OSCILLATIONS

1

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Q.1 Define simple harmonic motion. Write down its equation of motion and solve for displacement, hence calculate its time period, velocity, acceleration and energy? (PU. 2007, 2011,2014, SU. 2014)

SIMPLE HARMONIC MOTION

The to and fro motion of a body in which acceleration is directly proportional to the displacement and always directed towards mean position is called **simple harmonic motion.** The body executing simple harmonic motion is called **simple harmonic oscillator.**

EXAMPLES

1. The motion of simple pendulum is SHM.
2. The motion of spring mass system is SHM.

SIMPLE HARMONIC OSCILLATOR

Consider a block of mass m is attached with one end of a spring. The other end of spring is fixed to a support. The block is free to move to and fro over a frictionless horizontal surface as shown in fig.

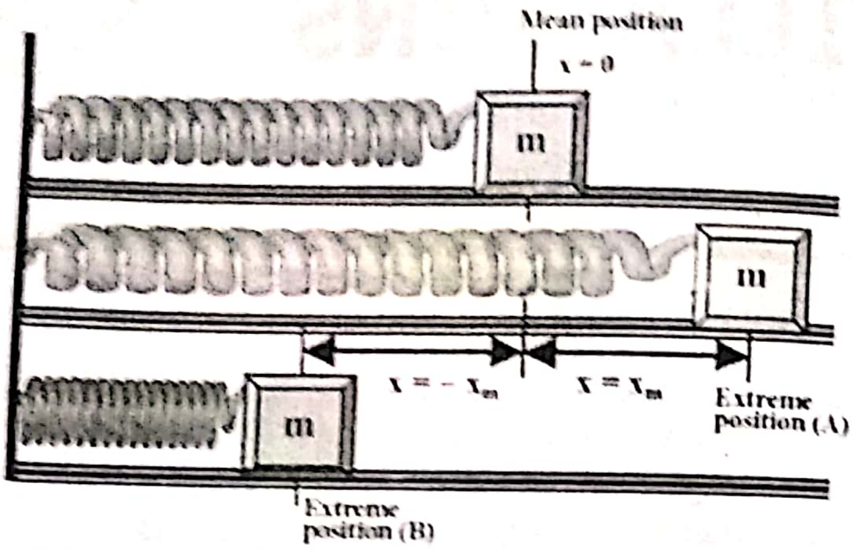
MEAN POSITION

The position $x = 0$ when block is at rest is called mean position because spring is not exerting any no force on the block.

RESTORING FORCE

Now apply external force on the block towards right which extends the spring through distance x_m(extreme position). The spring exerts a force on the block towards mean position when applied force is removed.

This force is equal in magnitude to the applied force but opposite in direction called **spring force or restoring force.**



VIBRATORY MOTION

The block moves towards mean position under spring force. It gets maximum velocity when reaches at mean position and does not stop there due to inertia but continues to move towards extreme position(B). The velocity of block becomes zero at extreme position(B) due to restoring force.

The acceleration is directed from extreme position(B) to mean position because velocity of block is going to decrease when block is moving from mean position to extreme position(B).

Now block moves from extreme position(B) towards mean position due to spring force. The velocity of block increases from B towards mean position and becomes maximum when reaches at mean position. The acceleration is again directed towards mean position. In this way acceleration of block is always directed towards mean position during its to and fro motion.

EQUATION OF MOTION

The block attached with spring having spring constant k takes to and fro motion under restoring force F given as

$$F = -kx \qquad \text{------(1)}$$

The negative sign means restoring force F and displacement x are always in posite direction. Now apply Newton's second law of motion

$$F = ma$$

$$F = m\frac{d^2x}{dt^2} \quad \text{------ (2)}$$

Compare eq (1) and eq (2)

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

This is called **equation of motion of simple harmonic oscillator.**

LOCITY OF HARMONIC OSCILLATOR

The equation of motion of simple harmonic oscillator is

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\boxed{\omega^2 = \frac{k}{m}}$$

$$\frac{d}{dt}\left(\frac{dx}{dt}\right) + \omega^2 x = 0$$

$$\frac{dV}{dt} + \omega^2 x = 0$$

$$\frac{dV}{dt}\frac{dx}{dx} + \omega^2 x = 0$$

$$V\frac{dV}{dx} = -\omega^2 x$$

$$V\,dV = -\omega^2 x\,dx$$

Integrating on both sides

$$\int V\,dV = -\omega^2 \int x\,dx$$

$$\frac{V^2}{2} = -\omega^2\left(\frac{x^2}{2}\right) + C$$

$$V^2 = -\omega^2 x^2 + 2C \quad \text{------ (3)}$$

Where C is integration constant. Its value can be determined by initial boundary itions. The velocity $V = 0$ when $x = x_m$

$$(0)^2 = -\omega^2 x_m^2 + 2C$$

$$2C = \omega^2 x_m^2$$

Put this value in eq(3)

$$V^2 = \omega^2(x_m^2 - x^2)$$

The velocity of harmonic oscillator is

$$V = \pm\omega\sqrt{(x_m^2 - x^2)}$$

MAXIMUM VELOCITY

The velocity of harmonic oscillator is maximum at mean position($x = 0$)

$$V_{max} = \pm\omega\sqrt{(x_m^2 - 0)}$$

$$V_{max} = \pm x_m\sqrt{\frac{k}{m}}$$

MINIMUM VELOCITY

The velocity of harmonic oscillator is minimum at extreme position($x = x_m$)

$$V_{min} = \pm\omega\sqrt{(x_m^2 - x_m^2)}$$

$$V_{min} = 0$$

DISPLACEMENT OF HARMONIC OSCILLATOR

The velocity of harmonic oscillator is

$$V = \pm\omega\sqrt{(x_m^2 - x^2)}$$

$$\frac{dx}{dt} = \pm\omega\sqrt{(x_m^2 - x^2)}$$

$\frac{-dx}{\sqrt{(x_m^2 - x^2)}} = \omega dt$ Integrate on both sides $\cos^{-1}\left(\frac{x}{x_m}\right) = \omega t + \phi$ Where ϕ is integration constant $\frac{x}{x_m} = \cos(\omega t + \phi)$ $x = x_m \cos(\omega t + \phi)$	$\frac{dx}{\sqrt{(x_m^2 - x^2)}} = \omega dt$ Integrate on both sides $\sin^{-1}\left(\frac{x}{x_m}\right) = \omega t + \phi$ Where ϕ is integration constant $\frac{x}{x_m} = \sin(\omega t + \phi)$ $x = x_m \sin(\omega t + \phi)$

The eq $x = x_m \cos(\omega t + \phi)$ is the displacement of simple harmonic oscillator when motion starts from extreme position. The eq $x = x_m \sin(\omega t + \phi)$ is the displacement of simple harmonic oscillator when motion starts from mean position. Here we shall consider motion is started from extreme position.