

# Chapter # 01: Matrix and Determinant

## DEFINITION OF A MATRIX:

A matrix is a rectangular array of numbers. The numbers are the entries or elements of the matrix.

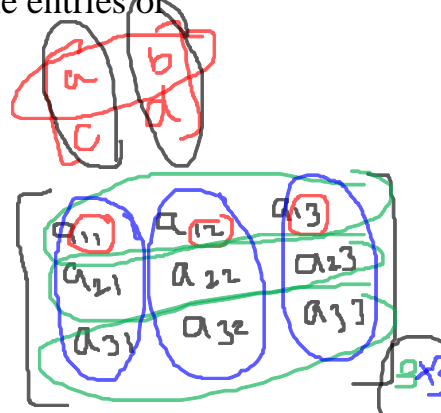
Rows:  $= m$

Horizontal lines of a numbers is called Rows

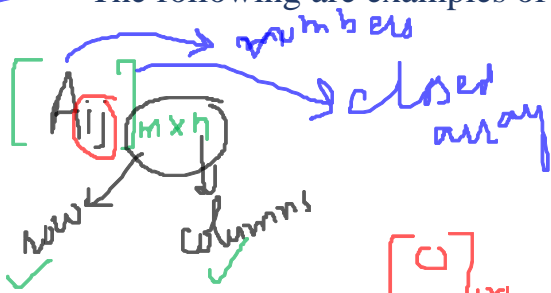
Columns:  $n$

Vertical lines of a numbers is called columns

The following are examples of matrices.

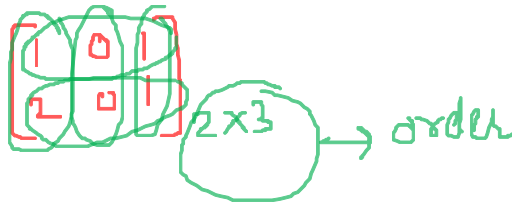


*Symbol*



$A, B, C, \dots$

*order = m x n*  
 $3 + 3$   
 $= 6$



*horizontal*

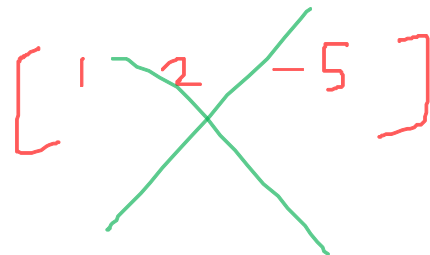
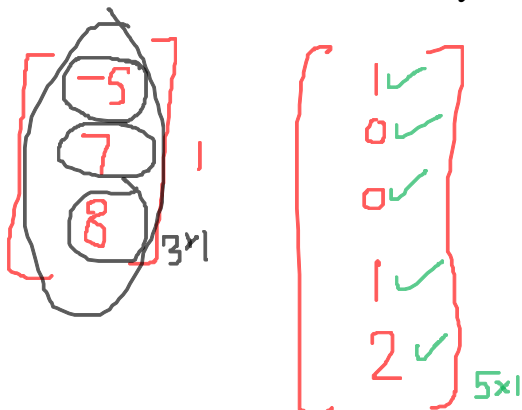
## Row Matrix or Row vector:

A matrix, which has only one row is called row vector. *row Matrix*



## Column Matrix or Column vector:

A matrix, which has only one column is called column vector. *column Matrix*



$m \times n$

column  $\neq$  row

• **Rectangular Matrix:**

If  $n \neq m$ , then the matrix is Rectangular matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 5 \end{bmatrix}_{3 \times 2}$$

$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 2 \end{bmatrix}_{2 \times 3}$

*Handwritten notes:* "row" (pointing to rows), "column" (pointing to columns), "column" (pointing to columns), "row" (pointing to rows)

column = row

• **Square Matrix:**

if  $n = m$ , then the matrix is called square matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3 \times 3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}_{2 \times 2}$$

• **Diagonal Matrix:**

All elements are zero except main diagonal elements

Principal diagonal

Square matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{k=1} = \text{Scalar Matrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*Handwritten notes:* "k=1", "k=1, 2, 3, ..."

• **Scalar Matrix:**  $k=1$

If any scalar number multiply with diagonal matrix

$$kD = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3 \times 3}$$

if  $k=2 \Rightarrow$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

• **Unit matrix or Identity matrix:**

If main diagonal element is 1 is called identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

- **Null or Zero Matrix:**

Each element is zero is called zero matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3 \times 3} \quad , \quad [0]_{1 \times 1}$$

- **Equal Matrix:**

①

If two matrix has same order & same representation is called equal matrix

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \quad B = \begin{bmatrix} 4 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \quad A = B$$

~~$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$A \neq B$~~

- **Triangular Matrix**

1) upper triangular matrix

2) Lower triangular matrix

upper

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 1 & 5 & 3 \end{bmatrix}_{3 \times 3} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3 \times 3} = \text{Lower}$$