If the slope of *AC* is positive, *MC* must be greater than *AC*. If the slope of *AC* is negative, *MC* must be less than *AC*. If the slope of *AC* is zero, *AC* is at its minimum and *MC* must equal *AC*. The reader can verify that the same relationship must hold between *MC* and *AVC*.

AFC is a rectangular hyperbola. Draw a straight line from any point on the *AFC* curve to the corresponding vertical ($) and horizontal (*y*) axis. The area of the enclosed rectangle is equal to *FC* which is the constant *k* (Figure 4.2). To the point of maximum output, as *y* becomes larger and larger, *AFC* comes closer and closer to the horizontal axis but does not reach it. Similarly, as *y* becomes smaller and smaller, *AFC* becomes larger and larger and gets closer and closer to the vertical axis. Again *AFC* never reaches the vertical axis.

Since *AC* is the sum of *AVC* + *AFC*, and *AFC* becomes smaller and smaller to the point of maximum output, as output increases, *AC* should be drawn closer and closer to *AVC*. The minimum slope of a line drawn from the origin of the graph to the *TC* curve occurs at an output level larger than the output level associated with the minimum slope of a line drawn from the origin to the *VC* curve. Therefore, minimum *AVC* occurs at an output level smaller than the level at which minimum *AC* occurs.

The behavior of average and marginal cost curves beyond the point of output maximization is somewhat complicated. Beyond the point of output maximization, *y* is reduced. Since *FC* remains constant, *AFC* returns along the exact same curve. *AVC* and *AC* are increasing even as *y* is reduced, when inputs are used beyond the point of output maximization. Moreover, if there are any fixed costs, *AC* must remain above *AVC*. Both *AVC* and *AC* must turn back on themselves to represent the new higher average costs associated with the reduction in output when inputs are used beyond the point of output maximization. If this is to occur, *AC* must cross over *AVC*. At the point of output maximum, both *AC* and *AVC* have a perfectly vertical or infinite slope (Figure 4.3). In stage III, *MC* goes into the negative quadrant when *MPP* is negative and forms a mirror image of its appearance in the positive quadrant.

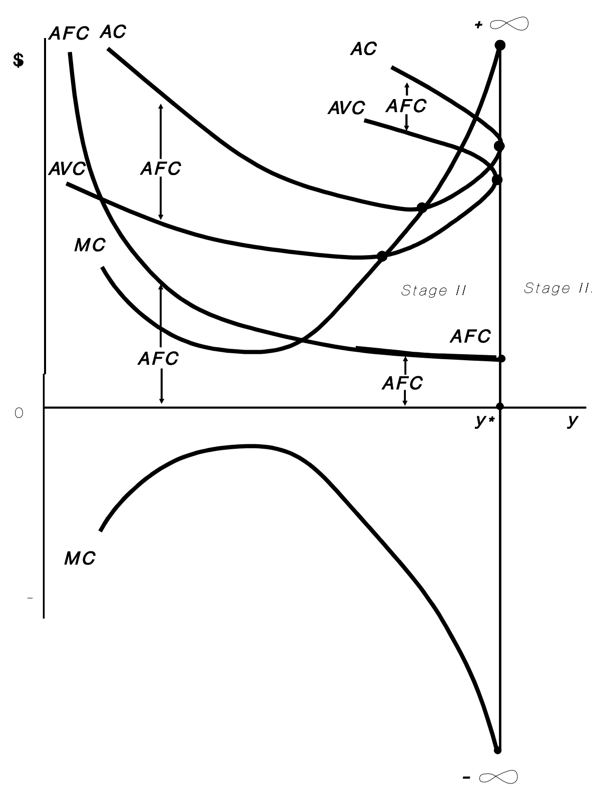

Figure 4.3 Behavior of Cost Curves as Output Approaches a Technical Maximum *y**

4.2 Simple Profit Maximization from the Output Side

Perhaps no criterion is more famous in economics than the expression "marginal cost equals marginal revenue." This simple rule is the basic requirement for selecting the level of output that maximizes profit.

If a farmer can sell all the output that he or she produces at the going market price, the resulting total revenue (*TR*) function is a line with a constant positive slope of $p°$

(4.16) $$TR = p°y$$

where $p°$ is some constant market price and y is the output.

The farmer's profit is equal to total revenue (*TR*) minus total cost (*TC*)

(4.17) $$\Pi = TR - TC$$

The greatest profit will be achieved when the difference between *TR* and *TC* is greatest (Figure 4.4). Superimpose the *TR* function on the previously defined *TC*. The greatest vertical distance between *TR* and *TC* occurs at points where the slope of *TR* and *TC* are the same. There are two points where this occurs. At the first point, *TC* is above *TR*, so this point represents the minimum profit. The second point represents maximum profit, which is the desired point.

Maximum (or minimum) profit is achieved at the points where the slope of the profit function is equal to zero. Thus

(4.18) $$d\pi/dy = dTR/dy - dTC/dy = 0$$

Notice that dTR/dy represents the slope of *TR*, and dTC/dy is the slope of *TC*. The slope of *TR* is referred to as marginal revenue (*MR*). The slope of *TC* has already been defined as marginal cost (*MC*) Hence equation (4.18) can be rewritten as

(4.19) $$MR - MC = 0$$

or, the famous

(4.20) $$MR = MC$$

Under the assumptions of pure competition, the output price is constant. Incremental units of the output can be sold at the going market price $p°$. Hence *MR* must be $p°$.

(4.21) $$dTR/dy = p° = MR$$

Figure 4.4 illustrates the average and marginal cost curves with marginal revenue included. Marginal cost equals marginal revenue at two points. The first point corresponds to the point of profit minimization, the second to the point of profit maximization. The second derivative test can be used to confirm this.

Differentiate the equation

(4.22) $$MR - MC = 0$$

which results in

(4.23) $$dMR/dy - dMC/dy = + \text{ or } - \ ?$$

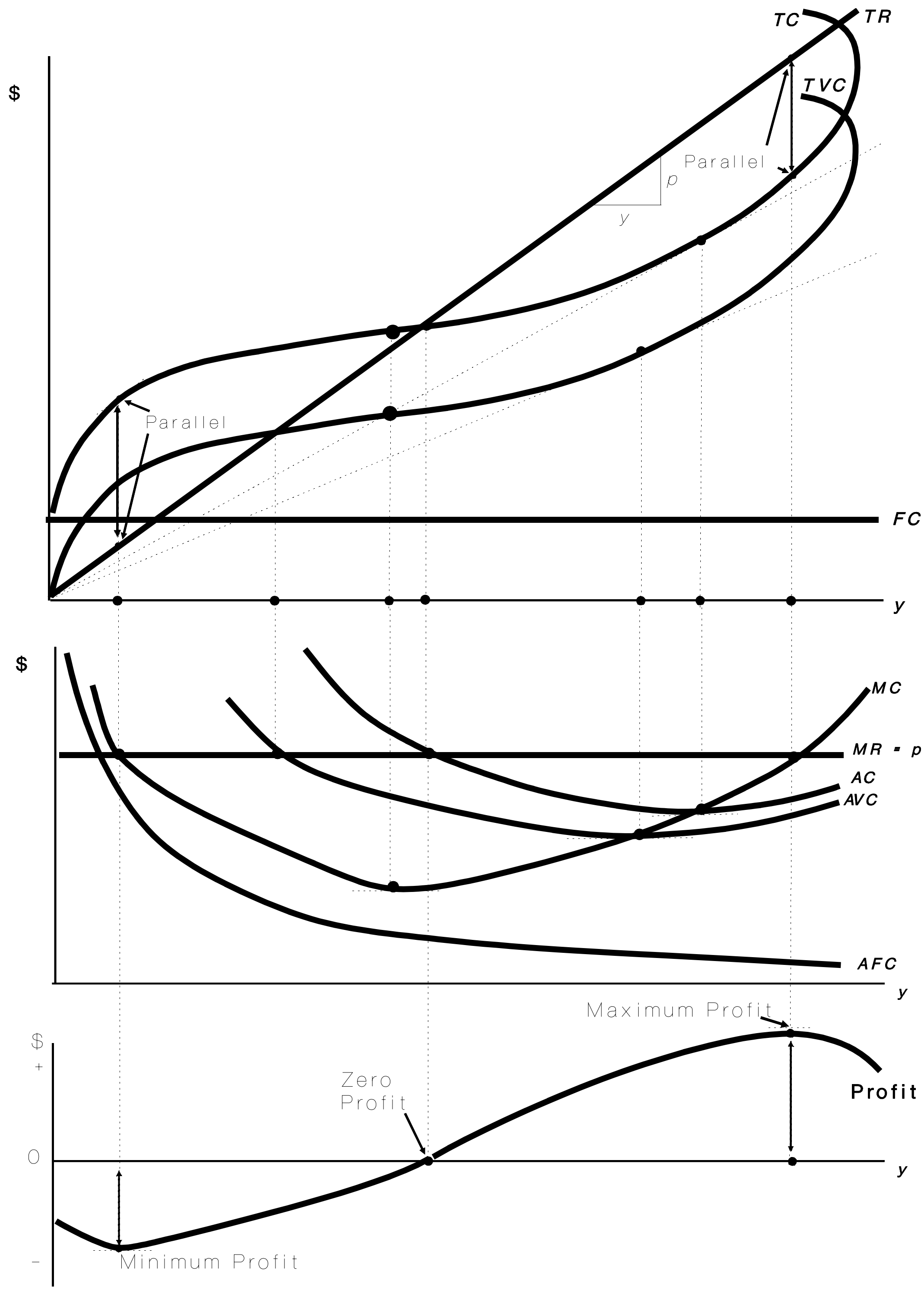

Figure 4.4 Cost Functions and Profit Functions

The sign on equation (4.23) tells if the point is a maximum or a minimum on the profit function. A negative sign indicates a maximum, and a positive sign is a minimum. Another way of looking at equation (4.23) is that the slope of *MC* must be greater than the slope of *MR* for profits to be maximized.

The term *dMR*/*dy* represents the slope of the the marginal revenue curve. In this case, marginal revenue is a constant with a zero slope. The sign on equation (4.23) is thus determined by the slope of *MC*, which is *dMC*/*dy*. If the slope of *MC* is negative, equation (4.23) will be positive. This condition corresponds to the first point of intersection between *MC* and *MR* in Figure 4.4. If the slope of *MC* is positive, then equation (4.23) is negative, and a point of profit maximization is found corresponding to the second point of intersection between *MC* and *MR* in Figure 4.4. The minimum point on the profit function represents the maximum loss for the farmer.

The farmer has an option not recognized by the mathematics. Suppose that *MC* has a positive slope, but *MR* = *MC* at a price level so low that it is below *AVC*. In this instance, the farmer would be better off not to produce, because he or she would lose only his fixed costs (*FC*). This would be less than the loss incurred at the point where *MR* = *MC*. If, however, *MC* = *MR* at a level between *AVC* and *AC*, the farmer would be better off to produce. In this instance, the farmer, by producing, would cover all the variable costs plus a portion of the fixed costs. The total loss would be less than if production ceased and all the fixed costs had to be paid. This explains why farmers might continue to produce corn even though the market price is less than the total costs of production. With a high ratio of fixed to variable costs (as would often be the case in grain production), the farmer is better off to produce and incur only the partial loss, at least in the short run.

Of course, in the long run, the farmer can make major adjustments, and all costs should be treated as variable. Farmers can buy and sell land and machinery in the long run, making these costs variable. If the length of run is sufficiently long, a farmer will continue to produce only insofar as all costs are covered. A farmer cannot continue to lose money indefinitely without going bankrupt.

Table 4.1 illustrates some hypothetical total cost data for corn production and shows the corresponding average and marginal costs. Corn is assumed to sell for $4.00 per bushel.The relationships represented in the data contained in Table 4.1 are the same as those illustrated in Figure 4.4. Marginal cost (*MC*) is the change in cost over the 10-bushel increment obtained by calculating the change in *TC* (or *VC*) and dividing by the change in output. Marginal cost equals marginal revenue at between 110 and 120 bushels of corn per acre. Profits are maximum at that output level. It is not possible to determine the exact output level without first knowing the exact mathematical function underlying the data contained in Table 4.1.

Figure 4.5 illustrates the data contained in Table 4.1, and confirms the profit-maximizing output level at approximately 115 bushels of corn per acre. Thic corresponds withthe point where the slope of *TR* equals the slope of *TC*, or *MR* = *MC*. Notice also that the *TR* curve intersects *TC* at exactly the output level at which *AC* equals *MR* equals *Average Revenue* (*AR*) per unit of output equals the price (*p*) of the output, which in this example is $4.00 per bushel. An increase in the price of the output would increase the profit-maximizing output level beyond 115 bushels per acre; a decrease would reduce the profit-maximizing output level below the 115 bushel level. An increase in the variable input price(s) would reduce the profit-maximizing output level, whereas adecrease inthe input price(s) would increase the profit-maximizing output level.

Table 4.1 Hypothetical Cost Data for Corn Production

Yield, (*y*)	*TVC*	*FC*	*TC*	*AVC*	*AFC*	*AC*	*MC*	*MR*
40	89	75	164	2.23	1.88	4.11		
							2.10	4.00
50	110	75	185	2.20	1.50	3.70		
							2.00	4.00
60	130	75	205	2.17	1.25	3.42		
							1.00	4.00
70	140	75	215	2.00	1.07	3.07		
							1.50	4.00
80	155	75	230	1.94	0.94	2.88		
							2.00	4.00
90	175	75	250	1.94	0.83	2.78		
							2.50	4.00
100	200	75	275	2.00	0.75	2.75		
							3.00	4.00
110	230	75	305	2.09	0.68	2.77		
							4.00	4.00
120	270	75	345	2.25	0.63	2.88		
							5.00	4.00
130	320	75	395	2.46	0.58	3.04		
							6.00	4.00
140	380	75	455	2.71	0.54	3.25		

4.3 The Duality of Cost and Production

The shape of the total variable cost function is closely linked to the shape of the production function that underlies it. If input prices are constant, all the information about the shape of the *VC* function is contained in the equation for the underlying production function. Moreover, if the *VC* function and the prices for the inputs are known, so is the shape of the underlying production function. If input prices are constant, then all of the needed information for determining the shape of the *VC* is given by the production function, and all the information for determining the shape of the production function is given by the *VC* function.

In Chapter 2, the law of diminishing returns was stated "As units of a variable input are added to units of a fixed input, after a point, each additional unit of variable input produces less and less additional output." Another way of stating this law is that after a point, incremental or additional units of input each produce less and less additional output.

The law of diminishing returns might also be interpreted from the output side. From the output side, the law states that as output is increased by 1 unit at a time, after a point, each incremental or additional unit of output requires more and more additional units of one or more variable inputs. Another way of saying this is that if output is increased incrementally, after a point, each incremental or additional unit of output becomes more and more costly with respect to the use of inputs. Another unit of output is produced but only at the expense of using more and more additional input.

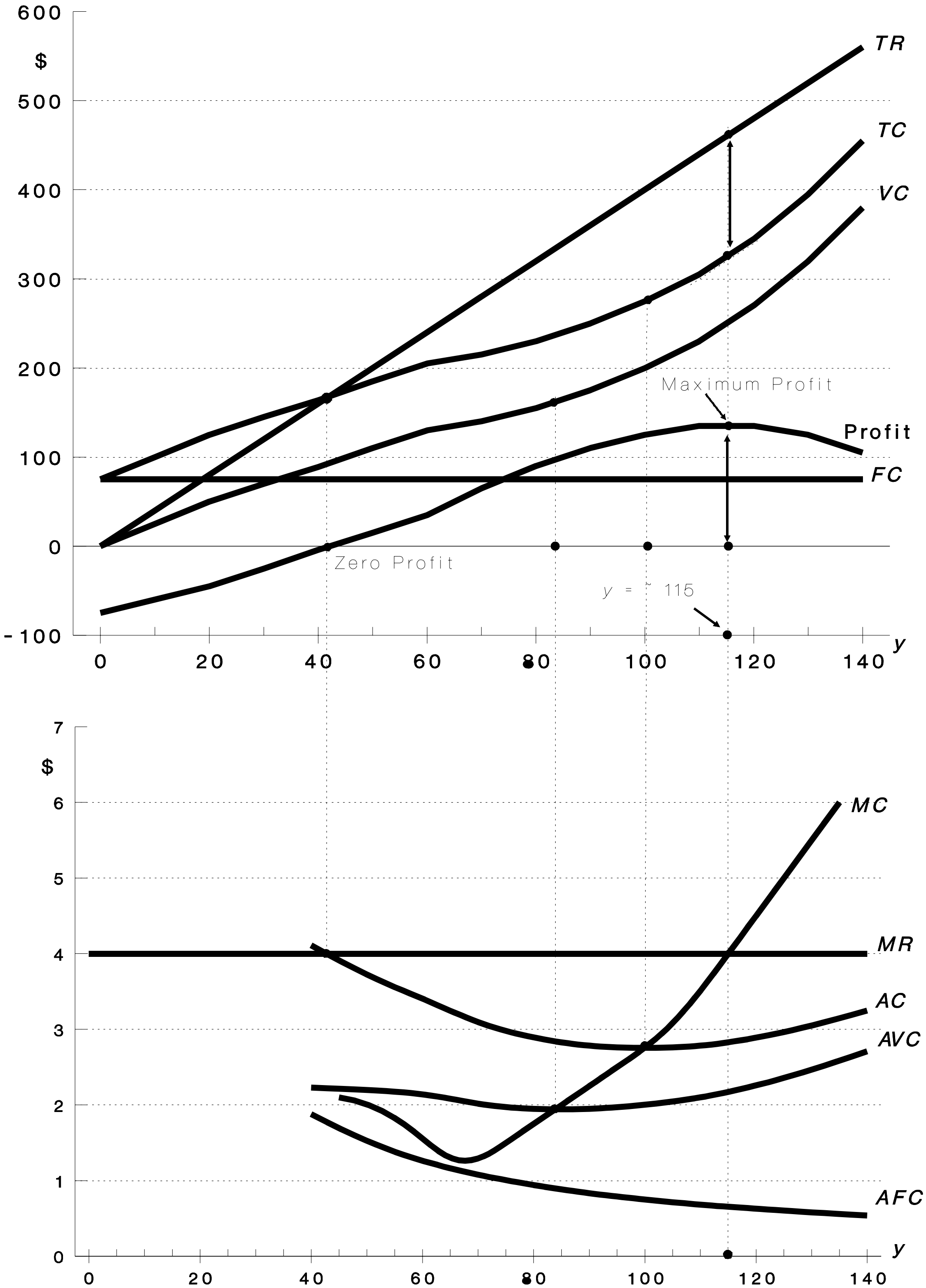

Figure 4.5 The Profit-Maximizing Output Level Based on Data Contained in Table 4.1