predetermined assumption with regard to the fixed level of x_2.