# 4

# Costs, Returns and Profits on the Output Side

In this chapter the concept of a cost function defined in terms of units of output is introduced. Total, variable and marginal cost curves are illustrated using graphics and derived using mathematics. The necessary conditions for determining the level of output that maximizes profits are derived. The cost functions are shown to be closely linked to the parameters of the underlying production function. The supply function for the firm is derived.

## Key terms and definitions:

Total Cost (*TC*)
Total Variable Cost (*VC*)
Marginal Cost (*MC*)
Total Fixed Cost (*FC*)
Average Cost (*AC*)
Average Fixed Cost (*AFC*)
Average Variable Cost (*AVC*)
Inverse Production Function
Duality of Cost and Production

## 4.1 Some Basic Definitions

In Chapter 3, a very simple cost equation was defined. This cost equation was

$$TFC = v°x \tag{4.1}$$

Equation (4.1) states that the total cost for an input or factor of production is the constant price of the input ($v°$) multiplied by the quantity that is used.

However, the costs of production might also be defined not in terms of the use of the input, but in terms of the output. To do this, some basic terms need to be explained.

Variable costs (*VC*) are the costs of production that vary with the level of output produced by the farmer. For example, in the production of corn, with the time period being a single production season, variable costs might be thought of as the costs associated with the purchase of the variable inputs used to produce the corn. Examples of variable costs include the costs associated with the purchase of inputs such as seed, fertilizer, herbicides, insecticides, and so on. In the case of livestock production within a single production season, a major variable cost item is feed.

Fixed costs (*FC*) are the costs that must be incurred by the farmer whether or not production takes place. Examples of fixed-cost items include payments for land purchases, and depreciation on farm machinery, buildings, and equipment.

The categorization of a cost item as fixed or variable is often not entirely clear. The fertilizer and seed a farmer uses can only be treated as a variable cost item prior to the time in which it is placed in the ground. Once the item has been used, it is sometimes called a sunk, or unrecoverable, cost, in that a farmer cannot decide to sell seed and fertilizer already used and recover the purchase price.

Although depreciation on farm machinery is normally treated as a fixed cost, given sufficient time, the farmer does have the option of selling the machinery so that the depreciation would no longer be incurred. Payments for the purchase of land would not be made if the farmer elected to sell the land. The categorization of farm labor is very difficult. A farm laborer on an annual salary might be treated as a fixed cost which the farmer incurs whether or not production takes place. But if the laborer is laid off, the cost is no longer fixed. Temporary workers hired on an hourly basis might be more easily categorized as a variable cost.

The categorization of a particular input as a fixed cost or variable cost item is thus closely intertwined with the particular period involved. Over very long periods, a farmer is able to buy and sell land, machinery, and other inputs into the production process that would normally be considered fixed. Thus, over very long periods, all costs are normally treated as variable.

Over a very short period of time, perhaps during a few weeks within a single production season, a farmer might not be able to make any adjustment in the amounts of any of the inputs being used. For this length of time, all costs could be treated as fixed. Thus the categorization of each input as a fixed- or variable-cost item cannot be made without explicit reference to the particular period involved. A distinction between fixed and variable costs has thus been made on the basis of the period involved, with the proportion of fixed to variable costs increasing as the length of time is shortened, and declining as the length of time increases.

Some economists define the long run as a period of time of sufficient length such that the size of plant (in the case of farming, the farm) can be altered. Production takes place on a short-run average cost curve (*SRAC*) that is U shaped, with the manager equating marginal revenue (the price of the output in the purely competitive model) with short-run marginal cost (*SRMC*). There exists a series of short-run marginal and average cost curves corresponding to the size of the particular plant (farm). Given sufficient time, the size of the plant can be altered. Farmers can buy and sell land, machinery, and equipment. Long−run average cost (*LRAC*) can be derived by drawing an envelope curve which comes tangent to each short run average cost curve (Figure 4.1).

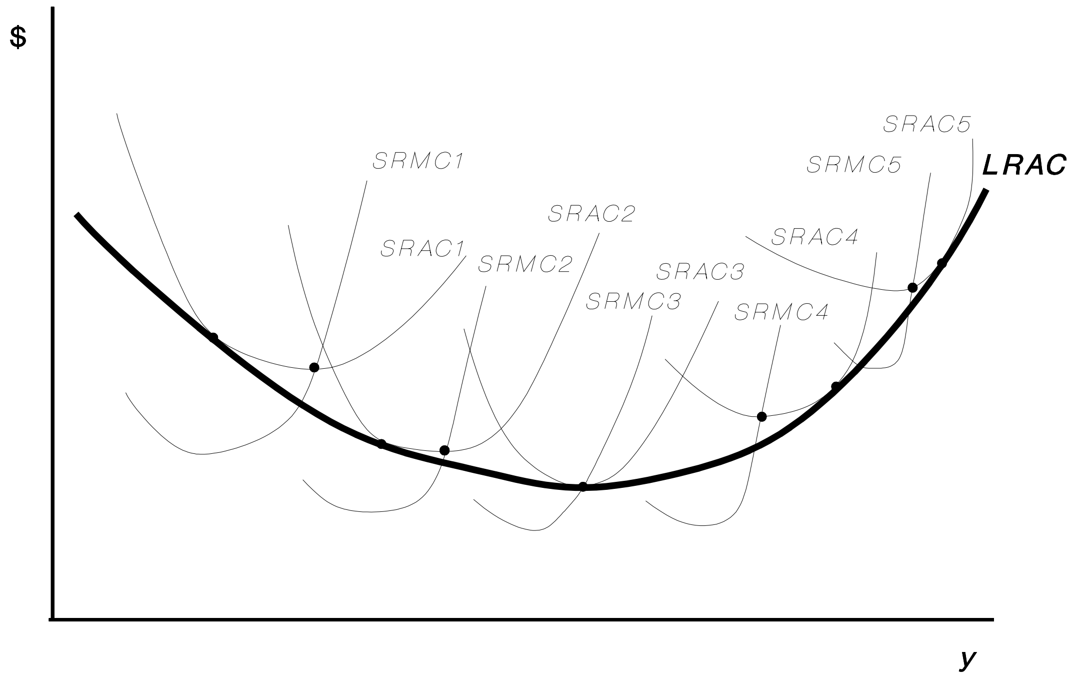


**Figure 4.1 Short- and Long-Run Average and Marginal Cost with Envelope Long-Run Average Cost**

A classic argument in economics was that between the economist Jacob Viner and his draftsman. Viner insisted that such a long-run average cost curve must necessarily come tangent to the minimum points on each short run average cost curve. The draftsman argued that this was impossible−that plants operating with less capacity than that represented by the minimum point on the *LRAC* curve must necessarily be tangent to a point on the *LRAC* at higher than minimum *SRAC*. Plants operating at greater than the capacity suggested by the minimum *LRAC* would have a *SRAC* tangent to *LRAC* at a point at greater than minimum *SRAC*. Only for the plant operating with its *SRAC* curve at the point of minimum *LRAC* would the *LRAC* be tangent to the minimum point on the *SRAC*. The draftsman was, of course, correct (Figure 4.1).

In long-run equilibrium, producers discover and select a plant size at the minimum point on *LRAC*. Hence *MR* equals *LRMC* and there is no profit. In the short run, however, *MR* can exceed *MC*. Each producer would equate *MR* to his own *SRMC*. For the producers operating in the short run, this would entail using the plant beyond its point of minimum *SRAC*. No producer would ever be observed operating at the minimum *SRAC* and *LRAC*, save the firms in long run equilibrium.

Variable costs are normally expressed per unit of output ($y$) rather than per unit of input ($x$). This is because there is usually more than one variable cost item involved in the production of agricultural commodities. A general expression for a variable cost function is

$$VC = g(y) \tag{4.2}$$

Since fixed costs do not vary with output, fixed costs are equal to some constant dollar value $k$; that is

$$FC = k \tag{4.3}$$

Total costs ($TC$) are the sum of fixed plus variable costs.

$$TC = VC + FC, \text{ or} \tag{4.4}$$

$$TC = g(y) + k \tag{4.5}$$

Average variable cost ($AVC$) is the variable cost per unit of output

$$AVC = VC/y = g(y)/y \tag{4.6}$$

Average fixed cost is equal to fixed cost per unit of output

$$AFC = FC/y = k/y \tag{4.7}$$

The output level $y$ is divided into the constant $k$, where $k$ represents total fixed costs ($FC$).

There are two ways to obtain average cost ($AC$), sometimes also called average total cost ($ATC$). One way is to divide total cost ($TC$) by output ($y$)

$$AC = ATC = TC/y \tag{4.8}$$

Another way is to sum average variable cost ($AVC$) and average fixed cost ($AFC$)

$$AC = AVC + AFC, \text{ or} \tag{4.9}$$

$$TC/y = VC/y + FC/y \tag{4.10}$$

Marginal cost is defined as the change in total cost, or total variable cost, resulting from an incremental change in output.

$$MC = \Delta TC/\Delta y = \Delta VC/\Delta y \tag{4.11}$$

Since the value for fixed costs ($FC$) is a constant $k$, $MC$ will be the same irrespective of whether it is based on total costs or total variable cost.

Marginal cost ($MC$) at a particular point is the slope of the total cost function. Marginal cost can be defined in terms of derivatives. In this instance

$$MC = dTC/dy = dVC/dy \tag{4.12}$$

The marginal cost function is a function representing the slope of the total cost function. For example, a value for $MC$ of \$5.00 indicates that the last or incremental unit of output cost an additional \$5.00 to produce.

Figure 4.2 illustrates the cost functions that have been defined. The illustration of *VC* looks like a production function that has been inverted. Output, rather than input, is on the horizontal axis. The vertical axis is dollars, not units of input. Moreover, the slope of the *VC* function appears to be exactly the inverse of the slope on a production function. The production function increased initially at an increasing rate until the inflection point was reached, then it increased at a decreasing rate. The cost function first increases at a decreasing rate until the inflection point is reached. Then the cost function increases at an increasing rate.

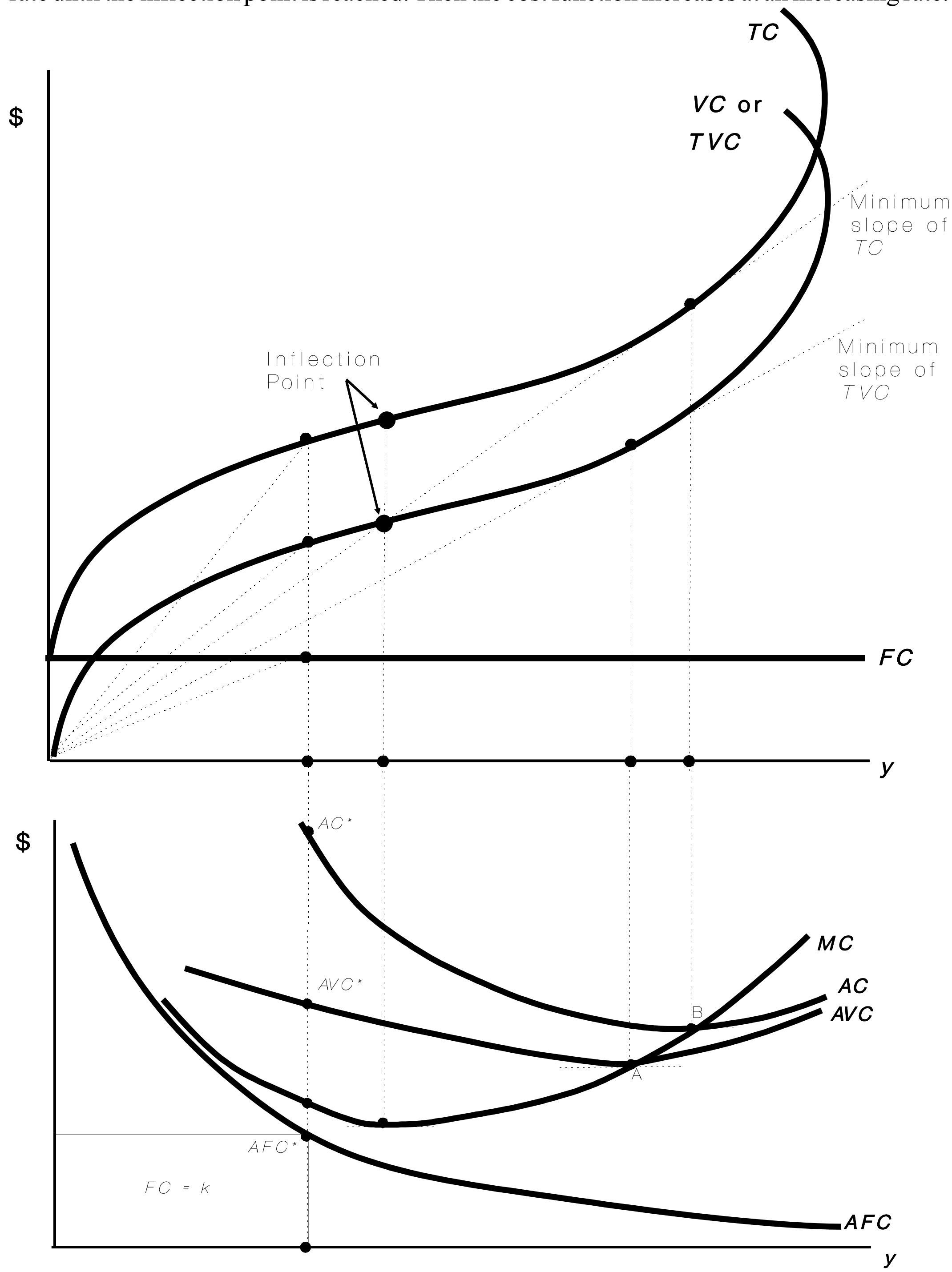


**Figure 4.2 Cost Functions on the Output Side**

The cost curves look rather strange when output reaches its technical maximum. Suppose that the maximum yield a farmer can achieve in the production of corn is 140 bushels per acre. Suppose that despite the farmer's best efforts to increase yields further by applying more seed, fertilizer, and pesticides, the additional yield is just not there. The additional seed results in more plants that become overly crowded in the field, and the additional plants become so crowded that yield is reduced. The additional fertilizer starts to do damage to the crop. The additional herbicides kill the corn plants. As more and more variable inputs are used, yield starts to drop off to 130, 120 or even 110 bushels per acre. Costs for the additional variable inputs are incurred even at yield levels that could have been achieved with a much lower level of input use and a corresponding reduction in the cost for seed, fertilizer, and pesticide. The variable-cost function must turn back on itself once the maximum yield is achieved. This is actually stage III of variable cost.

Once variable cost turns back on itself, it is no longer technically a function. This is because for some yield levels, two rather than one value for variable cost is assigned. Thus *VC* might be thought of in this case as a cost correspondence rather than a cost function.

Fixed cost (*FC*), being constant, is a horizontal line positioned at the corresponding dollar value on the vertical axis. Total cost (*TC* ) appears nearly the same as variable cost (*VC*). Total cost has been shifted vertically by the fixed-cost amount. The difference between *TC* and *VC* at any point is *FC*. *TC* and *VC* are not parallel to each other, because *FC* is represented by the vertical distance between *TC* and *VC*. At each level of output, however, the slope of *TC* equals the slope of *VC*.

Any point on the average cost curves (*AC*, *AVC*, and *AFC*) can be represented by the slope of a line drawn from the origin of the graph to the corresponding point on *TC*, *VC* or *FC*. Suppose that the value for *AC*, *AVC* and *AFC* at some output level called y* is to be determined. Draw a vertical line from y* to the corresponding point on *TC VC* and *FC*. Call these points *TC**, *VC**, and *FC**. Now draw a line from each of these points to the origin of the graph. Three triangles will result. The slope of each of these triangles represents the corresponding *AC**, *AVC** and *AFC** for the output level y* (Figure 4.2).

Marginal cost (*MC*) at any point is represented by the slope of a line drawn tangent to either *TC* or *VC*. The minimum slope for both *TC* and *VC* occurs at the respective inflection points of *TC* and *VC*. The inflection points for both *TC* and *VC* correspond to the same level of output. Thus *MC* is minimum at the inflection point of either the *TC* or the *VC* curve, and there is but one *MC* curve that can be derived from either the *TC* or the *VC* curve.

Minimum *AVC* occurs where a line drawn from the origin comes tangent to *VC*. Minimum *AC* occurs where a line drawn from the origin comes tangent to *TC*. The point of tangency on *TC* occurs to the right of the point of tangency on *VC*. Thus the minimum *AC* will occur to the right of the minimum *AVC*. Since these lines are tangent to *TC* and *VC*, they also represent the slopes of the curves at the two points. Hence they also represent *MC* at the two points. Therefore, *MC* must be equal to and cut *AVC* and *AC* at each respective minimum (points A and B, Figure 4.2).

The relationship that must exist between *AC* and *MC* can be proven

(4.13) $TC = (AC)y$

(4.14) $dTC/dy = AC{\cdot}1 + y(dAC/dy)$

(4.15) $MC = AC + y(\text{the slope of } AC)$