Partial Market Equilibrium —— A Linear Model

The major issue of static equilibrium model is to find the set of values of endogenous variables which could satisfy the equilibrium conditions of the model. As such values are attained the equilibrium situation is identified. It is told that the partial equilibrium is concerned with determination of prices in a specific market. Here the influence of other variables on this market is not shown. Thus, in partial analysis the demand and supply of a particular good in a particular market is kept in view. Hence, in this model three variables are included: (1) quantity demanded (Qd), (2) quantity supplied (Qs) and (3) price (P). Then we make certain following assumptions regarding working of the market.

1. We shall have to determine the equilibrium conditions which occur when Qd – Qs = 0 OR Qd = Qs. This is called market clearing equation.
2. Regarding demand's functional equation it shows a linear negative relationship. It means when price rises demand contracts and when price falls the demand expands. The general demand function is Qd = f (P) and the restriction over the model is P↑, Qd ↓, P ↓, Qd ↑.
3. Regarding supply's functional equation it shows a linear positive relationship. It means when price rises supply expands and when price falls the supply contracts. The general supply function is Qs = f (P) and the restriction over the model is P↑, Qs↑, P↓, Qs↓.

The supply function also shows this fact when price does not exceed a specific level nothing will be sold. Thus, in the model there will be an equilibrium condition and two behavioural equations which would represent demand and supply in the market. They are as: Qd = Qs.

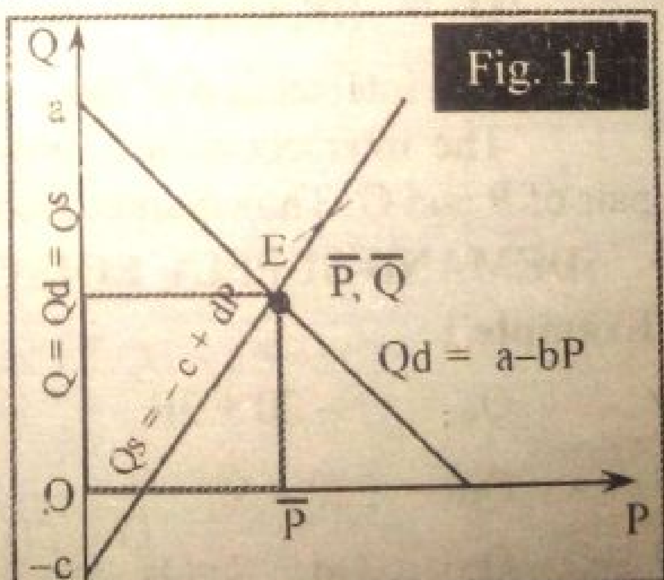

Fig. 11

The standard demand function is: Qd = a – bP. The a is the vertical intercept of demand curve while – b is the slope of demand curve. The standard supply function is: Qs = – c + dP. The '– c' is the vertical intercept of supply curve while d is the slope of supply curve. The both supply and demand functions in their standard form are shown in Fig. 11. Here on x–axis the independent variable (P) and on y–axis the dependent variable (Q) has been demonstrated. If we follow Marshall (as we do) the reverse is adopted where we show P on y–axis and Q on x–axis.

After constructing this model, the solution set will be found out, i.e., the values of three endogenous variables P, Qs and Qd will have to be found. They are shown by $\overline{Q}$, $\overline{Qd}$, $\overline{Qs}$, $\overline{P}$. These values will satisfy the equation. These values are attained with the help of elimination method.

As at equilibrium Q = Qd = Qs while Qd = $a - b$P and Qs = – c + dP. Then putting them $a - b\text{P} = -\text{c} + \text{dP} \Rightarrow a + c = \text{bP} + \text{dP} \Rightarrow \text{bP} + \text{dP} = a + c \Rightarrow \text{P(b + d)} = a + c \Rightarrow \text{P} = \dfrac{a + c}{\text{b + d}}$ ...... (1).

Here the equilibrium price has been represented into parameters. As, the parameters are positive hence equilibrium price ($\overline{P}$) is positive. Putting the value of equation (1) in demand equation.

$$Q = a - bP = a - b\left(\frac{a + c}{b + d}\right) \Rightarrow \overline{Q} = \frac{a}{1} - \frac{ba - bc}{b + d}$$

Taking L. C. M. and solving

$$\overline{Q} = \frac{ab + ad - ba - bc}{b + d} \Rightarrow \overline{Q} = \frac{ad - bc}{b + d}$$

Here equilibrium quantity ($\overline{Q}$) has been represented into parameters. Here the denominator (b + d) is positive, while the numerator (ad – bc) is negative. As $\overline{Q}$ is positive, hence the numerator must be positive. Therefore, a restriction is imposed on the model as : ad > bc. Now we present the above discussion in a set form.

$$D = \{(P, Q) \mid Q = a - bP\}, S = \{(P, Q) \mid Q = -c + dP\}$$

The intersection of these sets will be : $D \cup S = (\overline{P}, \overline{Q})$

The intersection set shows a single element, i.e., it is an ordered pair of P and Q. Thus market model represents a unique solution.

	Problem	Ans		Problem	Ans
(1)	Qd = 24 − 2P Qs = −5 + 7P	$\bar{P}$=3.2 $\bar{Q}$=17.5	(2)	Qd = 51− 3P Qs = 6P − 10	$\bar{P}$=6.78 $\bar{Q}$=30.67
(3)	Qd 30 − 2P Qs = −6+5P	$\bar{P}$=5.14 $\bar{Q}$=19.7	(4)	13P − Qs = 27 Qd + 4P − 24 = 0	$\bar{P}$=3 $\bar{Q}$=12
(5)	Qd = 20 − 5P Qs = 4 + 3P	$\bar{P}$=2 $\bar{Q}$=10	(6)	Qd = 15 − 0.2 P	$\bar{P}$=20 $\bar{Q}$=11
(7)	Qd = 8 − 2P Qs = 2 + 2P	$\bar{P}$=1.5 $\bar{Q}$=5	(8)	Q + P =5 Qs = −1 + 0.6P	$\bar{P}$=3.5 $\bar{Q}$=1.5

(9)	Qd = 1200 – 2PQs = 4P	$\bar{P}$=200 $\bar{Q}$=800	(10)	Q = 2 – .02 P Q = 2 + .07P	$\bar{P}$=0 $\bar{Q}$=2
(11)	Qd =50 – 8/7P Qs= 10 + 2/3P	$\bar{P}$=22.10 $\bar{Q}$=24.74	(12)	Q = 20 + 3P Q = 160 – 2P	$\bar{P}$=28 $\bar{Q}$=104
(13)	P= 50 – 7/8Q P=15 + 3/4Q	$\bar{P}$=31.15 $\bar{Q}$=21.53	(14)	x = 500 + 2P x = – 100 + 40P	$\bar{P}$=15.78 $\bar{Q}$=531.56
(15)	P = 7 – x/2 P = 7/6 + x	$\bar{P}$=5.05 $\bar{x}$=3.89	(16)	P = 5 – x/2 6P = 6 + x	$\bar{P}$=2 $\bar{Q}$=6

Effect of Imposition of Excise Tax on Market Model

In order to enhance its revenues, govts. may impose excise duty on goods and services. Again, the excise duty is levied with the aim of discouraging the consumption of certain goods and services. For example, a govt. may decide that for each bottle of whisky the suppliers sell, they must pay the govt. $1. Thus, here the tax on each unit of the taxed good is fixed. However, we shall also show the effect when the imposed excise duty is in some percentage.

1. The Case of Fixed Excise Tax

Here we start with the assumption that the govt. imposes an excise tax of T per unit on the producer. Here the selling price of the supplier will not be P rather it would be P – T. Accordingly, the supply function will be as: $Qs = f(P - T)$. While the demand function will be the usual one, $Qd = f(P)$

The standard supply and demand functions will be as:

$Q's = -c + (P - T)$, $Qd = a - bP$.

On this pattern we utilize Example – 1.

$Qs = -20 + 3P$... (1)and $Qd = 220 - 5P$ ... (2)

Equating Qd and Qs we found that equilibrium price is 30, while equilibrium quantity is 70.

Now we assume that Rs.5 per unit tax is imposed on the supplier. As a result, the supply function assumes the following form :

$$Q's = -20 + 3(P - 5) \Rightarrow Q's = -20 + 3P - 15 \Rightarrow Q's = 3P - 35$$

While the demand function is: $Qd = 220 - 5P$ Thus, at equilibrium $Qd = Q's$. Putting values,

$$220 - 5P = 3P - 35 \Rightarrow \quad -5P - 3P = -35 - 220$$

$$\Rightarrow \quad -8P = -255 \Rightarrow \quad P = 31.875$$

Putting $P = 30.187$ in Qd and Qs, we get

$$Qd = 220 - 5P = 220 - 5(31.875) = 60.625$$

and $$Q's = 3P - 35 = 3(31.875) - 35 = 60.625$$

It is, therefore, obvious that because of imposition of sales tax per unit the equilibrium price has gone up to 31.875 from 30, while the equilibrium quantity has decreased to 60.625 from 70.

2. The Case of Percentage Excise Duty

If $Qd = 20 - 5P$, while $Qs = 4 + 3P$ and 20% tax is imposed on the supplier. It means that imposed tax will 20% of the price charged, i.e., 0.2P. The equilibrium quantity and equilibrium price without tax will be: $Q = 10$, $P = 2$.

The supply function, after imposition of 20% excise duty will be as:

$$Q_S' = 4 + 3(P - 0.2P) = 4 + 3P - 0.6P = 4 + 2.4P. \text{ Thus } Q_S' = 4 + 2.4P$$

Thus at equilibrium $Qd = Q's \Rightarrow 20 - 5P = 4 + 2.4P$

or $-5P - 2.4P = 4 - 20 \Rightarrow -7.4P = -16$

$\Rightarrow P = 2.16$ Then $Q_S' = 4 + 2.4P = 4 + 2.4(2.16) = 9.2$

and $Qd = 20 - 5P = 20 - 5(2.16) = 9.2$

It is, therefore, obvious that because of imposition of percentage excise duty, the equilibrium price has increased to 2.16 from 2, while equilibrium quantity has decreased to 9.2 from 10.

Example 2: The demand and supply equations for a particular product are:

$Qd = 200 - 4P$, $Qs = -10 + 26P$

(a) A flat rate tax of 5 per unit is imposed on each unit sold, determine the new equilibrium position, the tax revenue at the equilibrium and the producer's revenue.

(b) Instead of the flat-rate tax of part (a) of the above, a tax of 20% of the price is imposed on each item sold. Determine the new equilibrium position, the tax revenue at the equilibrium and the producer's revenue.

The basic equilibrium values of $\bar{Q}$ and $\bar{P}$.

$Qd = 200 - 4P$, $Qs = -10 + 26P \Rightarrow 200 - 4P = -10 + 26P \Rightarrow$
$-4P - 26P = -10 - 200 \Rightarrow -30P = -210 \Rightarrow P = 7$

$Qd = 200 - 4(7) = 172$, $Qs = -10 + 26(7) = 172$ Thus $\bar{P} = 7$, $\bar{Q} = 172$

(a) The values of $\bar{P}$ and $\bar{Q}$ when flat rate of 5 per unit is imposed

$Q's = -10 + 26(P - 5) \Rightarrow Q's = -10 + 26P - 130 \Rightarrow Q's = -140 + 26P.$
$Qd = Q's \Rightarrow 200 - 4P = -140 + 26P \Rightarrow -4P - 26P = -140 - 200 \Rightarrow$
$-30P = -340 \Rightarrow P = 11.33.$
$Qd = 200 - 4(11.34) = 200 - 45.33 = 154.67$

$Q's = -140 + 26(11.34) = -140 + 294.58 = 154.58 \Rightarrow \boxed{154.6}$

Total revenue at equilibrium $= P \times Q = 11.34 \times 154.6 = 1753.16$
Tax revenue at equilibrium $= T \times Q = 5 \times 154.6 = 773$
Producer revenue at equilibrium = Total revenue – Tax Payments =
$1753.16 - 773 = 980.16$

(b) The values of $\bar{P}$ and $\bar{Q}$ when 20% tax is imposed

$Q''s = -10 + 26(P - 0.2P) \Rightarrow Q''s = -10 + 26P - 5.2P \Rightarrow$

$Q''s = -10 + 20.8P.$

$Qd = Q''s \Rightarrow 200 - 4P = -10 + 20.8P \Rightarrow -4P - 20.8P = -10 - 200 \Rightarrow$

$-24.8P = -210 \Rightarrow P = \frac{210}{24.8} = 8.47$

$Q''s = -10 + 20.8(8.47) = -10 + 176.17 = 166.17$

$Qd = 200 - 4(8.47) = 200 - 33.88 = 166.12$

Total revenue at equilibrium = $P \times Q = (8.47)\ (166.1) = 1406.9$

Tax revenue at equilibrium = $t \times Q = 0.20P \times Q = 0.20(8.47) \times 166.17 = 281.5$

Producer revenue at equilibrium = Total revenue – Tax Payments = $(1406.9) - (281.5) = 1125.40$

GENERAL EQUILIBRIUM OF MARKET

Earlier we presented partial equilibrium of the model where the demand depends upon the price of the good which is purchased by consumers, as: $Qd_X = f(P_X)$. In the same way, the supply depends upon the price of the good which is sold by producers, as: $Qs_X = f(P_X)$. But practically, we have to face the prices of complementary and substitute goods. In this way, the demand for any good depends upon price of substitute, in addition to its own price. The same like situation is faced in case of supply. Thus, when we introduce prices of other goods the market model is extended. Accordingly, we can get the equilibrium prices of other goods. Hence, the prices of different goods and their quantities were exogenous in the partial market model. In partial model, at equilibrium $Qd = Qs$ or $E = Qd - Qs = 0$ where E represents excess demand. But when we include goods depending upon each other the equilibrium would require that none of the good should face excess demand. Therefore, in the market of n goods there will be n equations and for each good the following equation will hold true. $E_i = Qd_i - Qs_i = 0$ where $i = 1, 2, 3, \ldots n$. Here when we attain any solution value we will be getting a set of equilibrium prices ($\overline{P}_i$) and equilibrium quantities ($\overline{Q}_i$). The equations will be simultaneously satisfied if these values are put into equations.

Example 4:

(1) $Qd_1 = 410 - 5P_1 - 2P_2$, (2) $Qs_1 = -60 + 3P_1$

(3) $Qd_2 = 295 - P_1 - 3P_2$, (4) $Qs_2 = -120 + 2P_2$

At Equilibrium in (2)

$295 - P_1 - 3P_2 = -120 + 2P_2$

$415 - P_1 - 5P_2 = 0$

At Equilibrium in (1)

$410 - 5P_1 - 2P_2 = -60 + 3P_1$

$470 - 8P_1 - 2P_2 = 0$

Here we get two equations.

$470 - 8P_1 - 2P_2 = 0$ ... ... (5)

$415 - P_1 - 5P_2 = 0$ ... ... (6)

Multiplying Eq. (6) by. 8.

$3320 - 8P_1 - 40P_2 = 0$ ... ... (7)

$470 - 8P_1 - 2P_2 = 0.$

$- \quad + \quad +$

$2850 - 38P_2 = 0$ Subtracting Eq. (5) from (7)

$-38P_2 = -2850$ $\overline{P}_2 = 75$ $\boxed{\overline{P_2} = 75}$

Putting $\overline{P_2} = 75$ in Eq. (5)

$470 - 8P_1 - 2P_2 = 0 \Rightarrow 470 - 8P_1 - 2(75) = 0$

$-8P_1 = -470 + 150 \Rightarrow -8P_1 = -320 \Rightarrow \overline{P_1} = 40$ $\overline{P}_2 = 75$

Putting $\overline{P_1} = 40$ and $\overline{P_2} = 75$ in Eq. (1), (2), (3) and (4).

At $\overline{P}_1 = 40$ the equilibrium in market I.

$Qd_1 = 410 - 5(40) - 2(75) = 60$, $Qs_1 = -60 + 3(40) = 60$

$\overline{Q}_1 = Qd_1 = Qs_1 = 60$, $Qd_2 = 295 - 40 - 3(75) = 30$

$Qs_2 = -120 + 2(75) = 30$ $Qs_2 = Qd_2 = Qs_2 = 30$

Thus at $\overline{P}_2 = 75$, the market II is in equilibrium.