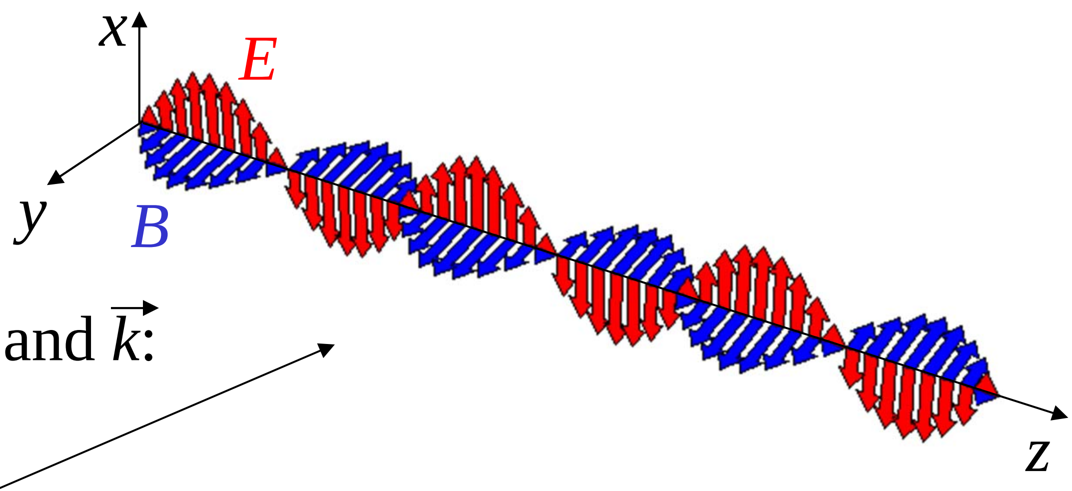
Phys 322
Lecture 21

Chapter 8
Polarization

Plane of polarization

Transverse EM wave



Plane of polarization -
plane defined by vector $\vec{E}$ and $\vec{k}$:

Plane of polarization *xz*:

$$\vec{E}_x(z,t) = \hat{\mathbf{i}}E_x(z,t) = \hat{\mathbf{i}}E_{0x}\cos(kz - \omega t)$$

Can create another wave with polarization along *y*:

$$\vec{E}_y(z,t) = \hat{\mathbf{j}}E_y(z,t) = \hat{\mathbf{j}}E_{0y}\cos(kz - \omega t + \xi)$$

Linearly (plane) polarized light: Electric field orientation is constant, though its amplitude can vary in time.

Superposition: in-phase & out-of-phase

in-phase:

$\xi = 0, \quad 2\pi, \ldots$

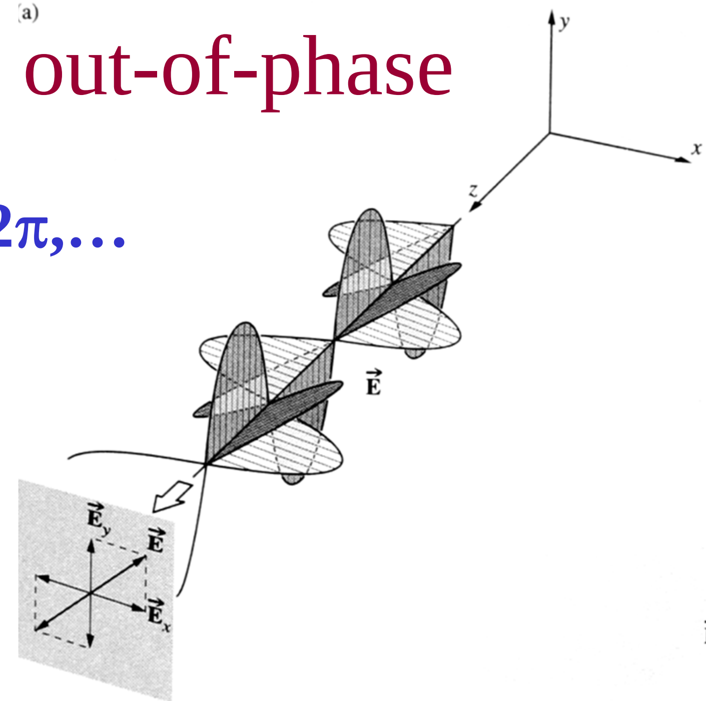
$$\vec{E}_x(z,t) = \hat{\mathbf{i}} E_{0x} \cos(kz - \omega t)$$

$+$

$$\vec{E}_y(z,t) = \hat{\mathbf{j}} E_{0y} \cos(kz - \omega t + \xi)$$

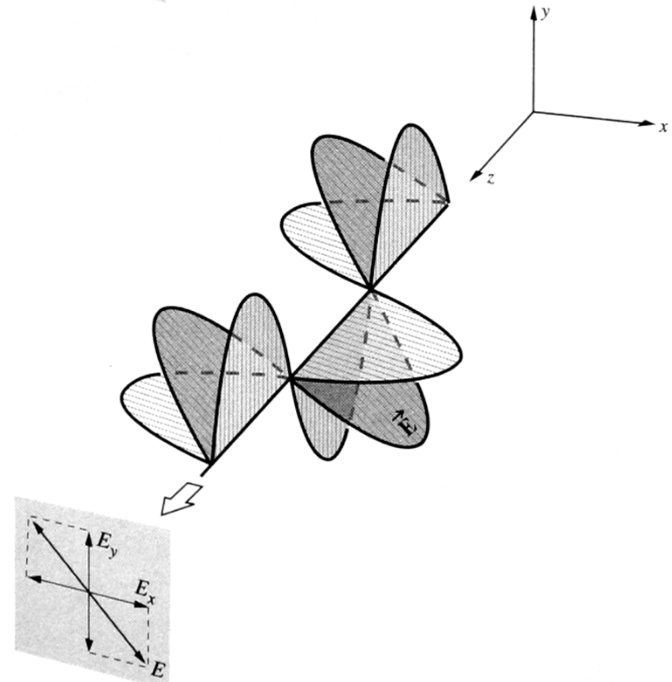
$=$

$$\vec{E}(z,t) = \left(\hat{\mathbf{i}} E_{0x} + \hat{\mathbf{j}} E_{0y}\right)\cos(kz - \omega t)$$



out-of-phase: $\xi = \pi, \quad 3\pi \ldots$

$$\vec{E}(z,t) = \left(\hat{\mathbf{i}} E_{0x} - \hat{\mathbf{j}} E_{0y}\right)\cos(kz - \omega t)$$



By changing E_{x0}/E_{y0} can create **linearly** polarized light along any direction in *xy* plane.

Circular polarization

$$\vec{E}_x(z,t) = \hat{\mathbf{i}} E_0 \cos(kz - \omega t)$$

$\xi = 2\pi m \pm \pi/2$, where m=0,1,2,…

$+$

$$\vec{E}_y(z,t) = \hat{\mathbf{j}} E_0 \cos(kz - \omega t + \xi) \xrightarrow{\xi = -\pi/2} \vec{E}_y(z,t) = \hat{\mathbf{j}} E_{0y} \sin(kz - \omega t)$$

$=$

$$\vec{E}(z,t) = E_0 \left[\hat{\mathbf{i}} \cos(kz - \omega t) + \hat{\mathbf{j}} \sin(kz - \omega t) \right]$$

What is the magnitude of the electric field?

$$\left(\vec{E}\right)^2 = E_0^2 \left[\cos^2(kz - \omega t) + \sin^2(kz - \omega t) \right] = E_0^2$$

Magnitude is constant in time!

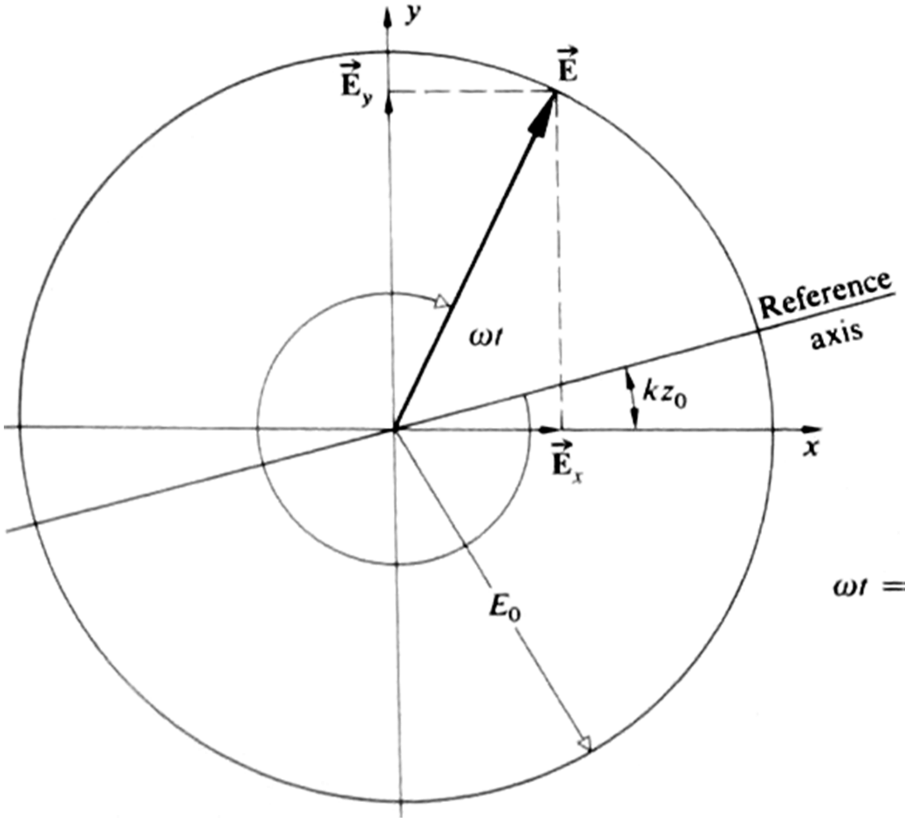
Is it a wave?

Circular polarization

$$\vec{E}(z,t) = E_0\left[\hat{\mathbf{i}}\cos(kz-\omega t) + \hat{\mathbf{j}}\sin(kz-\omega t)\right]$$

x and y components oscillate: $E_x = E_0\cos(kz-\omega t)$

$E_y = E_0\sin(kz-\omega t)$



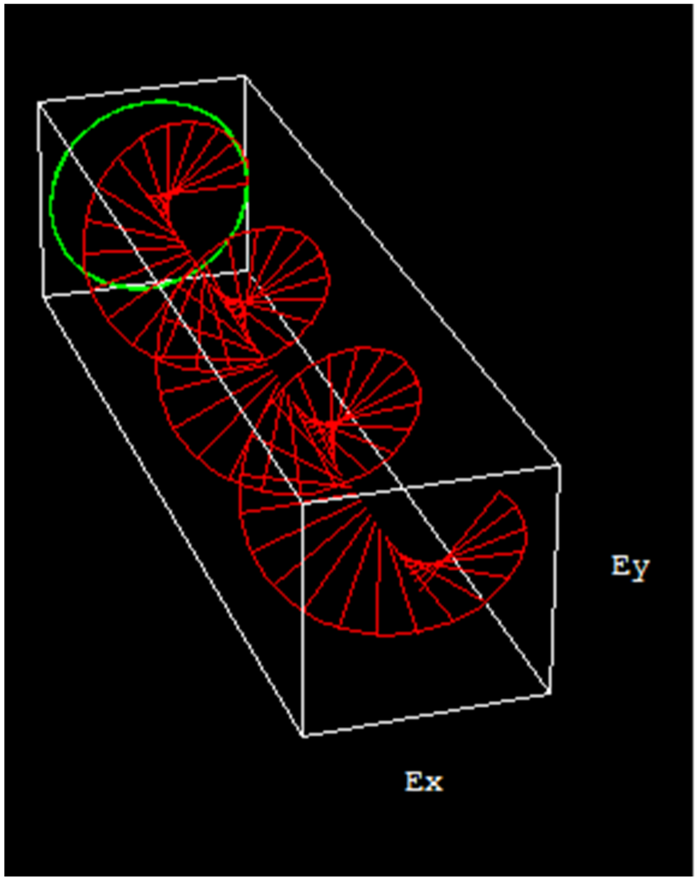
Angle $\alpha = kz_0 - \omega t$

Vector E rotates in time with angular frequency $-\omega$

Vector E rotates in space with angular spatial speed k

Circular polarization

$$\vec{E}(z,t) = E_0\left[\hat{\mathbf{i}}\cos(kz-\omega t) + \hat{\mathbf{j}}\sin(kz-\omega t)\right]$$



Right circularly polarized light

E rotates clockwise as seen by observer

Vector makes full turn as wave advances one wavelength

Left circularly polarized light

E rotates counter clockwise

$$\vec{E}(z,t) = E_0\left[\hat{\mathbf{i}}\cos(kz-\omega t) - \hat{\mathbf{j}}\sin(kz-\omega t)\right]$$

What if we have a superposition of left and right circularly polarized light of equal amplitude?

$$\vec{E}(z,t) = 2E_0\hat{\mathbf{i}}\cos(kz-\omega t)$$ - linearly polarized light

Elliptic polarization

General case: $$E_x = E_{0x}\cos(kz-\omega t)$$ $$E_y = E_{0y}\cos(kz-\omega t+\xi)$$

E changes direction and magnitude

What is the trajectory of the tip of *vector* (E_x, E_y) in *xy* plane?

$$E_y / E_{0y} = \cos(kz-\omega t)\cos\xi - \sin(kz-\omega t)\sin\xi$$

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\xi = -\sin(kz-\omega t)\sin\xi$$

$$\sin^2(kz-\omega t) = 1-\left(\frac{E_x}{E_{0x}}\right)^2$$

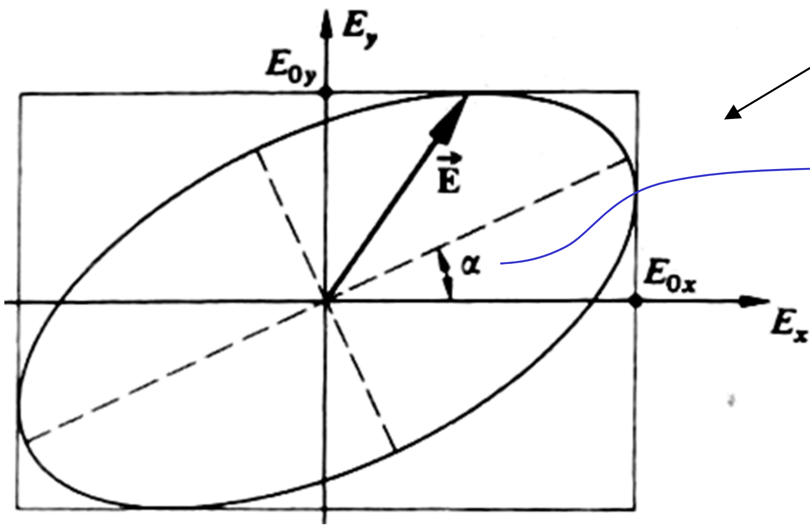
$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\xi\right)^2 = \left[1-\left(\frac{E_x}{E_{0x}}\right)^2\right]\sin^2\xi$$

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos\xi = \sin^2\xi$$

Elliptic polarization

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos\xi = \sin^2\xi$$

Equation of ellipse



$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\xi}{E_{0x}^2 - E_{0y}^2}$$

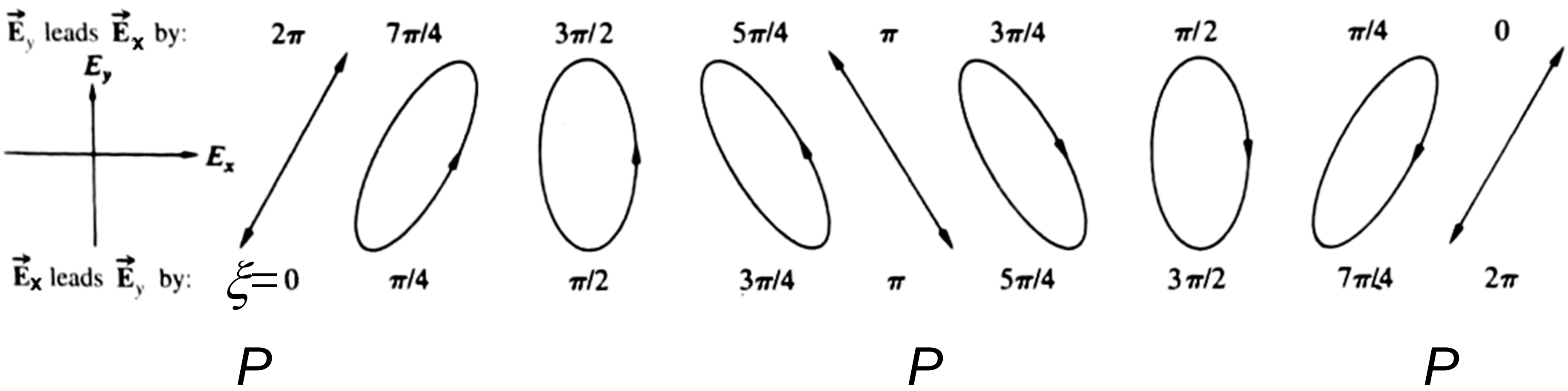
Can always rotate axes to get convenient form (α=0, or ξ= π/2,…):

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 = 1$$

Circular polarization: $E_{0x}=E_{0y}$

Elliptic polarization

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)^2 \cos\xi = \sin^2\xi$$



State of polarization:

P - linearly polarized

R - right circular polarization

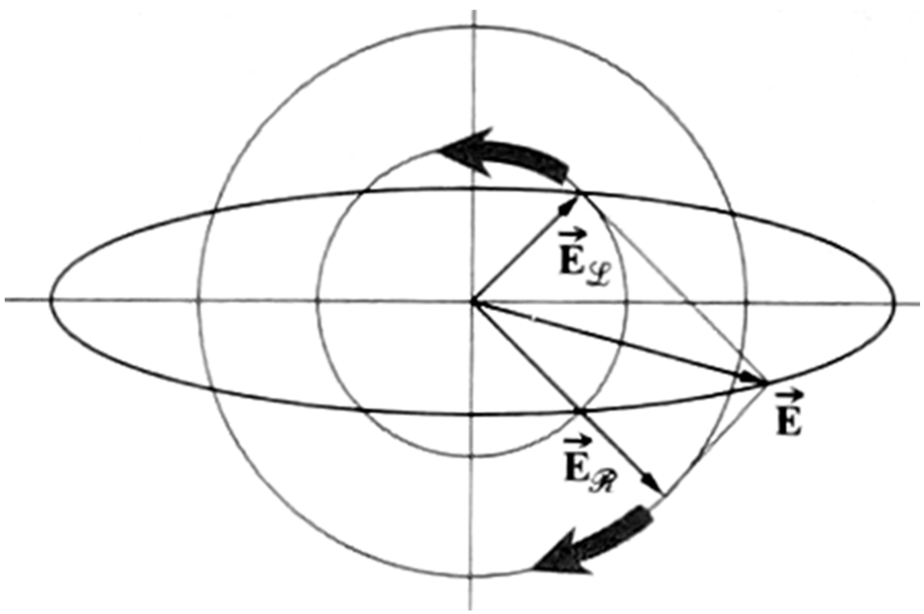
L - left circular polarization

E - elliptical polarization

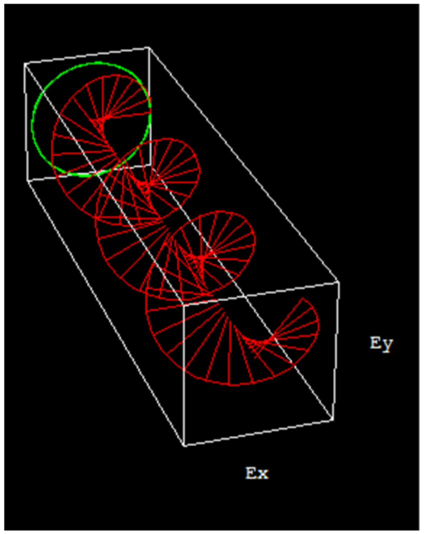
Superposition of *L* and *R*

P -state can be represented as superposition of *L*- and *R* -states of the same amplitude

E-state can be represent as superposition of *L* and *R*-states:



Circular polarization and angular momentum



What would happen with an electron under circularly polarized light?

Angular velocity ω - angular momentum

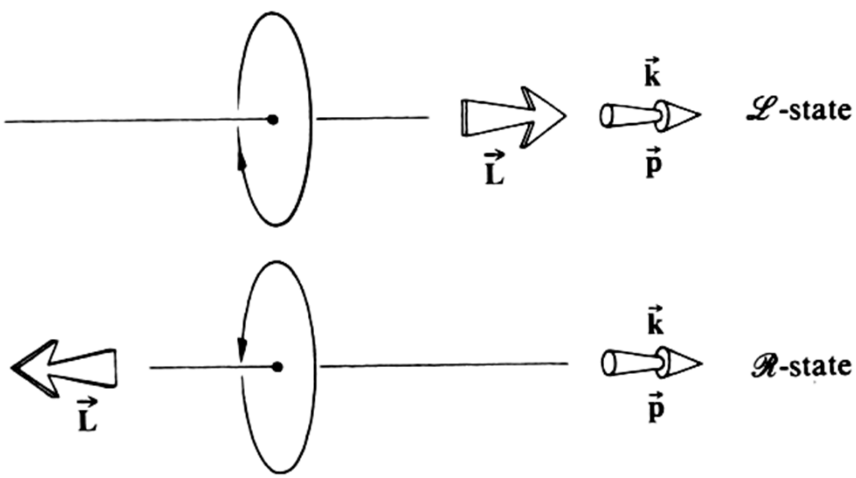
Light is absorbed, and if it was circularly polarized:

$$L = \frac{E}{\omega}$$

angular momentum → L

energy of light → E

frequency of light → ω



Photon and angular momentum

$$L = \frac{E}{\omega}$$

Photon has energy: $E = h\nu = \frac{h}{2\pi}\omega = \hbar\omega$

Angular momentum of a photon is independent of its energy:

$$L = \pm\hbar$$

Photon has a *spin*, $+\hbar$ - *L*-state
$-\hbar$ - *R*-state

Whenever a photon is absorbed or emitted by a charged particle, along with the change in its energy the electron will undergo a change in its angular momentum

First measured in 1935 by Richard Beth

Linearly polarized light: photons exist in either spin state with equal probability

Polarizer

An optical device that transmits (or reflects) only light polarized in a certain way.

Linear polarizer: passes (reflects) only light that is linearly polarized in certain direction (plane).

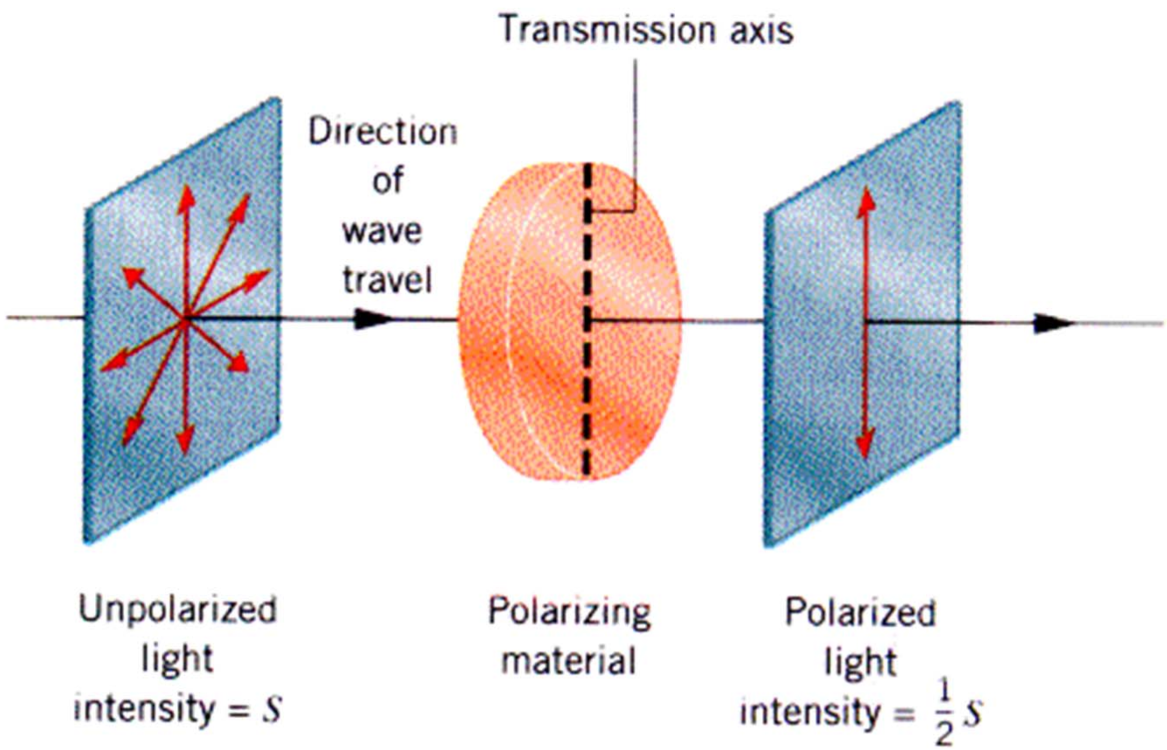
Unpolarized light

Is sun light polarized?

Unpolarized light = randomly polarized

Atoms emit wavepackets ~10 ns long

Unpolarized light on polarizer



- Most light comes from electrons accelerating in random directions and is unpolarized.
- Averaging over all directions $I_{transmitted} = \frac{1}{2} I_{incident}$

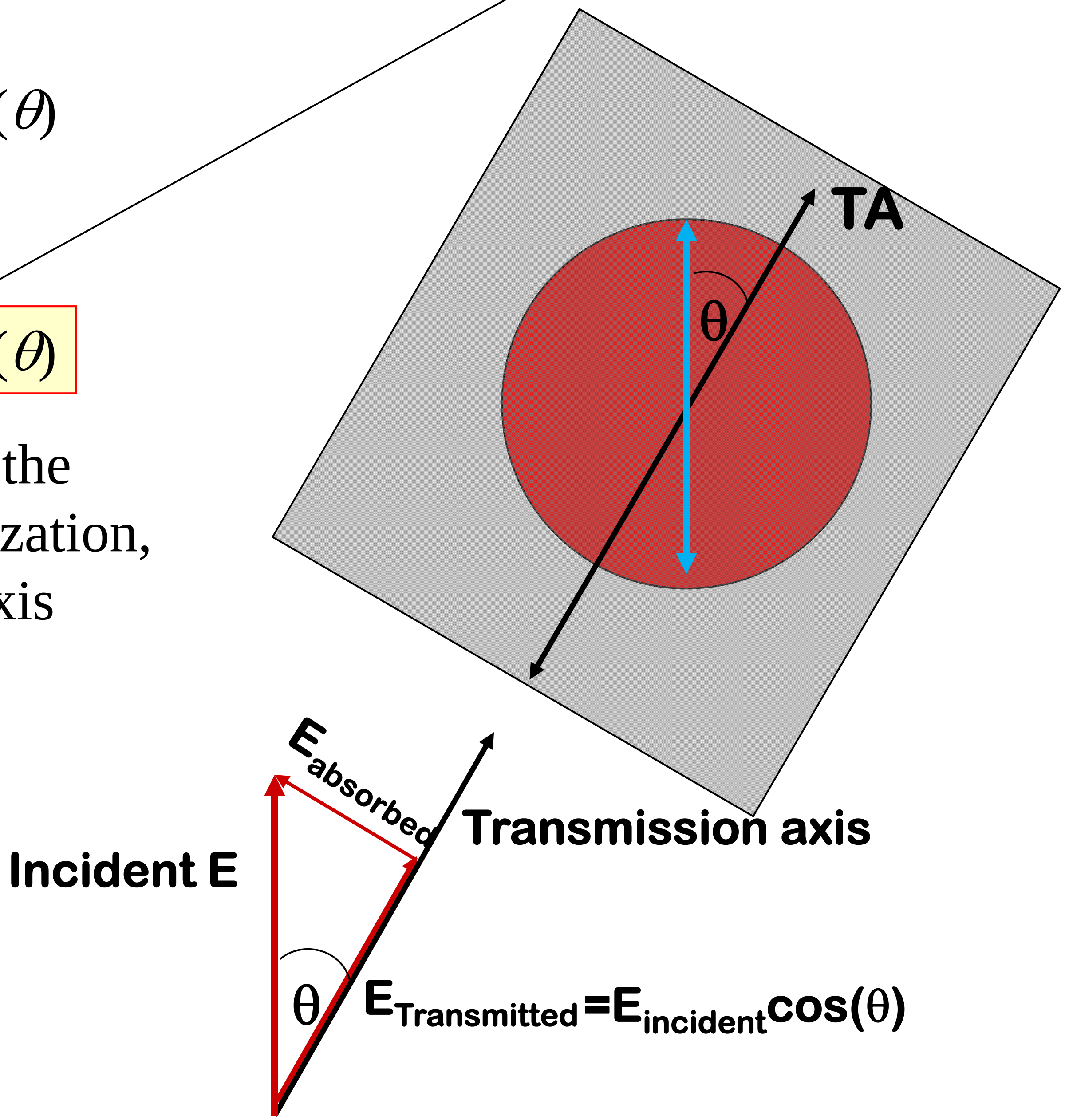
Polarized light on polarizer: Malus's law

$$E_{transmitted} = E_{incident}\cos(\theta)$$

$$\downarrow \; I \sim E^2$$

$$I_{transmitted} = I_{incident}\cos^2(\theta)$$

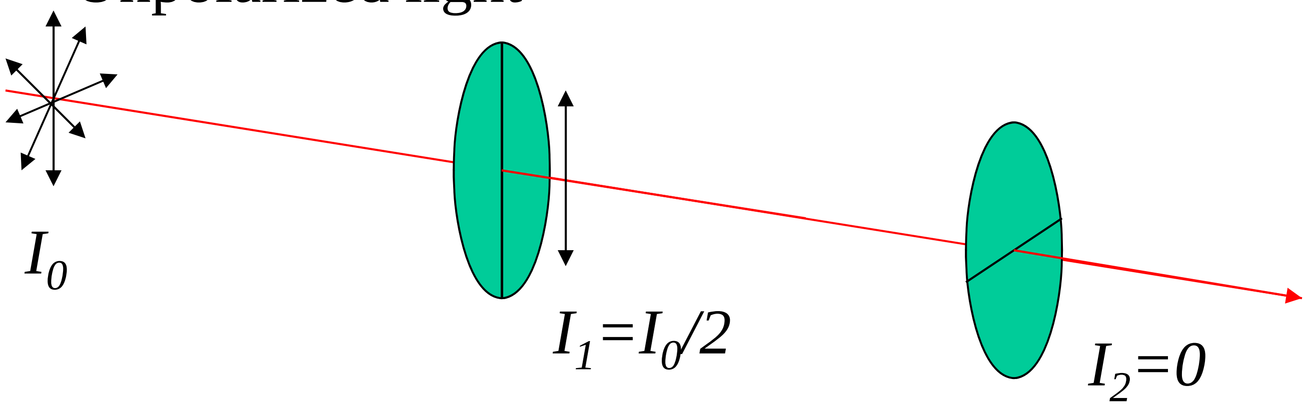
θ is the angle between the incoming light's polarization, and the transmission axis



Example: crossed polarizers

$$I_{transmitted} = I_{incident}\cos^2(\theta)$$

Unpolarized light



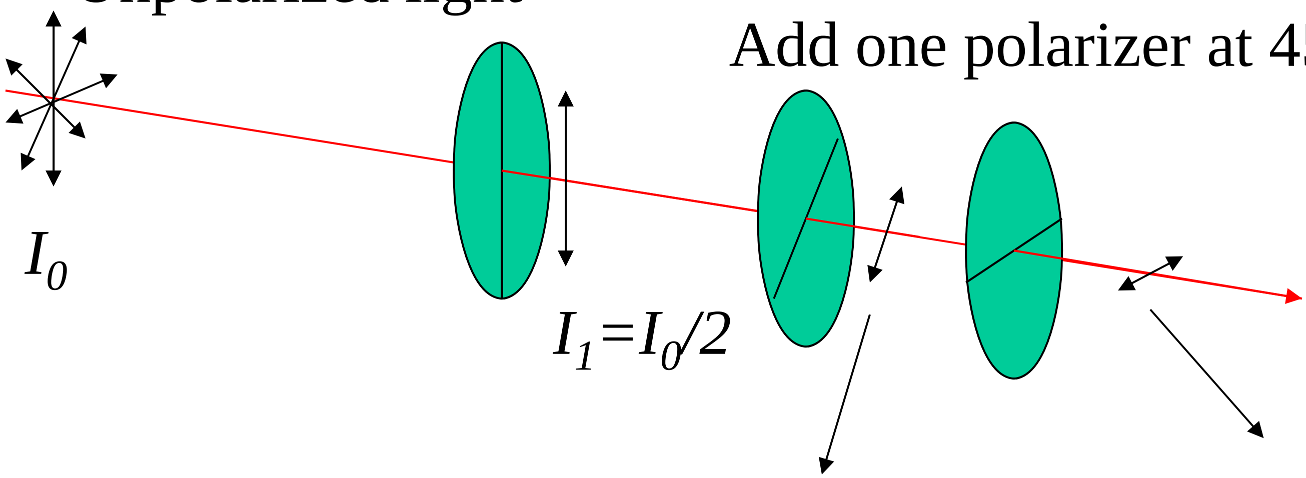
How much light passes through two crossed polarizers?

Example: three polarizers

$$I_{transmitted} = I_{incident} \cos^2(\theta)$$

Unpolarized light

Add one polarizer at 45^0 in between...



$$I_2=I_1\cos^2(45^0)$$

$$I_2 = \frac{I_0}{4}$$

$$I_3 = I_2 \cos^2\left(45^o\right)$$

$$I_3 = \frac{I_0}{4}\cos^2\left(45^o\right)$$

$$I_3 = \frac{I_0}{8}$$

Dichroism

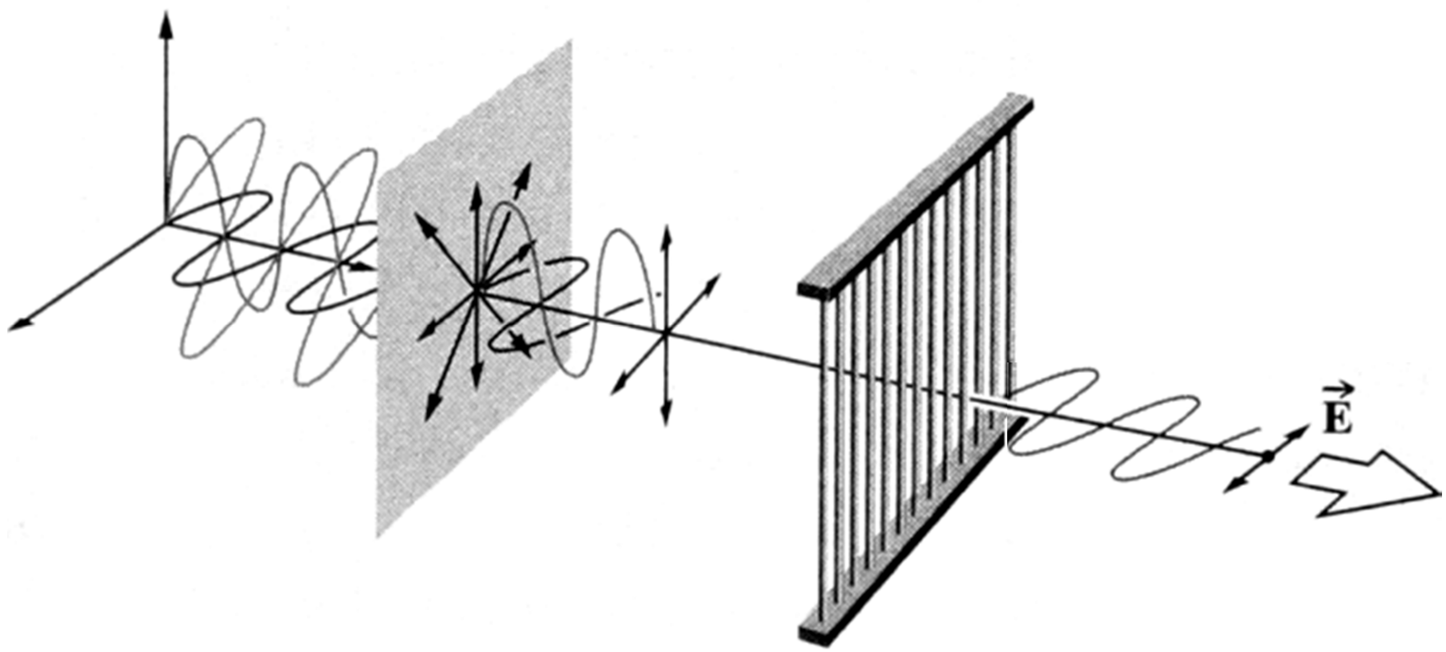
= selective absorption of light of certain polarization

Linear dichroism - selective absorption of one of the two *P*-state (linear) orthogonal polarizations

Circular dichroism - selective absorption of *L*-state or *R*-state circular polarizations

Using dichroic materials one can build a polarizer

Wire-grid polarizer



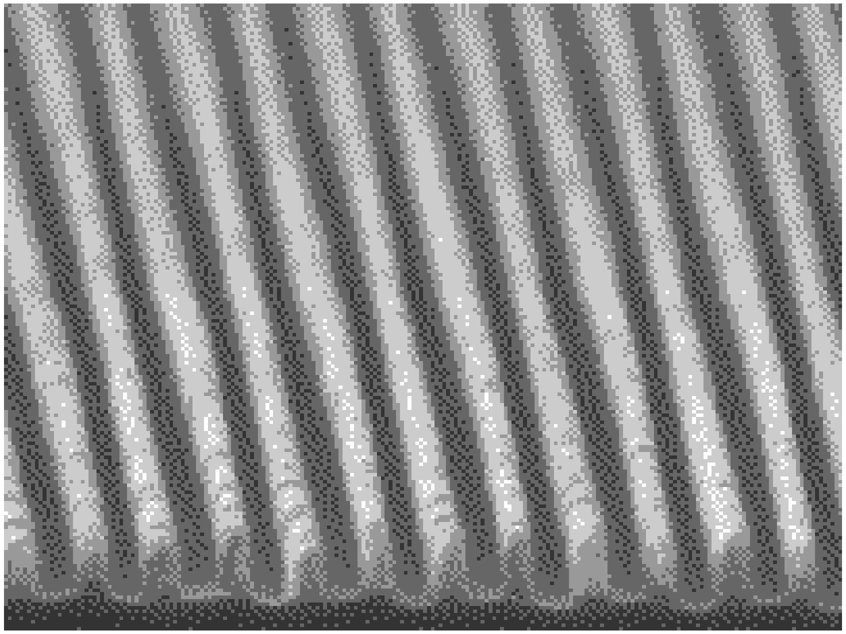
What is the transmission axis of this wire-grid polarizer

Can we use such a polarizer for light?

1960, George R. Bird and Maxfield Parish: 2160 wires per mm

Wire grid polarizer in the visible

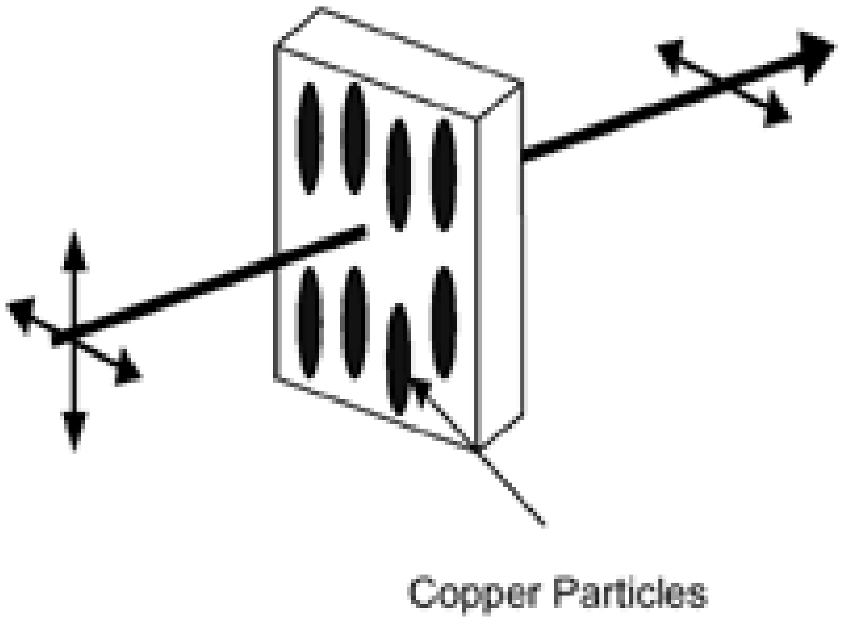
Using semiconductor fabrication techniques, a wire-grid polarizer was recently developed for the visible.

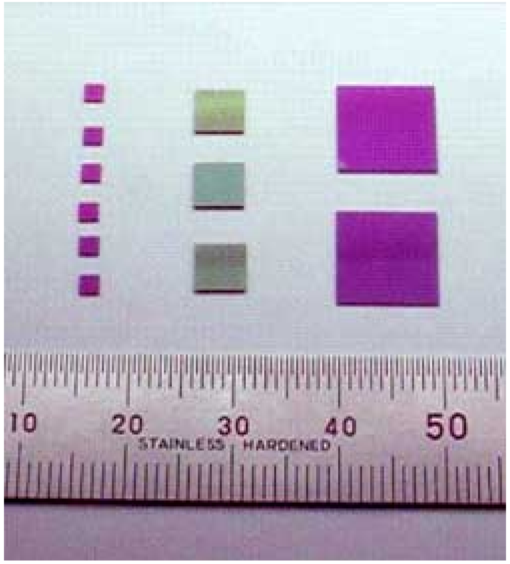


The spacing is less than 1 micron.

The wires need not be very long.

Hoya has designed a wire-grid polarizer for telecom applications that uses small elongated copper particles.

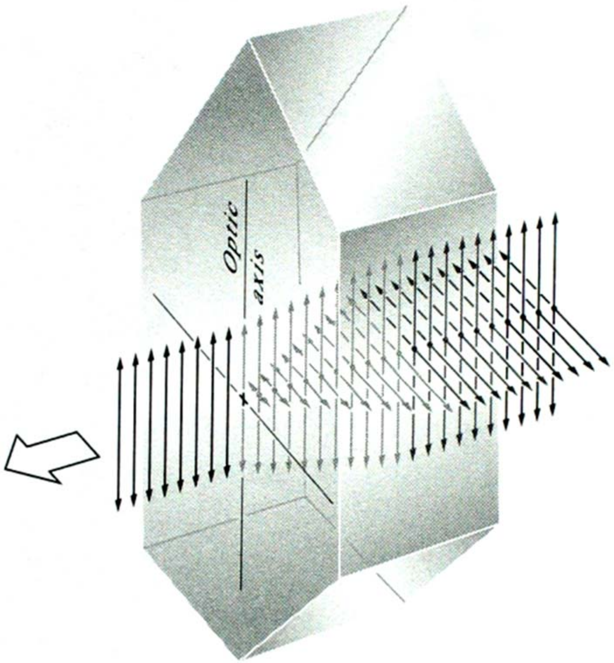




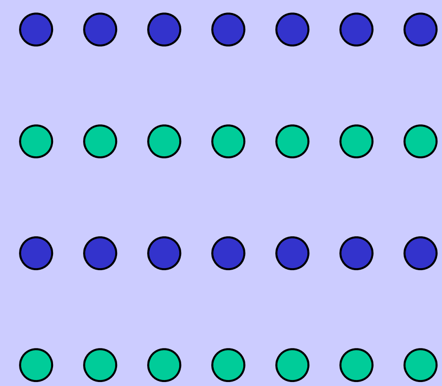
Extinction coefficient > 10,000

Transmission > 99%

Dichroic crystals



Anisotropic crystal structure: one polarization is absorbed more than the other



Elastic constants for electrons may be different along two axes

Example: *tourmaline*