Chapter **3**

EQUILIBRIUM ANALYSIS

STRAIGHT LINE AND ITS SLOPE

Here we shall explain straight line and its slope graphically. In this respect, we define straight line. "If any equation is linear and the coefficients of the variables (x, y) of the equation are non–zero real numbers the graphical representation of such equation is called straight line". To explain straight line we take following linear equation having two variables. $ax + by + c = 0$. On this pattern we take the equation $4x + 5y - 20 = 0$. To represent it graphically we solve its intercepts. It is told that the intercepts of the curve represent those distances where from the curve intersects the axis. Thus y–intercept is attained by putting $x = 0$ in the equation while x–intercept is attained by putting $y = 0$ in the equation.

If $x = 0$, then

$4x + 5y - 20 = 0$

$4(0) + 5y - 20 = 0$

$5y = 20 \Rightarrow y = 4$

Thus y–intercept is = 4.

If $y = 0$, then

$4x + 5y - 20 = 0$

$4x + 5(0) - 20 = 0$

$4x = 20 \Rightarrow x = 5$,

then x–intercept is = 5.

By joining these two points (0, 4) and (5, 0) we get a graph — as shown in the Fig. 1. The straight line can be constructed with the help of these points. However, for more accuracy the third point can also be taken. As if $y = 2$, then $4x + 5y - 20 = 0 \Rightarrow 4x + 5(2) - 20 = 0 \Rightarrow 4x = 20 - 10 \Rightarrow x = 2.5$. Thus the third point is (5/2, 2).

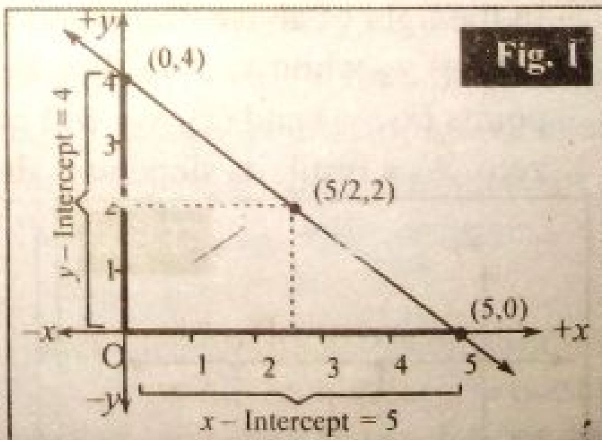


Fig. 1

Slope of a Straight Line

The direction of a straight line is represented by its inclination (θ) while the slope ($m = \tan\theta$) is determined by the angle in between x–axis and straight line.

If any straight line intersects the x–axis its trend is depicted by its angle (θ). It is shown in Fig. 2.

Here the slope of the curve is tangent to the trend of the angle. We suppose that the value of angle is less than 180° and greater than zero ($0 < \theta < 180$). The straight line whose trend angle is shown by θ intersects x – axis at A. We take another point on thisstraight line like B and draw a perpendicular which intersects x–axis on C. Now the tangent of angle (θ) which is known as Tan θ will be as: $\tan\theta = \frac{CB}{AC}$. On the same line if we take two points on straight line as (x_1, y_1) and (x_2, y_2), the slope of straight line will be as: Slope $= m = \tan\theta$ $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$. As we get similar triangles on a straight line, the slope the curve between two points on a straight line will remain the same. As, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$. Thus the above discussion shows that "The ratio of vertical change to horizontal change is called the slope of a straight line". As $\Delta y = y_2 - y_1$ and $\Delta x = x_2 - x_1$. Then

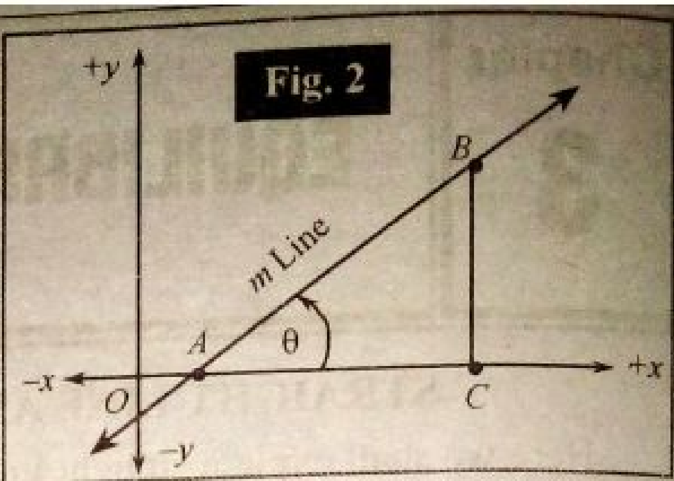


Fig. 2

$$\text{the slope of curve} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}.$$

In the light of above discussion we present following possibilities:

1. If $y_1 = y_2$ while $x_1 \neq x_2$ then the straight line derived through the points (x_1, y_1) and (x_2, y_2) will be horizontal. Its trend angle will be zero. As a result its slope will also be zero. It is shown in Fig. 3.

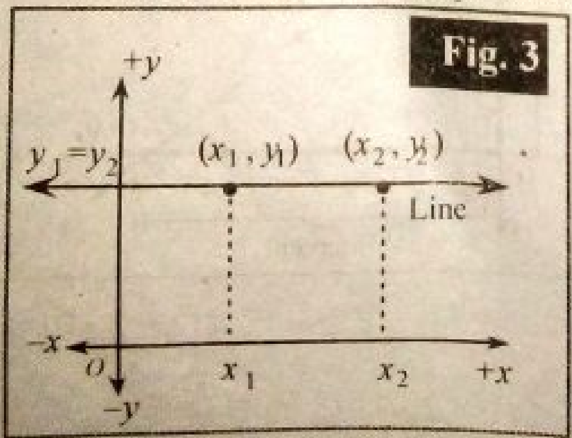


Fig. 3

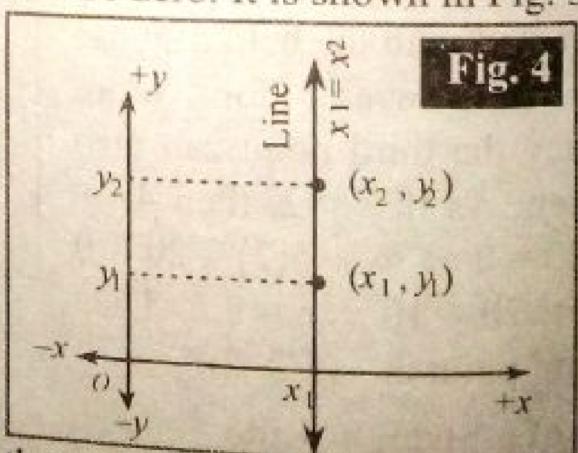


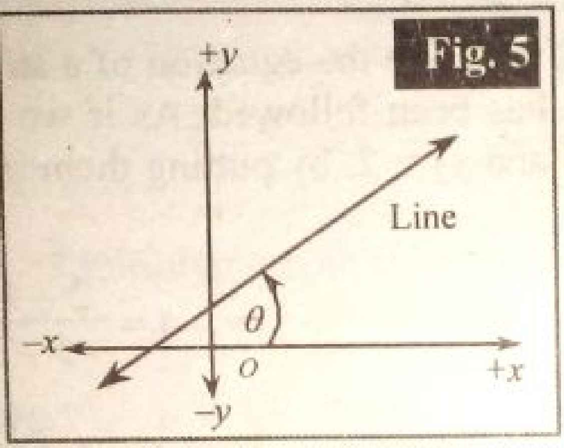
Fig. 4

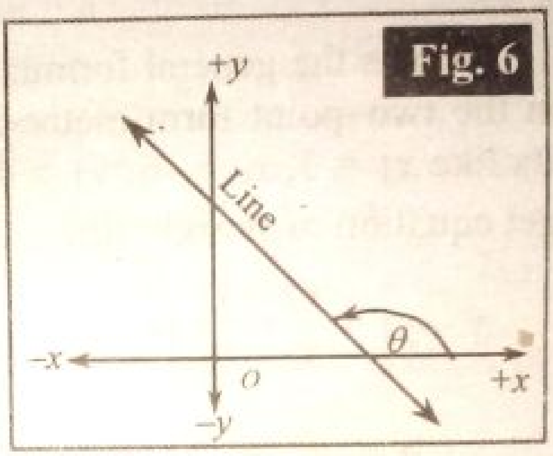
2. If $x_1 = x_2$ while $y_1 \neq y_2$ then the straight line derived through the points (x_1, y_1) and (x_2, y_2) will be vertical or parallel to y–axis. Here, the trend angle will be of 90°. As a result, the slope of the

curve will be undefined i.e., $\tan\theta = \frac{\Delta y}{\Delta x}$ = undefined. It is shown in Fig. 4.

3. If a straight line slopes upward and signs of Δy and Δx are similar (i.e., positive), the value of trend angle (θ) will be less than 90° and greater than zero ($0 < \theta < 90°$). Here, the slope of the curve will be positive.

$\tan\theta = \frac{\Delta y}{\Delta x} > 0$. It is shown in Fig. 5.





4. If a straight line slopes downward and signs of Δy and Δx are opposite to each other the value of trend angle (θ) will be greater than 90° and less than 180° ($90° < \theta < 180°$). Here the slope of the curve will be negative. $\tan\theta = \frac{\Delta y}{\Delta x} < 0$. It is shown in Fig. 6.

DERIVATION OF EQUATION OF A STRAIGHT LINE.

The straight line is determined by the followings: (1) Two points on the line, (2) The slope of the curve and (3) One point on the curve. As a result, different methods are adopted to derive straight line. In the situation of two points on the curve the method of two–points form and intercept form are used. Again the methods of point–slope form and intercept form are used to derive straight line. These formulas will correspond to the standard form of straight line, i.e., $ax + by + c = 0$.

1. **Two–Point Form** The salient feature of a straight line is that its slope remains same. As a result, its slope can be obtained between any two points on the line. Such slope will remain same. On such basis we can find the equation of straight line with the help of two points on the straight line. If there are two points like (x_1, y_1) and (x_2, y_2) on a straight line then its slope will be: $m = \frac{y2 - y1}{x2 - x1}$. If we take a general point on the curve like (x, y), then this point and the point (x_1, y_1) will also provide us the slope of

the curve. It is as: $m = \frac{y - y1}{x - x1}$. The slope of all the points on a straight line is equal. Hence, $\frac{y - y1}{x - x1} = \frac{y2 - y1}{x2 - x1}$. By solving it with cross multiplication we get :

$$(y - y1)(x2 - x1) = (x - x1)(y2 - y1)$$

$$\Rightarrow y - y1 = \frac{y2 - y1}{x2 - x1}(x - x1)$$

This is the general formula to derive the equation of a straight line when the two–point form method has been followed. As if we have two points like $x_1 = 3$, $x_2 = -5$, $y_1 = 4$ and $y_2 = 2$, by putting them in formula we get equation of straight line.

$$y - y1 = \frac{y2 - y1}{x2 - x1}(x - x1) \quad \Rightarrow \quad y - 4 = \frac{2 - 4}{-5 - 3}(x - 3)$$

$$\Rightarrow \quad y - 4 = \frac{-2}{-8}(x - 3) \quad \Rightarrow \quad y - 4 = \frac{1}{4}(x - 3) \Rightarrow y - 4 = \frac{x - 3}{4}$$

$$\Rightarrow \quad 4(y - 4) = 4\left(\frac{x - 3}{4}\right) \quad \Rightarrow \quad 4y - 16 = x - 3 \Rightarrow 4y - 16 = x - 3$$

$$\Rightarrow \quad x - 4y - 3 + 16 = 0 \quad \Rightarrow \quad x - 4y + 13 = 0$$

Thus $x - 4y + 13 = 0$ is an equation of straight line. If we put $x = 3$ and $y = 4$ in the equation $x - 4y + 13 = 0 \Rightarrow 3 - 4(4) + 13 = 0 \Rightarrow 0 = 0$. Again if we put (– 5, 2) in the equation

$$x - 4y + 13 = 0 \quad \Rightarrow \quad -5 - 4(2) + 13 = 0 \quad \Rightarrow 0 = 0.$$

2. **Intercept Form**

If the point $(x1, y1)$ is a y–intercept which is shown by (0, b) where $b \neq 0$, while point $(x2, y2)$ is an x–intercept which is shown by $(a, 0)$ where $a \neq 0$ the straight line formula regarding two points will be as :

$$y - y_1 = \frac{y2 - y1}{x2 - x1}(x - x1)$$

If we introduce b for y1, a for $x2$, 0 for $x1$ and 0 for y2, then putting them in the formula.

$$y - b = \frac{0 - b}{a - 0}(x - 0) \Rightarrow y - b = -\frac{b}{a}(x) \Rightarrow y = -\frac{b}{a}(x) + b$$

$$\Rightarrow \quad y = \frac{(-bx) + ab}{a} \Rightarrow y = -\frac{bx}{a} + \frac{ab}{a}$$

Multiplying both the sides by a we get :

$$ay = -bx + ab \Rightarrow ay = -b[x-a] \Rightarrow \frac{ay}{b} = -[x-a]$$

$$\Rightarrow \frac{y}{b} = -\frac{[x-a]}{a} \Rightarrow \frac{y}{b} = -\frac{x+a}{a} \Rightarrow \frac{y}{b} = -\frac{x}{a} + \frac{a}{a}$$

$$\Rightarrow \frac{y}{b} + \frac{x}{a} = 1$$

This is the equation of a straight line in intercept form when we are given two intercepts. If we are given the following intercepts (0, – 6) and (4, 0) we can derive straight line with the help of above formula. Here b = –6 and $a = 4$, putting them in the formula.

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{4} + \frac{y}{-6} = 1$$

Multiplying both sides by 12 : $(12)\left(\frac{x}{4} + \frac{y}{-6}\right) = (1)(12)$

$3x - 2y = 12 \Rightarrow 3x - 2y - 12 = 0$ This is the equation of st. line. Putting (0, –6) in the above equation.

$$3(0) - 2(-6) - 12 = 0 \Rightarrow 12 - 12 = 0 \Rightarrow 0 = 0$$

Putting (4, 0) in the above equation.

$$3(4) - 2(0) - 12 = 0 \Rightarrow 12 - 12 = 0 \Rightarrow 0 = 0$$

3. Point – Slope Form

We know that the slope of a non–vertical curve is:

$$m = \frac{y2 - y1}{x2 - x1} = \frac{y - y1}{x - x1}$$

Writing the formula of two point form: $y - y1 = \frac{y2 - y1}{x2 - x1}(x - x1)$

Replacing the item of this formula by m, $y - y1 = m(x - x1)$. This is the equation of st. line with the point slope formula. Thus, if we are given one point and slope of a straight line we can derive equation of st. line. If the point is (–1, 2) and the slope is (–4), then putting them: $(x1, y1) = (-1, 2)$ and $m = -4$,

$$y - y1 = m(x - x1) \Rightarrow y - 2 = -4[x - (-1)]$$

$$y - 2 = -4(x + 1) \Rightarrow y - 2 = -4x - 4 \Rightarrow 4x + y - 2 + 4 = 0$$

$$4x + y + 2 = 0.$$

This is the equation of st. line. by putting (–1, 2) in the equation:

$$4(-1) + 2 + 2 = 0 \Rightarrow -4 + 4 = 0.$$

4. Slope – Intercept Form

If we have a point (x_1, y_1) and the y–intercept form like $(0, b)$ the two point formula is written as : $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\Rightarrow y - y_1 = m(x - x_1)$

As $\frac{y_2 - y_1}{x_2 - x_1} = m$. If $y_1 = b$ and $x - x_1 = x$ are assumed, then putting them in the above formula.

$y - y_1 = m(x - x_1)$ $y - y_1 = mx \Rightarrow y = mx + y_1 \Rightarrow y = mx + b$

This is a straight line equation in the slope intercept form. Thus if we are given y–intercept and slope of the curve we can find the straight line. If we assume y–intercept (0, 5) and the slope of the curve (3) — thus $b = 5$, $m = 3$, putting them in the formula.

$y = mx + b \Rightarrow y = 3x + 5 \Rightarrow 3x - y + 5 = 0$ Putting value of b and m $3(0) - y + 5 = 0$

Again putting (0, 5) $3(0) - 5 + 5 = 0 \Rightarrow 0 = 0$

ECONOMIC APPLICATION OF A STRAIGHT LINE

The demand function, the supply function, consumption function, saving function, IS curve, LM curve etc. are the representative of straight line. Accordingly, they can be explained with the help of above mentioned formulas. In the beginning we derive demand function with the help of two–point formula. Two point formula of straight line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Following it the demand function formula will be as :

$$Q - Q_1 = \frac{Q_2 - Q_1}{P_2 - P_1}(P - P_1)$$

If $P_1 = 80$, $Q_1 = 10$, $P_2 = 60$ and $Q_2 = 20$, the first point will be (80, 10) and the second point will be (60, 20). Putting them in the demand formula.

$$Q - 10 = \frac{20 - 10}{60 - 80}(P - 80) \Rightarrow Q - 10 = \frac{10}{-20}(P - 80)$$

$$\Rightarrow Q - 10 = \frac{1}{-2}(P - 80) \Rightarrow Q - 10 = \frac{1}{-2}P + 40$$

$$\Rightarrow Q = \frac{1}{-2}P + 50 \Rightarrow Q + \frac{1}{2}P - 50 = 0$$

equations of consumption, savings and IS curve.

CONSUMPTION FUNCTION

If we take two points on the consumption function (100, 100) and (150, 130), applying the two–point formula.

$C - C_1 = \frac{C_2 - C_1}{Y_2 - Y_1}(Y - Y_1)$. Putting the values in this formula:

$$C - 100 = \frac{130 - 100}{150 - 100}(Y - 100)$$

$$C - 100 = \frac{30}{50}(Y - 100) \quad \Rightarrow \quad C - 100 = \frac{3}{5}Y - \frac{3}{5}(100)$$

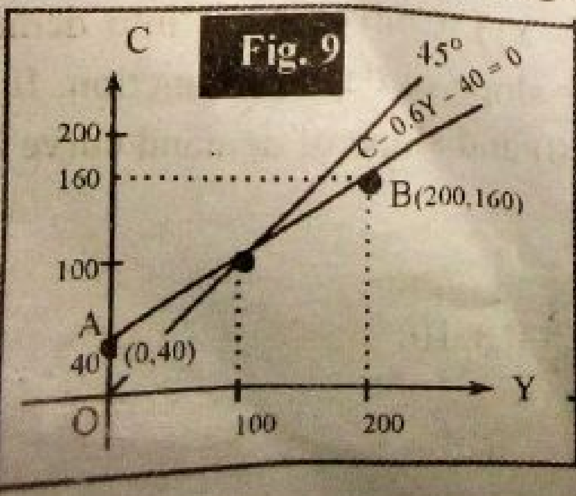
$$C = 0.6Y - 60 + 100 \Rightarrow C = 0.6Y + 40 \Rightarrow C - 0.6Y - 40 = 0.$$

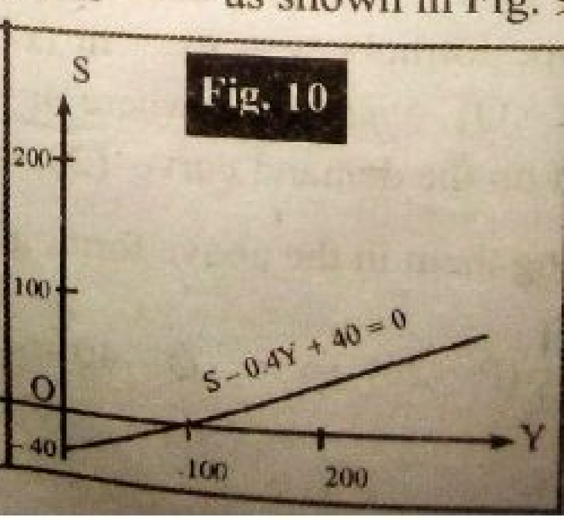
Putting the values Y = 0 and Y = 200

$$C - 0.6(0) - 40 = 0 \quad \Rightarrow \quad C - 40 = 0 \Rightarrow C = 40$$

$$C - 0.6(200) - 40 = 0 \Rightarrow C - 120 - 40 = 0 \Rightarrow C = 120 + 40 = 160$$

Thus the first point on consumption function is (0, 40) while the second point on consumption function is (200, 160). If we draw the consumption schedule we get a straight line —— as shown in Fig. 9.





SAVING FUNCTION

If we are given a point on saving schedule (150, 20) and slope of saving schedule (0.4) we can derive straight line equation of saving schedule.

$$S - S_1 = m(Y - Y_1) \Rightarrow S - 20 = 0.4(Y - 150)$$

$$\Rightarrow S - 20 = 0.4Y - 60 \Rightarrow S = 0.4Y - 60 + 20$$

$$\Rightarrow S = -40 + 0.4Y \Rightarrow S - 0.4Y + 40 = 0$$

If $Y = 0$, then $S = -40 + 0.4(0) = -40$. It means if $Y = 0$, then $S = -40$. Thus it is the first point on saving function (0, – 40). If $Y = 200$ then $S = 40$. It is calculated as:

$$S - 0.4(200) + 40 = 0 \Rightarrow S - 80 + 40 = 0 \Rightarrow S = 40.$$

With the help of two point (0, – 40) and (200, 40) the saving schedule is derived in Fig. 10.

THE *IS* CURVE

We know that IS curve shows the combinations of rate of interest and level of income where savings are equal to investment. If we are given two points on IS curve, i.e., (5%, 199.5) and (10%, 199), putting them in the formula.

$$Y - Y_1 = \frac{Y_2 - Y_1}{i_2 - i_1}(i - i_1) \Rightarrow$$

$$Y - 199.5 = \frac{199 - 199.5}{\frac{10}{100} - \frac{5}{1000}}\left(i - \frac{5}{100}\right)$$

$$\Rightarrow Y - 199.5 = \frac{-0.5}{\frac{5}{100}}\left(i - \frac{5}{100}\right) \Rightarrow Y - 199.5 = -10\left(i - \frac{5}{100}\right)$$

$$\Rightarrow Y - 199.5 = -10i + 0.5 \Rightarrow Y = -10i + 0.5 + 199.5$$

$$\Rightarrow Y = 200 - 10i \Rightarrow Y + 10i - 200 = 0$$

If we put $i = 0\%$, then $Y + 10(0) - 200 = 0 \Rightarrow Y = 200$

If we put $i = 20\%$, then $Y + 10\left(\frac{20}{100}\right) - 200 = 0 \Rightarrow Y = 198$

Accordingly, the first point on the IS schedule is (0%, 200) and the second point on the schedule (20%, 198). With the help of these points we can construct *IS* schedule.