Chapter # 01: Matrix and Determinant

- **DEFINITION OF A MATRIX:**

A matrix is a rectangular array of numbers. The numbers are the entries or elements of the matrix.

- **Rows**: = m

Horizontal lines of a numbers is called Rows

- **Columns:** n

Vertical lines of a numbers is called columns

The following are examples of matrices.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

order = m×n = 3×3

$[A_{ij}]_{m\times n}$ — row, columns; numbers, closed array

A, B, C, ...

$[0]_{1\times1}$ $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \end{bmatrix}_{2\times3}$ → order

- **Row Matrix or Row vector:** (horizontal)

A matrix, which has only one row is called row vector. / Row Matrix

$\begin{bmatrix} 1 & 2 & -5 \end{bmatrix}_{1\times3}$

- **Column Matrix or Column vector:**

A matrix, which has only one column is called column vector. / column Matrix

$\begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}_{3\times1}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}_{5\times1}$

- **Rectangular Matrix:** column $\neq$ row

If $n \neq m$, then the matrix is Rectangular matrix

$$A=\begin{bmatrix}1&2\\2&4\\3&5\end{bmatrix}_{3\times 2} \qquad \begin{bmatrix}1&2&4\\0&1&2\end{bmatrix}_{2\times 3}$$

- **Square Matrix:** column = row

if $n = m$, then the matrix is called square matrix.

$$\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}_{3\times 3} \qquad \begin{bmatrix}1&0&0&0\\0&0&0&0\\1&1&1&1\\0&0&1&0\end{bmatrix}_{4\times 4} \qquad \begin{bmatrix}2&1\\1&2\end{bmatrix}_{2\times 2}$$

- **Diagonal Matrix:**

All elements are zero except main diagonal elements (Principal diagonal) | Square matrix

$$A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix} \qquad D=\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$$

$$D=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \text{Scalar matrix} \quad k=1$$

$$D=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} \quad k=1,2,3,\dots$$

- **Scalar Matrix:** $k=1$

If any scalar number multiply with diagonal matrix

$$kD = k\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}_{3\times 3} \quad \text{if } k=2 \Rightarrow \begin{bmatrix}2&0&0\\0&4&0\\0&0&6\end{bmatrix}$$

- **Unit matrix or Identity matrix:**

If main diagonal element is 1 is called identity matrix

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = I$$

- **Null or Zero Matrix:**

Each element is zero is called zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3} , \quad [0]_{1\times 1}$$

- **Equal Matrix:**

If two matrix has same order & same representation is called equal matrix

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad A = B$$

$$\cancel{A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} , \quad \cancel{B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}}$$

$$A \neq B$$

- Triangular Matrix

1) upper Triangular Matrix

2) Lower Triangular Matrix

upper

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 1 & 5 & 3 \end{bmatrix}_{3\times 3} = U \quad ; \quad L = \begin{bmatrix} 1 & 8 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$$

lower