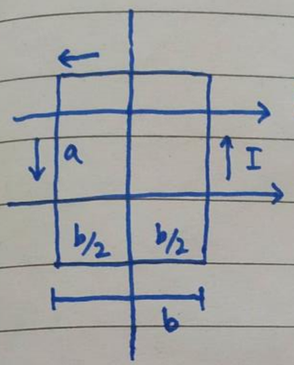
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$\vec{\mu} = I\vec{A}$



curl the fingures in the direction of current, thumb gives you direction of μ

Energy of current carying coil in external magnetic field

we know

$$W = F.\cancel{d}S$$

here 'S' is displacement

$$\Delta W = F.\Delta\cancel{d}S$$

$$dW = F.dS$$

$$\int dW = \int F.dS$$

$$W = \int F.dS$$

$\int dx = x$

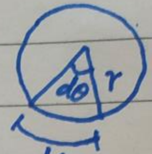
for angular motion work is

$$W = \int \tau . d\theta$$

her $d\theta$ is angular displacement

$$W = \int \tau \cdot d\theta$$

$$W = \int \tau \cdot \frac{ds}{r}$$



ds

as $\tau = rF$

$ds = r\,d\theta$

$$W = \int \cancel{r}F \cdot \frac{ds}{\cancel{r}}$$

$\frac{ds}{r} = d\theta$

$$W = \int F \cdot ds$$

which is the case of linear motion

as we know that work done is equal to change in potential energy or kinetic energy

so

$$W = \Delta U$$

$$W = \Delta U = \int_{\theta_i}^{\theta_f} \tau \cdot d\theta$$

as $\bar{\tau} = \bar{\mu} \times \bar{B}$

$\bar{\tau} = \mu B \sin\theta$

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$$\Delta U = \int_{\theta_i}^{\theta_f} \mu B \sin\theta \, d\theta$$

$\alpha = 0$

$\tau \parallel d\theta$

$\tau . d\theta$

$\tau d\theta \cos\alpha$

$\alpha = 0$

$\tau d\theta$

$$= \mu B \int_{\theta_i}^{\theta_f} \sin\theta d\theta$$

$$= \mu B (-\cos\theta)\Big|_{\theta_i}^{\theta_f}$$

$$\Delta U = -\mu B [\cos\theta_f - \cos\theta_i]$$

if we take initial angle $\theta_i = 90^\circ$ between μ & B

$$\Delta U = -\mu B [\cos\theta_f - \cos 90]$$

$$\Delta U = -\mu B \cos\theta_f$$

$\cos 90 = 0$

$$\Delta U = -\mu B \cos\theta$$

take $\theta_f = \theta$

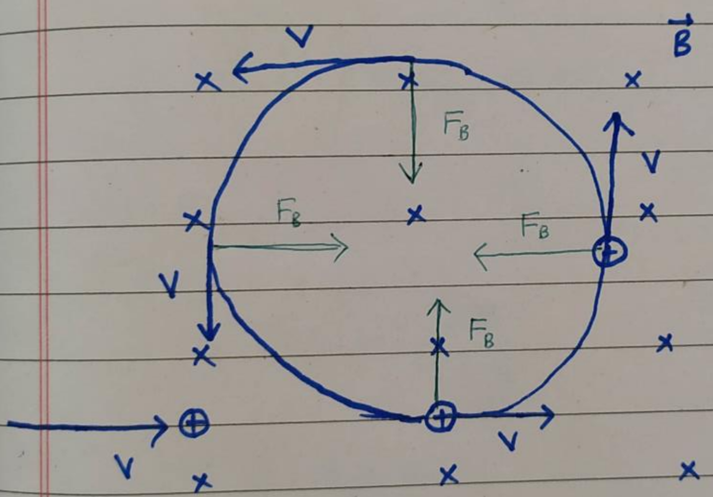
$$\Delta U = -\bar{\mu} . \bar{B}$$

$$\boxed{\Delta U = -\bar{u} . \bar{B}}$$

charge particle in a uniform magnetic field

sign of into the page ×
or away from you

sign of out of the page •
or towards you



$$F = qvB\sin\theta \qquad v \perp B$$
$$F = qvB \qquad \theta = 90$$
$$\sin 90 = 1$$

$$qv\cancel{v}B = \frac{mv^{\cancel{2}}}{r}$$

$$r = \frac{mv}{qB}$$

as $V = r\omega$

$$\omega = \frac{V}{r}$$

$$\omega = \frac{\cancel{V}}{\frac{m\cancel{V}}{qB}} = \frac{qB}{m}$$

so $$\omega = \frac{qB}{m}$$

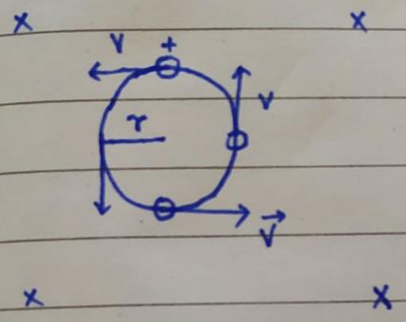
we also know that

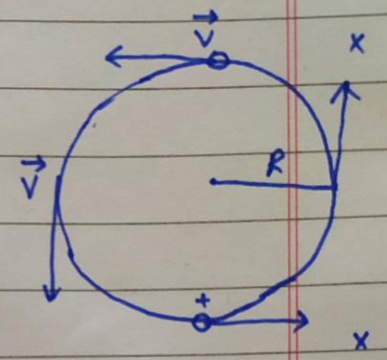
$$\omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\frac{qB}{m}}$$

$$T = \frac{2\pi m}{qB}$$

$\vec{B}$ (high) $\qquad\qquad$ $\vec{B}$ (low)





$$F_h = qvB_h$$

$$R > r$$

$$F_L = qvB_L$$

$$F_h = F_c$$

$$F_L = F_c$$

$$qvB_h = \frac{mv^2}{r}$$

$$qvB_L = \frac{mv^2}{R}$$

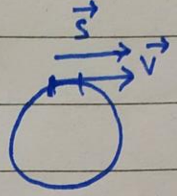
$$qB_h = \frac{mv}{r}$$

$$qB_L = \frac{mv}{R}$$

velocity of charge is same in both high field region and low field region

As $\qquad \bar{F} = q(\bar{v} \times \bar{B})$

$\vec{F}$ is perpendicular to both $\vec{V}$ and $\vec{B}$



velocity and displacement are parallel so

$\vec{F}$ ~~is~~ is also perpendicular to displacement $\vec{s}$

$$\vec{F} \perp \vec{S}$$

$$\theta = 90^\circ$$

$$W = F S \cos\theta$$

$$W = 0$$

so magnetic force should not change the energy of charge particle

we derived in last lecture

$$\omega = \frac{qB}{m}$$

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

$$\omega \propto B$$

But $v = r\omega$

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$$\omega = \frac{v}{r}$$

$$\frac{v}{r} = \frac{qB}{m}$$

as v is unchanged or constant so, r changes with change in B

$$\frac{1}{r} \propto B$$

it mean ω also changes ~~with~~ if v is constant

$$\omega = \frac{v}{r}$$

does it mean rotatational $K.E_{rot}$ also change

As

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

$$\omega_{large} = \frac{V}{r_{small}}$$

$$K.E_{rot} = \frac{1}{2} m r^2 \omega$$ $I = mr^2$

$$= \frac{1}{2} m r_s^2 \omega_l$$

$K.E_{rot}$ = unchanged

also

$$K.E_{rot} = \frac{1}{2} m r^2 \left(\frac{v^2}{r^2}\right)$$ $I = mr^2$

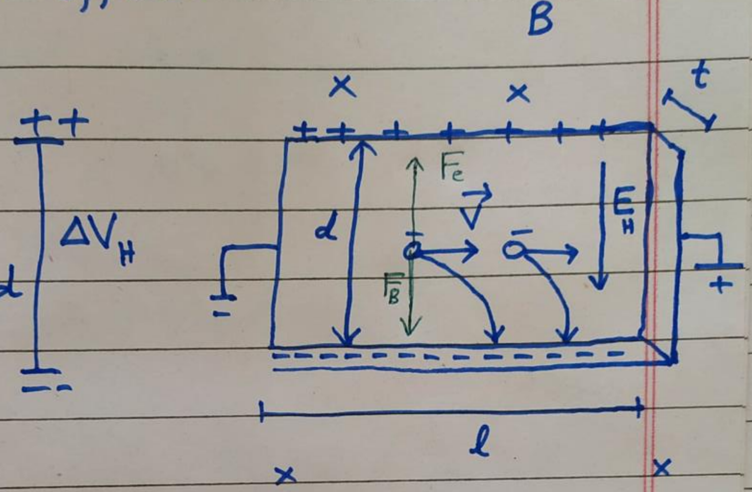
$\omega = \frac{v}{r}$

$$= \frac{1}{2} m v^2$$

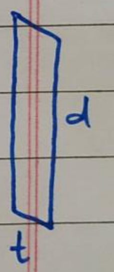
as v is constant

so $K.E_{rot}$ also constant

Hall effect



at equilibrium condition



$$F_e = F_B$$

$$\cancel{q} E_H = \cancel{q} V_d B$$

(i) —— $E_H = V_d B$

$A = td$

$V = l A$

here V_d is drift velocity of current carriers

we know

$$I = \frac{Q}{t}$$

$$I = \frac{n q V}{t}$$

here 'n' is charge carriers' per unit volume

$$I = nq\frac{lA}{t} \qquad V_d = \frac{l}{t}$$

$$I = nqV_dA$$

put in

$$E_H = \left(\frac{I}{nqA}\right)B$$

we also know that

$$-\frac{\Delta\vec{V}}{\Delta r} = \vec{E}$$

$$E = \frac{\Delta V}{d}$$

$$E_H = \frac{\Delta V_H}{d}$$

substitute in above

$$\frac{\Delta V_H}{d} = \frac{I}{nqA}B$$

$$\Delta V_H = \frac{Id}{nqA}B \qquad A = dt$$

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$$\Delta V_H = \frac{I \cancel{d}}{n q \cancel{d} t} B$$

$$\Delta V_H = \frac{IB}{nqt}$$

$$R_H = \frac{1}{nq}$$

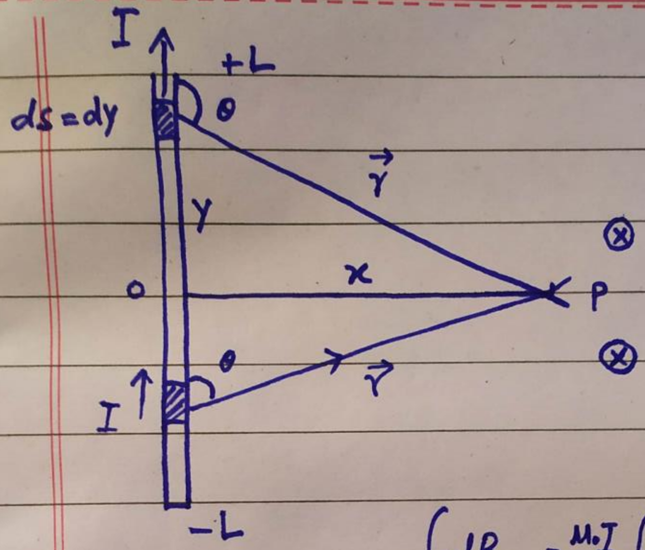
$$\Delta V_H = R_H \frac{IB}{t}$$

here 't' is thickness of conducting material

if R_H is +ve carrier of current are positive

if R_H is -ve the current is due to -ve charge carrier's

$$R_H = \frac{\Delta V_H \, t}{IB}$$



$$\int dB = \frac{\mu_0 I}{4\pi}\int \frac{\vec{dy} \times \hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{dy \sin\theta}{r^2}\,(-\hat{k}) \qquad |\hat{r}| = 1$$

$$B = I \int_{-L}^{+L} \frac{dy \;\; (x/r)}{(x^2+y^2)} \qquad r^2 = x^2 + y^2$$

$$= I \int_{-L}^{+L} \frac{x \; dy}{(x^2+y^2)^{3/2}}$$

$$y = \tan\theta . x$$

$$= 2I \int_{0}^{+L} \frac{x \; dy}{(x^2+y^2)^{3/2}} \qquad dy = x \sec^2\theta d\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$= 2I \int \frac{x^2 \sec^2\theta d\theta}{x^3 (1 + \tan^2\theta)^{3/2}}$$

$$B = 2I \int \frac{x^2 \sec^2\theta \, d\theta}{x^3 (\sec^2\theta)^{3/2}}$$

$$= 2I \int \frac{d\theta}{x \sec\theta} \qquad = \frac{\mu_0 2I}{4\pi x} \int \cos\theta \, d\theta$$

$$B = \frac{\mu_0}{4\pi} \frac{2I}{x} \int_0^{+L} \cos\theta \, d\theta \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

(i) $$= \frac{\mu_0}{4\pi} \frac{2I}{x} (\sin\theta)\Big|_0^{+L} \qquad \cos\theta \tan\theta = \sin\theta$$

we know that ~~sin θ~~ = 2

$y = x \tan\theta$

for $y \to 0$ $\qquad 0 = \tan\theta$

$0 = \theta$

for infinit long conducting wire

$y \to \infty$ $\qquad \frac{\infty}{x} = \tan\theta$

$\tan^{-1}(\infty) = \theta$

$\pi/2 = \theta$

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put in eq (i)

$$B = \frac{\mu_0 I}{2\pi x} \sin\theta \Big|_0^{\pi/2}$$

$$= \frac{\mu_0 I}{2\pi x}\left[\sin 90^\circ - \sin 0\right]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi x}}$$

for finite length wire eq (i)

$$B = \frac{\mu_0 I}{2\pi x} \sin\theta \Big|_0^L$$

(ii) $\longrightarrow$ $B = \frac{\mu_0 I}{2\pi x} \sin\left(\tan^{-1}\frac{y}{x}\right)\Big|_0^L$

$$\boxed{\begin{gathered}\frac{y}{x} = \tan\theta \\ \tan^{-1}\left(\frac{y}{x}\right) = \theta\end{gathered}}$$

here

$$\sin\left(\tan^{-1}\frac{y}{x}\right) = \frac{y^2/x^2}{\sqrt{\left(\frac{y}{x}\right)^2 + 1}}$$

$$\sin\left(\tan^{-1} y/x\right) = \frac{y^2}{x\sqrt{y^2 + x^2}}$$

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put this value in eq (ii)

$$B = \frac{\mu_0 I}{2\pi x} \left. \frac{y}{\sqrt{y^2+x^2}} \right|_0^L$$

$$= \frac{\mu_0 I}{2\pi x} \left[\frac{L}{\sqrt{L^2+x^2}} - \frac{0}{\sqrt{}} \right]$$

$$B = \frac{\mu_0 I}{2\pi x} \left[\frac{L}{\sqrt{L^2+x^2}} \right]$$

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$$\sin^2\theta + \cos^2\theta = 1$$

divide both sides by $\sin^2\theta$

$$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$1 + \frac{1}{\tan^2\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{\tan^2\theta + 1}{\tan^2\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{\tan^2\theta}{1 + \tan^2\theta} = \sin^2\theta$$

$\tan\theta = \frac{Y}{x}$

$$\frac{\tan^2[\tan^{-1} Y/x]}{1 + \tan^2[\tan^{-1} Y/x]} = \sin^2(\tan^{-1} Y/x]$$

$\theta = \tan^{-1}(Y/x)$

$$\frac{Y^2/x^2}{1 + Y^2/x^2} = \sin^2[\tan^{-1} Y/x]$$

square root both sides

$$\frac{\sqrt{Y^2/x^2}}{\frac{\sqrt{Y^2 + x^2}}{x}} = \sin[\tan^{-1} Y/x]$$

$$\frac{Y}{\sqrt{Y^2 + x^2}} = \sin[\tan^{-1} Y/x]$$