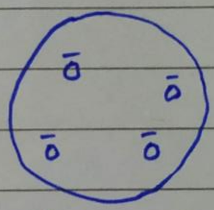
Atomic model

J.J Thomson model (1898)

plump pudding model of atom

uniformly distribution of positive charge plump, pudding with negative charge electrons just like kishmish in custurd or raisins in fruitcake

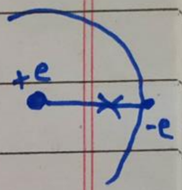


According to Thomson model alpha particles that pass through a thin foil ought to be deflected only slightly, 1° or less. B

Geiger and Marsden found that few α-particles scattered through very large angles

Rutherford explained the result by pictured an atom as being composed of a tiny

Rutherford model in view of [classical physics]



$$F_c = \frac{mv^2}{r}$$

$$F_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$$

$$v = \frac{e}{\sqrt{4\pi\varepsilon_0 m r}}$$

Energy of electron

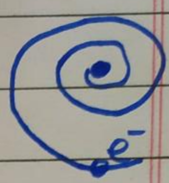
$$E = K.E + P.E$$

$$E = \frac{1}{2} mv^2 + \left(-\frac{e^2}{4\pi\varepsilon_0 r}\right)$$

$$E = \frac{-e^2}{8\pi\varepsilon_0 r}$$

Failure of classical physics

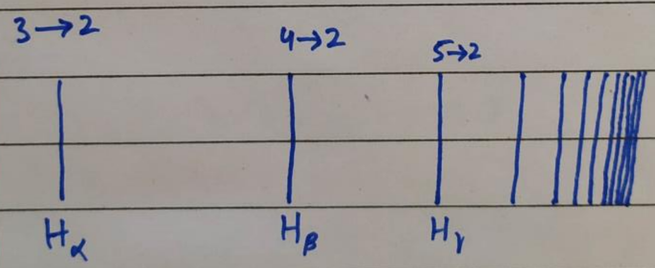
→ Newtons laws of motion



→ Columb's law of electric force

→ electromagnetic theory (accelerated charge particle emitts radiations)

Spectral series



$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$ Balmer Series

Bohar theory

$$\lambda = \frac{h}{mv}$$

Mon Tue Wed Thu Fri Sat Date: __/__/20

as $v = \dfrac{e}{\sqrt{4\pi\varepsilon_0 m r}}$

$$\lambda = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_0 r}{m}}$$

$$\lambda = 33 \times 10^{-11}$$

from circumferaence of electron orbit

$$2\pi r = 33 \times 10^{-11}\ m$$

So

$$n\lambda = 2\pi r$$

$$n\left(\frac{h}{mv}\right) = 2\pi r$$

$$\frac{nh}{2\pi} = mvr$$

$$n\hbar = L$$

↳ Bohar theory

$$n\lambda = 2\pi r$$

$$n\frac{h}{e}\sqrt{\frac{4\pi\varepsilon_0 r}{m}} = 2\pi r$$

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

$n = 1, 2, 3, \ldots$

$$r_n = a_0 n^2$$

Energy level and spectra

$$E = \frac{-e^2}{8\pi\epsilon_0 r_n}$$

$$E_n = \frac{-me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

$$E_n = -\frac{E_0}{n^2} = -\frac{E_1}{n^2}$$

$E_0 = E_1$

Mon Tue Wed Thu Fri Sat

Date: __/__/20

$$hf = E_f - E_i$$

$$f = \frac{-E_1}{h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{-E_1}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Nuclear motion

reduce mass

$$m' = \frac{mM}{m+M}$$

$$E_n' = -\frac{m'e^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

$$E_0 = \frac{me^4}{8\epsilon_0^2 h^2}$$

$$= -\frac{m'e^4}{8\epsilon_0^2 h^2}\,\frac{m}{m}\left(\frac{1}{n^2}\right)$$

$$= -\frac{me^4}{8\epsilon_0^2 h^2}\,\frac{m'}{m}\left(\frac{1}{n^2}\right)$$

$$E_n' = -\frac{m'}{m}\left(\frac{E_0}{n^2}\right)$$

$$\frac{m'}{m} = 1 \qquad m' = m$$

$$\frac{m'}{m} = 0.99945 \qquad m' \approx m$$

XIII. TOTAL ENERGY OF AN ELECTRON IN AN ELLIPTICAL ORBIT

The tangential velocity of the electron at any instant can be resolved into two components: one along the radius vector called the *radial velocity* $\left[\frac{dr}{dt}\right]$ and the other at right angle to the radius vector called the *trans - verse velocity* $r\left[\frac{d\phi}{dt}\right]$

Thus, the radial momentum, $$p_r = m\frac{dr}{dt}, \tag{2.28}$$

the orbital angular momentum, $$p_\phi = mr^2\frac{d\phi}{dt} \tag{2.29}$$

The kinetic energy of the revolving electron is given by

$$K \cdot E \cdot = \frac{1}{2}mv^2 = \frac{1}{2}m\left[\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2\right] \tag{2.30}$$

$$K \cdot E \cdot = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2}$$

Again

$$\left(\frac{dr}{dt}\right) = \left(\frac{dr}{d\phi}\right)\left(\frac{d\phi}{dt}\right)$$

Thus from equation (2.29)

$$\left(\frac{dr}{dt}\right) = \left(\frac{dr}{d\phi}\right)\left(\frac{p_\phi}{mr^2}\right)$$

and

$$p_\phi^2 = m^2r^4\left(\frac{d\phi}{dt}\right)^2$$

$$r^2\left[\frac{d\phi}{dt}\right]^2 = \frac{p_\phi^2}{m^2r^2}$$

Substituting these values in equation (2.30), we get

$$K \cdot E \cdot = \frac{1}{2}m\left[\frac{p_\phi^2}{m^2r^4}\left(\frac{dr}{d\phi}\right)^2 + \frac{p_\phi^2}{m^2r^2}\right]$$

$$= \frac{p_\phi^2}{2mr^2}\left[\left(\frac{1}{r}\frac{dr}{d\phi}\right)^2 + 1\right]$$

$$P \cdot E \cdot = -\frac{Ze^2}{4\pi \epsilon_0 r}$$

Thus the total energy of the electron

$$E_n = \frac{p_\phi^2}{2mr^2}\left[\left(\frac{1}{r}\frac{dr}{d\phi}\right)^2 + 1\right] - \frac{Ze^2}{4\pi \in_0 r} \tag{2.31}$$

or

$$\frac{p_\phi^2}{2mr^2}\left[\left(\frac{1}{r}\frac{dr}{d\phi}\right)^2 + 1\right] = E_n + \frac{Ze^2}{4\pi \in_0 r}$$

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = \frac{2mr^2E_n}{p_\phi^2} + \frac{mrZe^2}{2\pi \in_0 p_\phi^2} - 1 \tag{2.32}$$

From equation (2.21)

$$\frac{1}{r^2}\left[\frac{dr}{d\phi}\right]^2 = \frac{\varepsilon^2 r^2 \sin^2\phi}{[a(1-\varepsilon^2)]^2} = \frac{\varepsilon^2 r^2(1-\cos^2\phi)}{[a(1-\varepsilon^2)]^2} \tag{2.32a}$$

$$\frac{1}{r^2}\left[\frac{dr}{d\phi}\right]^2 = \left[\frac{r}{a(1-\varepsilon^2)}\right]^2 (\varepsilon^2 - \varepsilon^2\cos^2\phi) \tag{2.33}$$

The term $(\varepsilon^2 - \varepsilon^2\cos^2\phi)$ of this equation can be further simplified by referring to equation (2.20)

$$(1 + \varepsilon\cos\phi) = \frac{a(1-\varepsilon^2)}{r}$$

$$\varepsilon\cos\phi = \frac{a(1-\varepsilon^2)}{r} - 1$$

$$(\varepsilon\cos\phi)^2 = \left[\frac{a(1-\varepsilon^2)}{r} - 1\right]^2$$

Substituting this in equation (2.32*a*) we get

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = \left[\frac{r}{a(1-\varepsilon)^2)}\right]^2\left[\varepsilon^2 - \left\{\frac{a(1-\varepsilon^2)}{r} - 1\right\}^2\right]$$

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = \frac{r^2}{a^2(1-\varepsilon^2)^2}\left[\varepsilon^2 - \left\{\frac{a^2(1-\varepsilon^2)^2}{r^2} + 1 - \frac{2a(1-\varepsilon^2)}{r}\right\}\right]$$

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = \frac{r^2\varepsilon^2}{a^2(1-\varepsilon^2)^2} - 1 - \frac{r^2}{a^2(1-\varepsilon^2)^2} + \frac{2r}{a(1-\varepsilon^2)}$$

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = \frac{r^2\varepsilon^2}{a^2(1-\varepsilon^2)^2} - \frac{r^2}{a^2(1-\varepsilon^2)^2} + \frac{2r}{a(1-\varepsilon^2)} - 1$$

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = \frac{r^2\varepsilon^2 - r^2}{a^2(1-\varepsilon^2)^2} + \frac{2r}{a(1-\varepsilon^2)} - 1$$

$$\left[\frac{1}{r}\frac{dr}{d\phi}\right]^2 = -\frac{r^2(1-\varepsilon^2)}{a^2(1-\varepsilon^2)^2} + \frac{2r}{a(1-\varepsilon^2)} - 1 \tag{2.34}$$

Equating the coefficients of r^2 and r in equations (2.34) and (2.32), we get

$$\frac{2mE_n}{p_\phi^2} = -\frac{1}{a^2(1-\varepsilon^2)} \tag{2.35}$$

and

$$\frac{mZe^2}{2\pi \in_0 p_\phi^2} = \frac{2}{a(1-\varepsilon^2)} \tag{2.36}$$

Thus equation (2.35) becomes

$$E_n = -\frac{p_\phi^2}{2ma^2(1-\varepsilon^2)} \tag{2.37}$$

Substituting the value of $(1-\varepsilon^2)$ from equation (2.36) in equation (2.37), we get

$$E_n = -\frac{p_\phi^2}{2ma^2}\left[\frac{amZe^2}{4\pi \in_0 p_\phi^2}\right]$$

$$E_n = -\frac{Ze^2}{8\pi a \in_0} \tag{2.38}$$

Again substituting for a from equation (2.36), we get

$$E_n = -\left(\frac{Ze^2}{8\pi \in_0}\right)\left(\frac{mZe^2}{2\pi \in_0 p_\phi^2}\right)\left(\frac{1-\varepsilon^2}{2}\right)$$

$$E_n = -\left[\frac{mZ^2e^4}{32\pi^2 \in_0^2}\right]\frac{(1-\varepsilon^2)}{p_\phi^2}$$

Substituting for $(1-\varepsilon^2)$ and p_ϕ^2 from equations (2.25) and (2.19) respectively, we get

$$E_n = -\left[\frac{mZ^2e^4}{32\pi^2 \in_0^2}\right]\left(\frac{n_\phi}{n}\right)^2\left(\frac{2\pi}{n_\phi h}\right)^2$$

$$\boxed{E_n = -\frac{mZ^2e^4}{8 \in_0^2 h^2 n^2} = -\frac{mZ^2e^4}{8 \in_0^2 h^2}\left[\frac{1}{n_r + n_\phi}\right]^2} \tag{2.39}$$

which is identical with the expression for the energy of the electron in a circular orbit of quantum number n. Thus the introduction of elliptical orbits does not result in the production of new energy terms; hence no new spectral lines are to be expected because of this multiplicity of orbits. Thus the introduction of elliptical orbits gives no new energy levels and hence no new tranṣition. Hence Sommerfeld's attempt to explain the fine structure of spectral lines failed. But soon, on the basis of variation of mass of the electron with velocity, Sommerfeld was able to find a solution to the problem of fine structure of spectral lines. Here it is

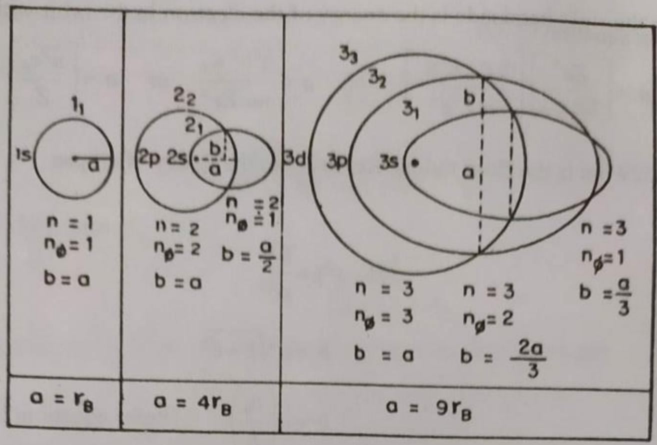


Fig. 2.6 Elliptical Bohr-Sommerfeld orbits for hydrogen.

XIV. SOMMERFELD'S RELATIVISTIC CORRECTION

The velocity of an electron moving in an elliptical orbit varies from point to point in the orbit, being a maximum when the electron is nearest to the nucleus and a minimum when it is farther away from the nucleus. Furthermore, this velocity is quite large $\left(\frac{c}{137}\right)$. According to the theory of relativity, the variation of velocity means variation of mass of the electron.

Sommerfeld, including the relativistic correction in the treatment of elliptical orbits, showed that equation of the path of the electron was not simply that for an ellipse but was of the form

$$\frac{1}{r} = \frac{1 + \varepsilon \cos \psi\phi}{a(1 - \varepsilon^2)}$$

where ψ is given by

$$\psi^2 = 1 - \left[\frac{Ze^2}{4\pi \in_0 pc}\right]^2$$

This is the equation of an ellipse which precesses, i.e., the major axis turns slowly about the focus (the nucleus) in the plane of the ellipse. The path of the electron is, therefore, a *rosette*.

It can be shown that the total energy with a principal quantum number n in the relativistic theory is

$$E_n, n_\phi = -\frac{mZ^2e^4}{8 \in_0^2 h^2 n^2} - \frac{mZ^2e^4\alpha^2}{8 \in_0^2 h^2}\left[\frac{n}{n_\phi} - \frac{3}{4}\right]\frac{1}{n^4} \qquad (2.42)$$

where $\alpha = \frac{e^2}{2 \in_0 ch} = \frac{1}{137}$. α is a dimensionless quantity and is called the *fine structure constant*.

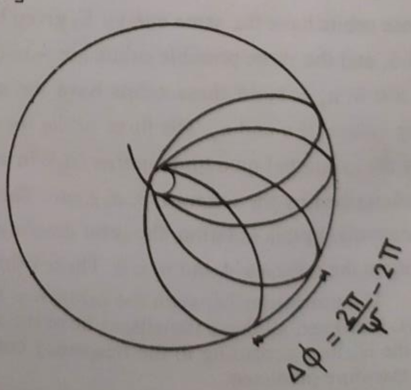


Fig. 2.7 Rosette path of the electron about the nucleus.

The first term on the right hand side is the energy of the electron in the orbit with the principal quantum number n according to Bohr's theory and the second term is Sommerfeld's relativity correction arising from the *rosette motion* of the electron orbit with principal quantum number n and azimuthal quantum number n_ϕ. The dependence of the total energy of the electron in its orbit as given by the equation (2.42) results in a splitting of energy levels in the atom. For a given value of n, there will be n components corresponding to the n permitted values of n_ϕ. Hence multiplicity of spectral lines should appear in hydrogen atom.

XV. FINE STRUCTURE OF H_α LINE

H_α *line* is due to the transition from $n = 3$ state to $n = 2$ state of hydrogen atom. For $n = 3$, there are three possible energy levels corresponding to the three values of $n_\phi = 1, 2$ and 3. Similarly, there are two possible levels for $n = 2$. Therefore, theoretically six transitions are possible:

$$3_3 \rightarrow 2_2;\ 3_3 \rightarrow 2_1;\ 3_2 \rightarrow 2_2;\ 3_2 \rightarrow 2_1;\ 3_1 \rightarrow 2_2;\ 3_1 \rightarrow 2_1$$

and these transitions are shown in Fig. 2.8. Actually, the H_α line has only three components. To make experiment and theory agree, some of the transitions have to be ruled out by some selection rule. The selection rule is that n_ϕ can change only by +1 or −1. i.e., $\Delta n_\phi = \pm 1$.

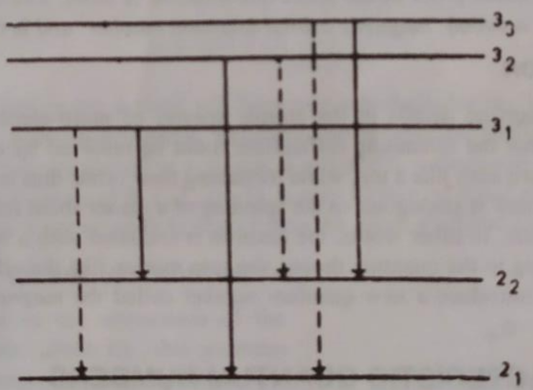
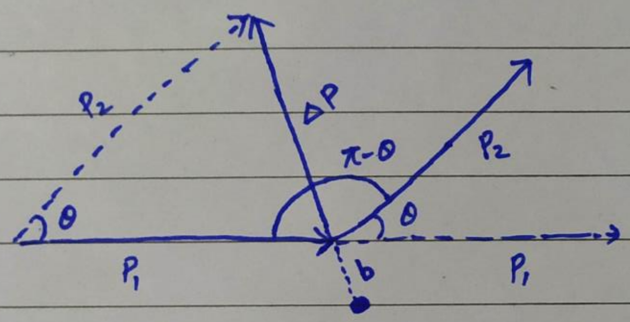


Fig. 2.8 Transitions and forbidden lines.

XVI. DRAWBACKS OF BOHR-SOMMERFELD ATOM MODEL

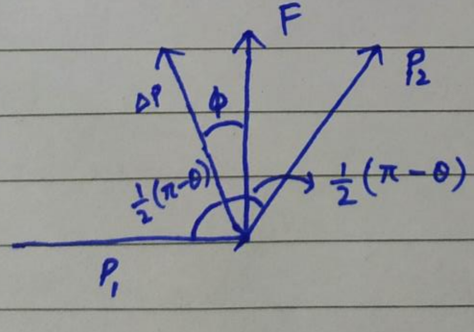
(i) Borhr's theory failed to explain the fine structure of spectral lines even in the simplest hydrogen atom.
(ii) In the case of complex atoms, Bohr-Sommerfeld theory failed to calculate the energy of the system and frequencies of radiation emitted.
(iii) Sommerfeld's modification, though gave a theoretical background of the splitting of individual spectral lines of hydrogen, still it could not predict the correct number of observed fine structure lines.
(iv) Both the models could not explain the distribution and arrangement of electrons in atoms.
(v) Both the models do not throw any light on the intensities of the spectral lines.

nucleus in which its +ve charge and almost all its mass are concentrated



$$P_2 = P_1 + \Delta P$$

$$\Delta P = P_2 - P_1$$



Mon Tue Wed Thu Fri Sat Date: __/__/20__

law of cosine

$$\Delta P \sin\left(\frac{\pi-\theta}{2}\right) = P_2 \sin\theta$$

as $P_1 = P_2 = mv$

$$\frac{\Delta P}{\sin\theta} = \frac{mv}{\sin\left(\frac{\pi-\theta}{2}\right)}$$

$$\Delta P = \frac{mv\sin\theta}{\sin\left(\frac{\pi-\theta}{2}\right)}$$

we know

$$\sin 2\theta = 2\sin\theta\cos\theta$$

also $\sin\theta = 2\sin\theta/2 \cos\theta/2$

$$\Delta p = \frac{mv\ 2\sin\theta/2\cos\theta/2}{\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}$$

As $\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right) = \cos\frac{\theta}{2}$

$$\Delta P = 2mv\sin\theta/2$$

impulse

$$\Delta P = \int_{-\infty}^{+\infty} F \cos\phi \, dt$$

(i) — $$2mv \sin\frac{\theta}{2} = \int_{-\left(\frac{\pi-\theta}{2}\right)}^{+\left(\frac{\pi-\theta}{2}\right)} F \cos\phi \frac{dt}{d\phi} d\phi$$

angular momentum

$$I\omega = mr^2\omega = mr^2 \frac{d\phi}{dt}$$

$\omega = \frac{d\phi}{dt}$

$$mr^2 \frac{d\phi}{dt} = rP$$

$r = b$

$$\cancel{m}r^2 \frac{d\phi}{dt} = b\cancel{m}v$$

$$\frac{d\phi}{dt} = \frac{bv}{r^2}$$

$$\frac{dt}{d\phi} = \frac{r^2}{bv}$$ put in (i)

$$2mv \sin\frac{\theta}{2} = \int F \cos\phi \frac{r^2}{bv} d\phi$$

$$2mv^2 b \sin\frac{\theta}{2} = \int F r^2 \cos\phi \, d\phi$$

as $F = K \frac{q_1 q_2}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0} \frac{(2e)(Ze)}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{r^2}$$

$$2mv^2 b \sin\frac{\theta}{2} = \int \frac{1}{\cancel{2}\,4\pi\varepsilon_0} \frac{2Ze^2}{\cancel{r^2}} \cancel{r^2} \cos\phi \, d\phi$$

$$\frac{4\pi m v^2 b \varepsilon_0}{Ze^2} \sin\frac{\theta}{2} = \int \cos\phi \, d\phi$$

$$= \sin\phi \Big|_{-\left(\frac{\pi-\theta}{2}\right)}^{+\left(\frac{\pi-\theta}{2}\right)}$$

$$= \left[\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right) - \sin\left[-\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right]\right]$$

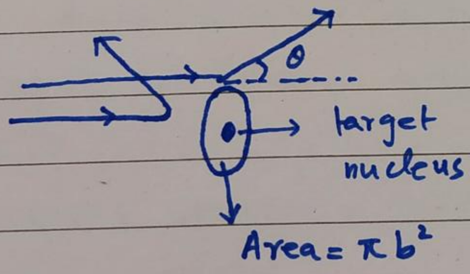
$$= \cos\frac{\theta}{2} + \cos\frac{\theta}{2}$$

$$\frac{4\pi m v^2}{Ze^2} b\varepsilon_0 \sin\frac{\theta}{2} = 2\cos\frac{\theta}{2}$$

$$\frac{2\pi\varepsilon_0 m v^2 b}{Ze^2} = \cot\frac{\theta}{2} \quad \text{——— (ii)}$$

$$\cot\frac{\theta}{2} = \frac{2\pi\varepsilon_0 \, 2\left(\frac{1}{2}mv^2\right)}{Z^2e^2} b$$

$$\cot\frac{\theta}{2} = \frac{4\pi\varepsilon_0 \, K.E}{Ze^2} b$$



area of cross section for interation $= \sigma = \pi b^2$

thickness of foil $= t$

numbr of atoms per unit volume $= n$

no of target nuclei per unit area $= nt =$

$$t \times n = \frac{N}{V} \times t = \frac{N}{A \times \cancel{t}} \times \cancel{t} = \frac{N}{A}$$

no of α-particles incident on area A = ntA = N nuclei

aggregate cross section for Scattering with angle θ is equal to

= ntA = number of nuclei = N

multiply by scattering cross section σ

aggreate cross section = $ntA\sigma$

Fraction of incident α-particle Scattered by θ

$$f = \frac{\text{aggregate cross section}}{\text{target area}}$$

$$= \frac{nt\cancel{A}\sigma}{\cancel{A}}$$

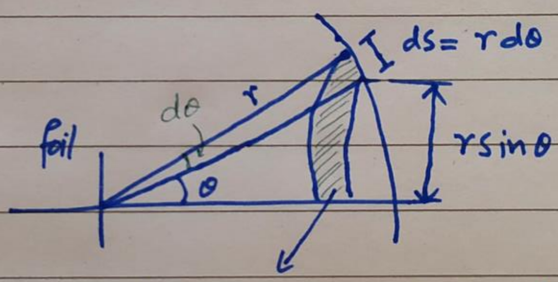
$$f = nt\pi b^2 \qquad \sigma = \pi b^2$$

put value of 'b' from eq (ii) in above

$$f = n\pi t\left(\frac{Ze^2}{4\pi\epsilon_0 KE}\right)^2 \cot^2\frac{\theta}{2}$$

detector measure α- particle
between θ and θ+dθ
So differentiate above eq

$$df = -n\pi t\left(\frac{Ze^2}{4\pi\epsilon_0 KE}\right)^2 \cot\frac{\theta}{2}\csc^2\frac{\theta}{2}$$



$$dA' = \text{Area} = 2\pi(r\sin\theta)\,ds$$

$$dA' = 2\pi r\left[2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]r\,d\theta$$

$$dA' = 4\pi r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}\,d\theta$$

N_i α- particle strikes the foil
no of scattered into dθ at θ
is $N_i df$
$N(\theta)$ α- particles strikes with
detector at angle θ

Mon Tue Wed Thu Fri Sat

Date: __/__/20__

$$N(\theta) = \frac{N_i |df|}{dA'}$$

$$N(\theta) = \frac{N_i \, n t \, Z^2 e^4}{(8\pi\epsilon_0)^2 \, r^2 \, KE^2 \sin^4(\theta/2)}$$