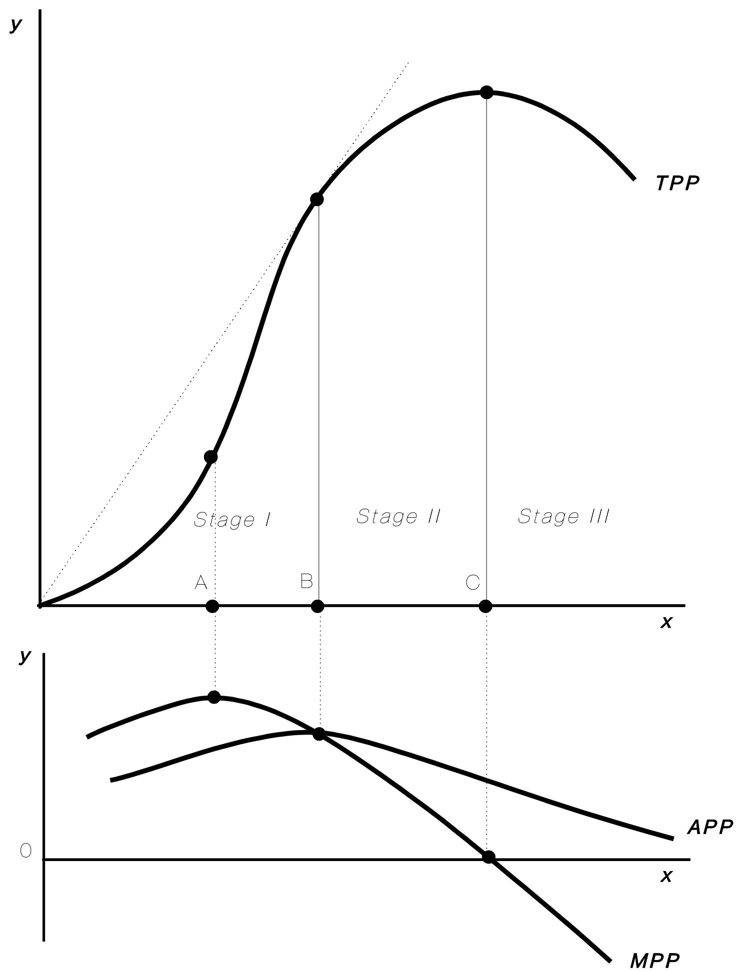

Figure 3.4 Stages of Production for a Neoclassical Production Function

chooses a goal inconsistent with the maximization of net returns, or profit. Stage II is sometimes called the rational stage, or economic region of production. This terminology suggests that rational farmers who have as their goal profit maximization will be found operating within this region. However, in certain instances, such as when dollars available for the purchase of inputs are limited, a rational farmer may not always operate in stage II of the production function.

Stage I of the neoclassical production function includes input levels from zero units up to the level of use where *MPP* = *APP*. *Stage II* includes the region from the point where *MPP* = *APP* to the point where the production function reaches its maximum and *MPP* is zero. *Stage III* includes the region where the production function is declining and *MPP* is negative.

The stages of production can also be described in terms of the elasticity of production. For the neoclassical production function, as the level of input use increases, the elasticity of production (E_p) also changes because the elasticity of production is equal to the ratio of *MPP* to *APP*. The value for the elasticity of production identifies the stage of production. If E_p is greater than 1, then *MPP* is greater than *APP* and we are in stage I. Stage I ends and stage II begins at the point where $E_p = 1$ and *MPP* = *APP*. Stage II ends and stage III begins at the point where E_p equals zero and *MPP* is also zero. Stage III exists anywhere that E_p is negative and hence *MPP* is also negative. Notice that the first stage of the neoclassical production function ends and the second stage begins at the point where the marginal product of the incremental or last unit of input *x* just equals the average product for all units of the input *x*.

It is easy to understand why a rational farmer interested in maximizing profits would never choose to operate in stage III (beyond point C, Figure 3.4). It would never make sense to apply inputs if, on so doing, output was reduced. Even if fertilizer were free, a farmer would never apply fertilizer beyond the point of output maximum.

Output could be increased and costs reduced by reducing the level of input use. The farmer would always make greater net returns by reducing the use of inputs such that he or she were operating instead in stage II.

It is also easy to see why a farmer would not choose to produce in the region where *MPP* is increasing (point A, Figure 3.4) in the first part of stage I, if output prices were constant and sufficient funds were available for the purchase of *x*. In this region, the marginal product of the input is increasing as more and more of the input is used. Diminishing marginal returns have not yet set in, and each additional unit of input used will produce a greater and greater additional net return. The additional return occurs despite the fact that for the first few units, the *MPP* for the incremental unit might still be below the cost of the incremental unit, as represented by the constant *MFC* function.

It is difficult to see why a farmer would not choose to operate in the second part of stage I, where *MPP* is declining but *APP* is increasing (line AB, Figure 3.4), if output prices were constant and sufficient funds were available to purchase additional units of *x*. However, using the definition

(3.67) $$AVP = p°APP = p°y/x.$$

the total value of the product (*TVP*) might then be defined as

(3.68) $$TVP = \mathrm{x}AVP = \mathrm{x}p°y/x = p°TPP$$

Look at Figure 3.5. Pick any level of input use and call that level x* corresponding with point A on Figure 3.5. Now draw a vertical line from the horizontal axis to the corresponding point on the *AVP* curve (point B). The value of the *AVP* curve at x* represents the average revenue obtained from the sale of output per unit of *x* used, assuming that the total amount of used was x*. With constant output prices, *AVP* might be thought of as the average revenue expressed per unit of *x* used. Now draw a horizontal line from the point on the *AVP* curve to the vertical axis. The length of the horizontal line represents the total amount of *x* used, or *x**. A rectangle has now been formed, with the lower sides being the axes of the graph. Thus, the rectangle OABE in Figure 3.5 represents the *TVP* for $x = x^*$. This is because the length of the rectangle is x* and its height is *AVP*.

Now draw a line from x^* to *MFC*. Another rectangle is formed by OACD. Input prices are assumed to be constant $v° = MFC$. Since $v°$ is constant, $v°$ is equal to the average cost of a unit of *x*; or $TFC = v°\mathrm{x}$ and $AFC = (v°\mathrm{x})/x = v° = dTFC/dx = MFC$. Then *TFC* at $x = x^*$ is equal to the area contained in the second rectangle as measured by OACD.

Profit equals returns less costs.

(3.69) $$\Pi = TVP - TFC.$$

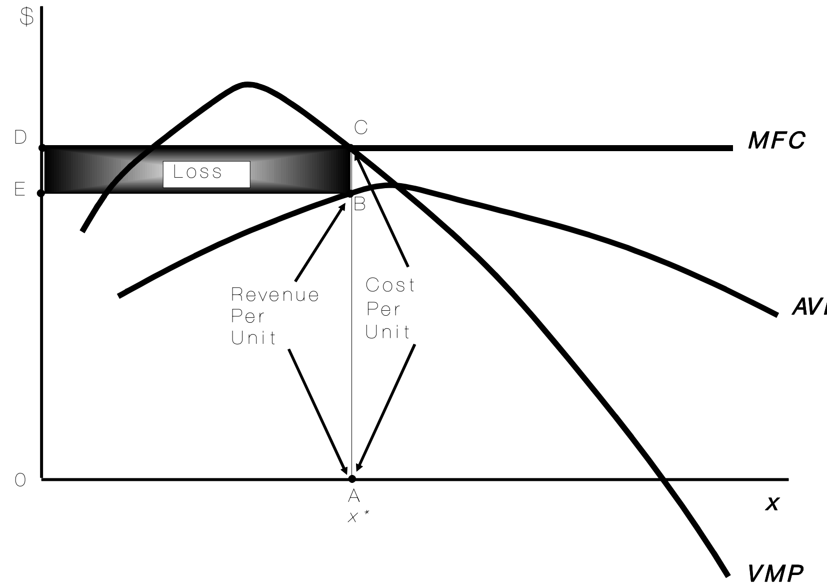

Figure 3.5 If *VMP* is Greater than *AVP*, the Farmer Will Not Operate

In Figure 3.5, the first rectangle is *TVP* and the second rectangle is *TFC*. Since *TVP* is less than *TFC*, the loss is represented by the rectangle EBCD. Suppose now that the input price is lower than the maximum value for *VMP* but higher than the maximum value for *AVP*. These conditions describe the second part of stage I. The farmer equates *VMP* and *MFC* and finds the resulting profit maximizing level of input use x^*. However, since *AVP* is less than *VMP*, the first rectangle representing *TVP* (OABE) would necessarily be less than the second rectangle representing *TFC* (OACD). This would imply that

$$\Pi = TVP - TFC < 0 \tag{3.70}$$

Moreover, *TVP* < *TFC* occurs everywhere in stage I of the production function. The farmer would lose money if operation were continued in stage I. If the price of the input is higher than the maximum *AVP*, there is no way that the incremental unit of input can produce returns sufficient to cover its incremental cost. Under such circumstances a rational solution would be to use zero units of the input. This situation will be remedied if either of two events occurs: (1) the price of the input declines to a level below the maximum *AVP*, or (2) the price of the output increases such that *AVP* rises. New technology might also cause *APP* to increase, and the result would be an increase in *AVP*.

If *MFC* were below *AVP* in stage I, the farmer could always increase profit by increasing the use of the input. However, a farmer might not be able to always get the funds needed for the purchase of the input. In the special case, the farmer could operate in stage I if funds for the purchase of input *x* were restricted or limited. In this instance, the profit-maximizing level of input use would occur in stage II. Revenues exceed costs at many points within stage I, and the farmer may be better off to use available revenue for the purchase of *x* and to produce in stage I, even if the profit-maximizing point in stage II cannot be achieved. However, the farmer would never want to operate in stage III of the production function, or, for that matter, to the right of the point in stage II representing the profit-maximizing level of input use, assuming positive input and output prices. The profit-maximizing point is most desired, but other points to the left of the profit maximizing point may also generate a positive profit for the farmer.

3.9 Further Topics on Stages of Production

One of the reasons for the popularity of the neoclassical production function is that it includes all three stages of production. It is worthwhile to examine some features of other production functions in an effort to determine whether or not the various stages of production are accurately represented. As a starting point, a simple function might be

$$(3.71) \qquad y = bx$$

As indicated earlier, The $MPP(dy/dx)$ for this function is b and the $APP(y/x)$ equal to b. The elasticity of production (MPP/APP) is b/b, which is 1 everywhere. This implies that this function does not have any identifiable stages. The curious conclusion is that the function is at the dividing point between stages I and II throughout its range. No wonder this function has not proven popular with economists. If py [output (y) times its price (p)] were greater than bx [input (x) times its marginal product], profit maximization would entail obtaining as much x as one could possibly obtain, and producing as much y as possible. At some point, input prices would not hold constant, and hence the purely competitive assumptions would break down.

A production function with a constant slope produces VMP and MFC curves that are both horizontal lines, with VMP above MFC. For a given level of input use, the area under VMP represents returns, and the area under the MFC represents costs. The portion of the rectangles that do not overlap represents profits. If py were less than bx, returns would not cover costs and the farm would maximize profits by shutting down and producing no output. If py exactly equaled bx, the farmer would be indifferent toward producing or shutting down, since zero profit would result in either case.

Now consider the case where the production function is

$$(3.72) \quad y = \sqrt{x}$$

As indicated earlier, the elasticity of production in this case is 0.5 throughout the range of the function, since the ratio MPP/APP is 0.5. This suggests that the farm is in stage II of the production function everywhere. Notice that this stage II is not a simple representation of stage II from the neoclassical production function. The elasticity of production for the neoclassical function decreases from 1 (at the start of stage II) to 0 (at the end of stage II) as the use of the input is increased.

For this production function, the elasticity of production remains constant. For any production function of the form $y = bx^{\alpha}$, the elasticity of production is equal to the constant α. If α is greater than 1, the production function is in stage I everywhere. If α is less than zero the function is in stage III everywhere. The function $y = bx$ is a special case of this function with α equal to 1.

3.10 The Imputed Value of an Additional Unit of an Input

For profits to be maximum, a necessary condition is that the slope of the TVP function be equal to the slope of the total factor cost function. This might also be expressed as

$$(3.73) \qquad VMP = MFC = v^{\circ}$$

$$(3.74) \qquad p^{\circ}MPP = MFC = v^{\circ}$$

(3.75) $p°dTPP/dx = dTFC/dx$

or

(3.76) $p°dy/dx = v°$

Equations (3.73) to (3.76) all describe the necessary condition for profit maximization. (The sufficient condition requires that *VMP* equal *MFC* and that the *VMP* curve intersect the *MFC* curve from above).

Another way of expressing the relationship *VMP* = *MFC* is

(3.77) $VMP/MFC = 1$

VMP is the return obtained from the incremental unit of *x*, or the value to the manager of the incremental unit of *x*. *MFC* is the cost of the incremental unit of *x*. The equation *VMP* = *MFC* is a decision rule that tells the farmer how much input should be used in order to maximize profits. This decision rule states that the use of the input should be increased until the point is reached whereby the last dollar spent on the input returns exactly its incremental cost. This is one of the fundamental marginal rules of economics. Many if not most of the previous incremental dollars spent on the input paid back more than the cost of the input. These units, taken together, generate the profit for the farm (Figure 3.6).

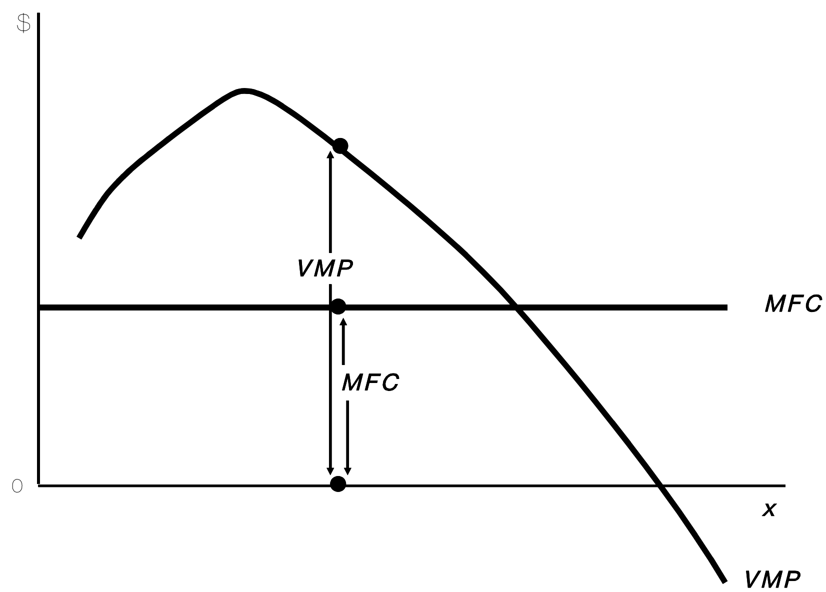

Figure 3.6 The Relationship Between *VMP* and *MFC* Illustrating the Imputed Value of an Input

Now suppose that

(3.78) $VMP/MFC = 3$

Equation (3.78) states that the value of the last dollar spent on the input in terms of its contribution to revenue to for the farm is three times its cost. Moreover, the last dollar spent on the input returns $3 to the farm. This number is sometimes referred to as the *imputed value* or *implicit worth* of the incremental dollar spent on the input. Both terms refer to the same concept.

There is no particular reason to believe that the imputed value, or implicit worth of the last dollar spent on an input should necessarily be a dollar. The implicit worth of the last dollar spent on an input may be greater than, equal to, or less than a dollar. However, a necessary condition for profit maximization is for *VMP* to equal *MFC*. Profit maximization requires that the value of the last dollar spent on the input be a dollar. If profits are max-imized, the imputed value of an input will be 1 since its contribution to revenue exactly covers its cost. If the imputed value is 3, as in this instance, profits could be further increased by increasing the use of the input until the imputed value is reduced to 1.

Now suppose that

$$VMP/MFC = 0.5 \tag{3.79}$$

Equation (3.79) states that the value of the last dollar spent on the input in terms of its contribution to revenue for the farm is only one-half its cost. This is a point to the right of the profit-maximizing point, although it is still in stage II. Revenue from the sale of the output produced by the last unit of input only covers 50 percent of the cost or price of the input. The last dollar spent returns only 50 cents. The other 50 cents is loss. In this case, profits to the farm could probably be increased by reducing the use of the input. Since the *MPP* of the input usually increases as its use decreases, this has the effect of raising *MPP* and thus increasing *VMP* for the input.

Now suppose that

$$VMP/MFC = 0 \tag{3.80}$$

Assuming constant positive prices for both the input and output the only way this could happen is if *MPP* were zero. In this instance, the last dollar contributes nothing to revenue. The only point where this could happen is at the maximum of the production (*TPP* or *TVP*) function, the dividing point between stages II and III.

Finally, suppose that

$$VMP/MFC = -5 \tag{3.81}$$

Assuming constant positive prices for both the input and the output the only way this could happen is for *MPP* to be negative. This implies stage III of the production function. In this case, the last dollar spent on the input results in a loss in revenue of $5. This is a point in stage III where the farmer would never produce.

The implicit worth or imputed value of an input or factor of production has also sometimes been called the *shadow price* for the input. It is called a shadow price because it is not the price that the farmer might pay for the input, but rather the value of a dollar spent on the input to the farmer in his or her operation. A farmer might be willing to purchase an input at prices up to but not exceeding the imputed value or shadow price of the input on the farm.

Diagrammatically, the shadow price or imputed value of an input can easily be seen (Figure 3.6). The *VMP* represents the value of the input: the *MFC*, its price or cost per unit. The shadow price is the ratio of value to price. If *MFC* and product prices are constant, the shadow price usually increases until *MPP* reaches its maximum and then decreases. The shadow price is 1 where *MPP* (and *VMP*) intersects *MFC*, and zero where *MPP* intersects the horizontal axis of the graph.

3.11 Concluding Comments

Profit-maximization conditions for the factor-product model have been introduced. Profits are maximum when the necessary and sufficient conditions for a maximum have been met. The necessary conditions for profit maximization require that the profit function have a slope of zero. The necessary condition for profit maximization can be determined by finding the point on the profit function where the first derivative is zero. The sufficient condition, ensuring profit maximization, holds if the first derivative of the profit function is zero and the second derivative of the profit function is negative.

Alternatively, the level of input use that maximizes profits can be found by equating the *VMP* of the input with the *MFC*, which in pure competition is the price of the input. The slope of the total value of the product curve will be equal to the slope of the total factor cost curve. The slope of the total value of the product curve is its derivative, which if output prices are constant is the *VMP* curve. If the price of x is constant, the slope of the total factor cost curve is the *MFC*.

Under the assumptions of pure competition, with constant, positive input and output prices, a farmer interested in maximizing profits would never operate in stage III of the production function, where *MPP* and *VMP* are declining. A farmer would operate in stage I of the production function only if sufficient funds were not available for the purchase of inputs needed to reach stage II. A farmer would not produce at all if the price of x exceeded the maximum average value of the product.

Problems and Exercises

1. Suppose that the output sells for \$5 and the input sells for \$4. Fill in the blanks in the following table.

x (input)	y (output)	*VMP*	*AVP*
0	0		——
		——	
10	50		——
		——	
25	75		——
		——	
40	80		——
		——	
50	85		——

2. In Problem 1, what appears to be the profit-maximizing level of input use? Verify this by calculating *TVP* and *TFC* for each level of input use as shown in the table.

3. Suppose that the production function is given by

$$y = 2x^{0.5}$$

The price of x is \$3 and the price of y is \$4. Derive the corresponding *VMP* and *AVP* functions. What is *MFC*? Solve for the profit-maximizing level for input use x.

4. When the input price is constant, the slope of the total factor cost function will also be constant. Is this statement true or false? Explain.

5. Whenever the total factor cost function and the total value of the product function are parallel to each other, profits will be maximized. Is this statement true or false? Explain.

6. Suppose that the production function is the one found in Problem 5, Chapter 2. Corn sells for $4.00 per bushel and nitrogen sells for $ 0.20 per pound. At what nitrogen application rate are profits maximized?

7. Explain the terms *necessary* and *sufficient*, in terms of a farmer seeking to maximize profits in the feeding of dairy cattle for milk production.

8. Is the shadow price of a dairy feed ration different from the price the farmer pays per pound of the ration? Explain. Of what importance is a shadow price to a farmer seeking to maximize profits from a dairy herd?

9. Explain the consequences to the farmer if the production function for milk were a linear function of the amount of feed fed to each cow.

10. Verify each of the numbers presented in Figure 3.3.