5

Production with Two Inputs

This chapter introduces the basics of the technical relationships underlying the factor-factor model, in which two inputs are used in the production of a single output. The concept of an isoquant is developed from a simple table containing data similar to that which might be available in a fertilizer response trial. The slope of the isoquant is defined as the marginal rate of substitution. Isoquants with varying shapes and slopes are illustrated. The shape of an isoquant is closely linked to the characteristics of the production function that transforms the two inputs into the output. The linkages between the marginal rate of substitution and the marginal products of each input are derived.

Key terms and definitions:

Isoquant
Marginal Rate of Substitution (*MRS*)
Diminishing Marginal Rate of Substitution
Constant Marginal Rate of Substitution
Increasing Marginal Rate of Substitution
Convex to the Origin
$\Delta x_2/\Delta x_1$
Asymptotic to the Axes
Concentric Rings
Synergistic Effect
Tangency
Infinite Slope
Zero Slope
Ridge Line
Family of Production Functions
Change in Output
Change in Input
Limit
Infinitesimally Small
Partial Derivative
Total Derivative
Total Differential

5.1 Introduction

The discussion in Chapters 2 to 4 centered on the problems faced by a farmer who wishes to determine how much of a single input should be used or how much of a single output should be produced to maximize profits or net returns to the farm. The basic assumption of this chapter is that two inputs, not one inputs, are allowed to vary. As a result, some modifications need to be made in the basic production function. The production function used in Chapters 1 to 4 was

$$y = f(x). \tag{5.1}$$

Suppose instead that two inputs called x_1 and x_2 are allowed to vary. The resulting production function is

$$y = f(x_1, x_2) \tag{5.2}$$

if there are no more inputs to the production process. If there are more than two, or n different inputs, the production function might be written as

$$y = f(x_1, x_2 | \; x_3, ..., x_n) \tag{5.3}$$

The inputs $x_3, ..., x_n$ will be treated as fixed and given, with only the first two inputs allowed to vary.

In the single-input case, each level of input used produced a different level of output, as long as inputs were being used below the level resulting in maximum output. In the two-input case, there may be many different combinations of inputs that produce exactly the same amount of output. Table 5.1 illustrates some hypothetical relationships that might exist between phosphate (P_2O_5) application levels, potash (K_2O) application levels, and corn yields. The nitrogen application rate was assumed to be 180 pounds per acre.

The production function from which these data were generated is

$$y = f(x_1, x_2 | \; x_3) \tag{5.4}$$

where y = corn yield in bushels per acre

x_1 = potash in pounds per acre
x_2 = phosphate in pounds per acre
x_3 = nitrogen in pounds per acre assumed constant at 180

Notice from Table 5.1 that potash is not very productive without an adequate availability of phosphate, The maximum yield with no phosphate is but 99 bushels per acre and that occurs at comparatively low levels of potash application of 20 to 30 pounds per acre. The production function for potash in the absence of any phosphate is actually decreasing at potash application rates of over 30 pounds per acre. In the absence of phosphate fertilizer, stage III for potash begins quite early.

Table 5.1 Hypothetical Corn Response to Phosphate and Potash Fertilizer

	Potash (lb/acre)								
Phosphate (lb/acre)	0	10	20	30	40	50	60	70	80
0	96	98	99	99	98	97	95	92	88
10	98	101	103	104	105	104	103	101	99
20	101	104	106	108	109	110	110	109	106
30	103	107	111	114	117	119	120	121	121
40	104	109	113	117	121	123	126	128	129
50	104	111	116	121	125	127	129	131	133
60	103	112	118	123	126	128	130	131	134
70	102	111	117	123	126	127	131	136	135
80	101	108	114	119	119	125	129	131	134

Phosphate in the absence of potash is more productive, but only slightly so. The maximum yield without any potash is 104 bushels per acre at between 40 and 50 pounds of phosphate. Stage III for phosphate begins at beyond 50 pounds per acre if no potash is applied.

Each of the rows of Table 5.1 represents a production function for potash fertilizer with the assumption that the level of phosphate applied is fixed at the level given by the application rate, which is the first number of the row. As the level of phosphate is increased, the productivity of the potash increases. The marginal product of an additional 10 pounds of potash is usually larger for rows near the bottom of the table than for rows near the top of the table. Moreover, production functions for potash with the larger quantities of phosphate typically achieve their maximum at higher levels of potash use.

Each of the columns of Table 5.1 represents a production function for phosphate fertilizer with the assumption that the level of potash remains constant as defined by the first number in the column. Again the same phenomenon is present. The productivity of phosphate is usually improved with the increased use of potash, and as the assumed fixed level of potash use increases, the maximum of each function with respect to phosphate occurs at larger levels of phosphate use.

These relationships are based on a basic agronomic or biological characteristic of crops. A crop would not be expected to produce high yields if an ample supply of all nutrients were not available. To a degree, phosphate can be substituted for potash, or potash for phosphate. In this example, there are several different combinations of phosphate and potash that will all produce the same yield.

But if the crops are to grow, some of both nutrients must be present, and the highest yields are obtained when both nutrients are in ample supply. This concept in economics is closely linked to Von Liebig's "Law of the Minimum," which states that plant growth is constrained by the most limiting nutrient.

Notice also that it is possible to use too much of both potash and phosphate. Yields using 70 pounds of each are greater than when 80 pounds of each are used. The law of diminishing returns applies to units of phosphate and potash fertilizer taken together when other inputs are held constant, just as it applies to each individual kind of fertilizer.

Table 5.1 contains data from nine production functions for phosphate, under nine different assumptions with regard to potash use. Table 5.1 also contains data from nine production functions for potash, each obtained from a different assumption with regard to the level of phosphate use.

Due to the biology of crop growth, a synergistic effect is present. This means that the presence of ample amounts of phosphate makes the productivity of potash greater. Ample amounts of potash makes the productivity of phosphate greater. The two fertilizers, taken together, result in productivity gains in terms of increased yields greater than would be expected by looking at yields resulting from the application of only one type of fertilizer.

This effect is not limited to crop production. The same phenomenon may be observed if data were collected on the use of the inputs grain (concentrate) and forage used in the production of milk. A cow that is fed all grain and no forage would not be a good milk producer. Similarly, a cow fed all forage and no grain would not produce much milk. Greatest milk production would be achieved with a ration containing a combination of grain and forage.

Each possible ration represents a particular combination or mix of inputs grain and forage. Some of these rations would be better than others in that they would produce more milk. The particular ration chosen by the farmer would depend not only on the amount of milk produced, but also on the relative prices of grain and forage. These ideas are fully developed in Chapter 7.

Figure 5.1 illustrates the production surface arising from the use of phosphate and potash. The x_1 and x_2 axes form a grid (series of agronomic test plots) with the vertical axis measuring corn yield response to the two fertilizers. The largest corn yields are produced from input combinations that include both potash and phosphate.

Data for yet another production function are contained in Table 5.1. From Table 5.1 it is possible to determine what will happen to corn yields if fertilizer application rates for potash and phosphate are increased by the same proportion. Suppose that 1 unit of fertilizer were to consist of 1 pound of phosphate and 1 pound of potash and that this proportion did not change. A table was constructed using numbers found on the diagonal of Table 5.1. These data points are illustrated on the production surface in Figure 5.1.

These data appear to be very similar to the data in the earlier chapters for single input production functions, and they are. The only difference here is that two types of fertilizer are assumed be used in fixed proportion to each other. Under this assumption, the amount of fertilizer needed to maximize profits could be found in a manner similar to that used in earlier chapters, but there is uncertainty as to whether or not the 1:1 ratio in the use of phosphate and potash is the correct ratio.

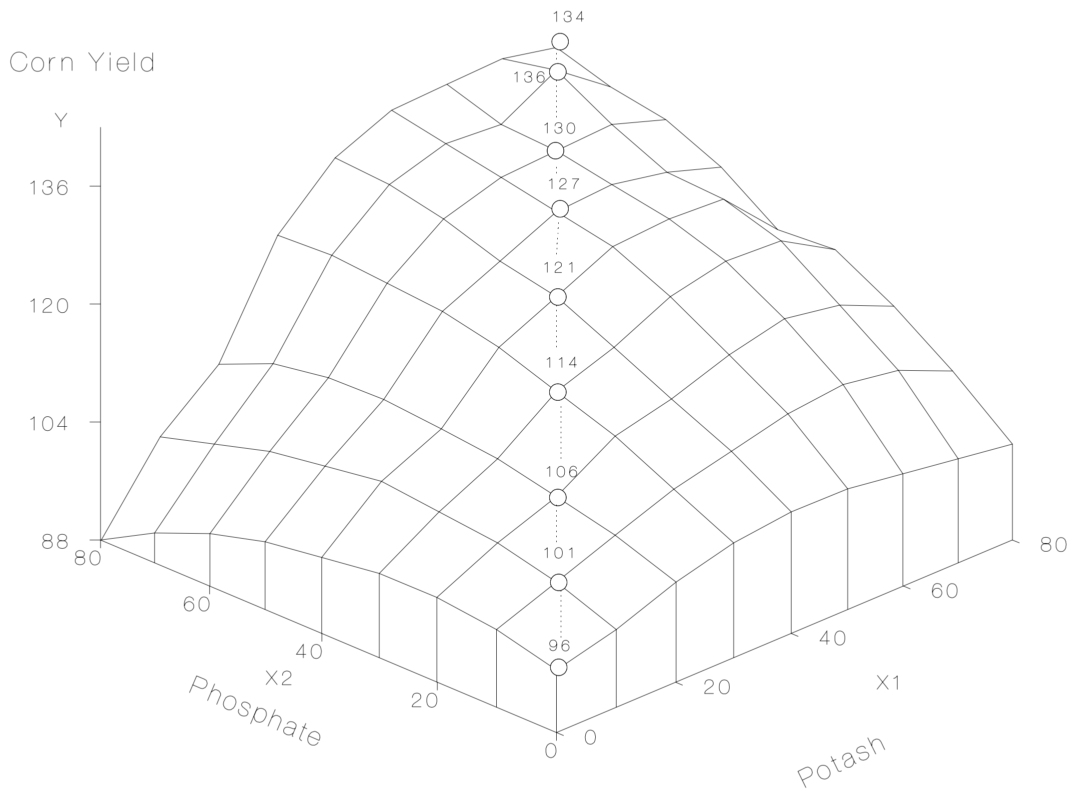

Figure 5.1 Production Response Surface Based on Data Contained in Table 5.1

Table 5.2 Corn Yield Response to 1:1 Proportionate Changes in Phosphate and Potash

Units of Fertilizer (1 Unit = 1 lb of Phosphate and 1 lb of Potash)	Corn Yield (bu/acre)
0	96
10	101
20	106
30	114
40	121
50	127
60	130
70	136
80	134

What would happen, for example, if phosphate were very expensive and potash were very cheap? Perhaps the 1:1 ratio should be changed to 1 unit of phosphate and 2 units of potash to represent a unit of fertilizer. Data for a production function with a 1:2 ratio could also be derived in part from Table 5.1. These data are presented in Table 5.3.

Table 5.3 Corn Yield Response to 1 : 2 Proportionate Changes in Phosphate and Potash

Units of Fertilizer (1 unit = 1 lb phosphate and 2 lb. potash)	Corn Yield (bu/acre)
10 -- 20	103
20 -- 40	109
30 -- 60	120
40 -- 80	129

Much of the next several chapters is devoted to the basic principles used for determining the combination of two inputs (such as phosphate and potash fertilizer) that represents maximum profit for the producer. Here the proper proportions are closely linked to the relative prices for the two types of fertilizer.

5.2 An Isoquant and the Marginal Rate of Substitution

Many combinations of phosphate and potash all result in exactly the same level of corn production. Despite the fact that Table 5.1 includes only discrete values, a bit of interpolation will result in additional combinations that produce the same corn yield. Take, for example, a corn yield of 121 bushels per acre (Table 5.1). This yield can be produced with the following input combinations:30 pounds of phosphate and 70 pounds of potash; 30 pounds of phosphate and 80 pounds of potash; 40 pounds of phosphate and 40 pounds of potash; and 50 pounds of phosphate and 30 pounds of potash.

Moreover, there are many more points that might also achieve approximately 121 bushels per acre-60 pounds of phosphate and approximately 27 pounds of potash; 70 pounds of phosphate and approximately 27 pounds of potash; and 80 pounds of phosphate and approximately 43 pounds of potash to name a few. All these combinations share a common characteristic in that they produce the same yield.

A line can be drawn that connects all points on Table 5.1 representing the same yield. This line is called an *isoquant*. The prefix *iso* comes from the Greek *isos* meaning equal. *Quant* is short for quantity. An isoquant is literally a line representing equal quantities. Every point on the line represents the same yield or output level, but each point on the line also represents a different combination of the two inputs. As one moves along an isoquant, the proportions of the two inputs vary, but output (yield) remains constant.

An isoquant could be drawn for any output or yield that one might choose. If it is possible to draw an isoquant for a yield of 121 bushels per acre it is also possible to draw one for a yield of 125.891 bushels per acre, if the data were sufficiently detailed, or an isoquant could be drawn for a yield of 120.999 bushels per acre, or any other plausible yield.

If isoquants are drawn on graph paper, the graph is usually drawn with the origin (0y, 0x) in the lower left-hand corner. The isoquants are therefore bowed toward the origin of the graph.

Figure 5.2 illustrates the isoquants based on the data contained in Table 5.1. These are the "contour lines" for the production surface illustrated in Figure 5.1. Notice that the isoquants are convex to the lower left hand corner, or origin, of Figure 5.2.

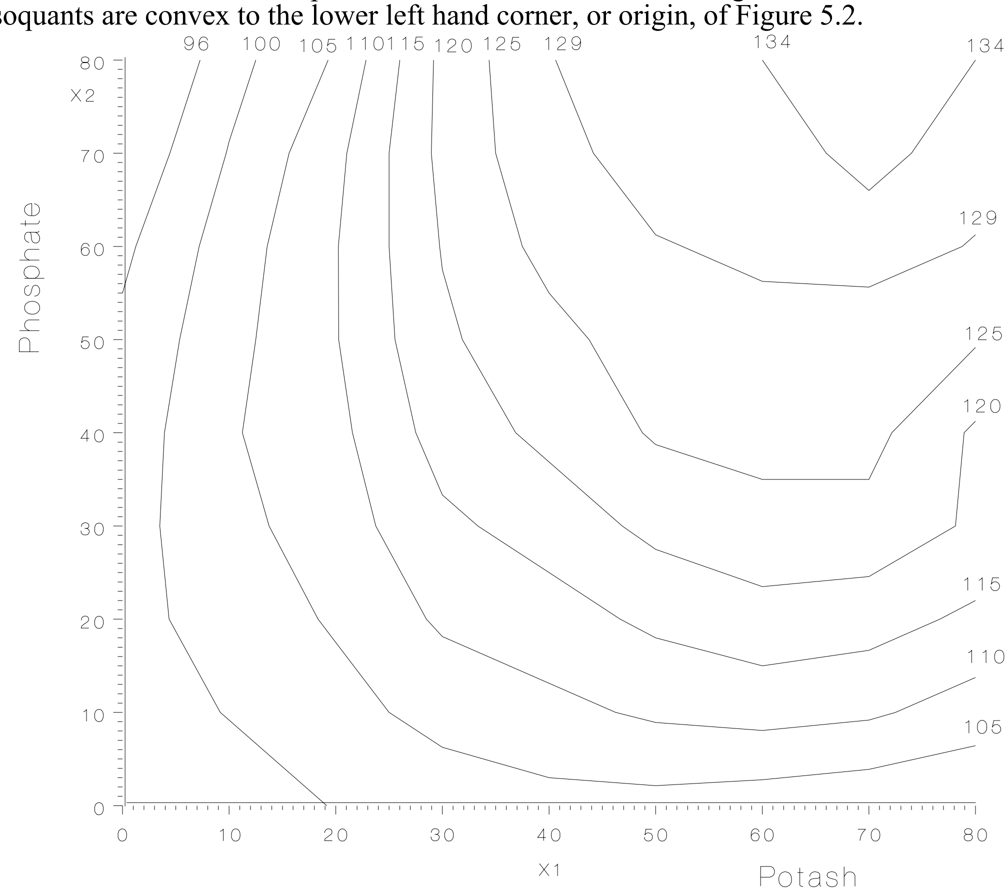

Figure 5.2 Isoquants for the Production Surface in Figure 5.1 Based on Data Contained in Table 5.1