# *Experiment 4: Charge to mass ratio (e/m) of the electron*

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*Office Hour: Monday, 5:30PM-6:30PM @ Pupin 1216*

INTRO TO EXPERIMENTAL PHYS-LAB 1494/2699

# Introduction

- ***Our first measurement of atomic structure***
- ***Charge-to-mass ratio of electron:***
  - Motivation and history of the first *e/m* measurement
  - Consequences Thomson's experiment
- ***The physics behind the experiment:***
  - The magnetic field generated by a single loop
  - Charged particle in constant magnetic field
- ***Measurements:***
  - Preliminary set-up: ambient magnetic field
  - Measure charge-to-mass ratio by bending electron through constant magnetic fields
- ***Analysis***

# Why measure *e/m*?

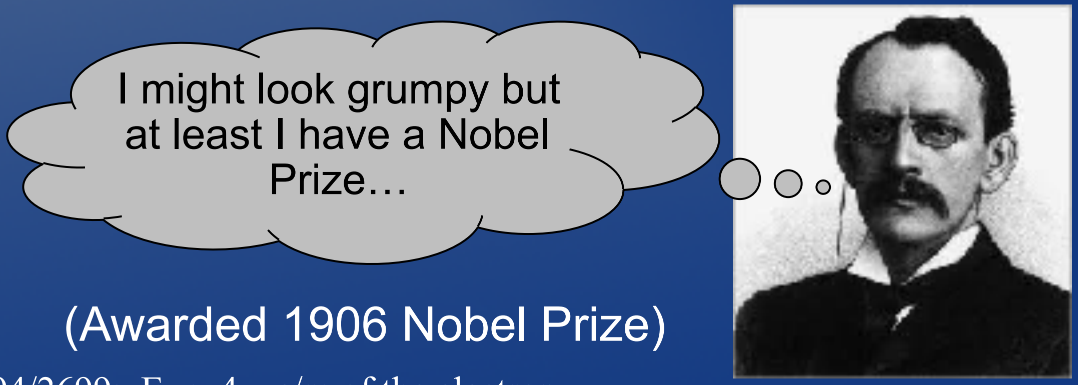
- Prior to this measurement *(~1897)*, what did we know about matter?
  - **ANSWER:** Very little!

- We understood the *classical* *macroscopic* forces between matter:
  - Gravity: $\vec{F}_g = -G\frac{m_1 m_2}{r^2}\hat{r}$
  - Electromagnetism:
    - Maxwell's equations
    - Coulomb and Lorentz forces: $\vec{F}_e = -k\frac{q_1 q_2}{r^2}\hat{r};\ \vec{F}_m = q\vec{v}\times\vec{B}$

- Nobody really knew what the constituents of matter were. Or if there were any!

# What is matter?

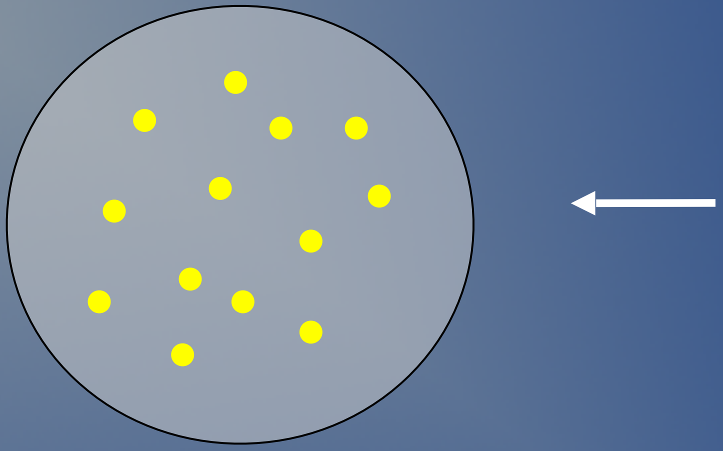
- What is matter really made of?
  - Infinitely divisible (*i.e.* continuous matter)?
  - Made up of individual constituents?
- **J.J. Thomson** provided experimental evidence of the *existence of discrete constituents of matter:*
  - He showed that matter has constituents that are negatively charged and whose charge/mass ratio is constant
  - This suggests that charge is not infinitely indivisible but comes in corpuscles (*e.g.* electrons)
  - This is the first example of **quantization** (existence of discrete components) of nature!




(Awarded 1906 Nobel Prize)

# Consequences

- This led Thomson to postulate the “plum pudding” model:



**Quanta (electrons) enclosed in a continuous positive distribution**

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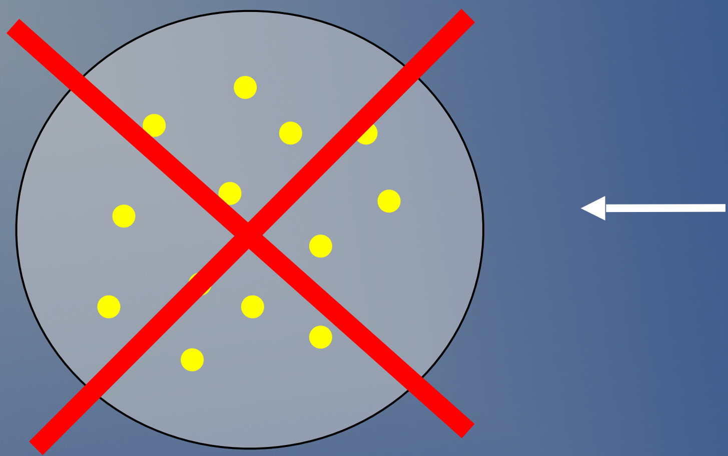

~~**Quanta (electrons) enclosed in a continuous positive distribution**~~

- Nice idea… too bad it is completely wrong…
- ***Rutherford scattering experiments (1911):***
  - Matter is made of <u>compact</u> <u>localized</u> **nuclei**
- ***Development of Quantum Mechanics (1920's):***
  - The microscopic constituents of matter follow novel and surprising laws of physics!

# We're not done yet!

- Several experiments suggest that electrons might indeed be elementary, indivisible particles
- However nucleons (protons or neutrons) are not!
  - 1968:  Scattering experiments at the Stanford Linear ACcelerator (SLAC) show that nucleons are made of more fundamental particles: **quarks**

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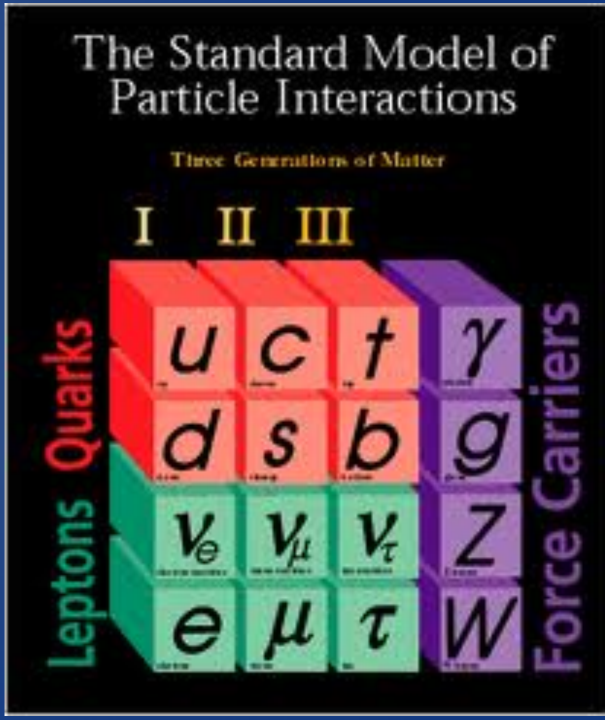
- This led to the development of the ***Standard Model of elementary particles***



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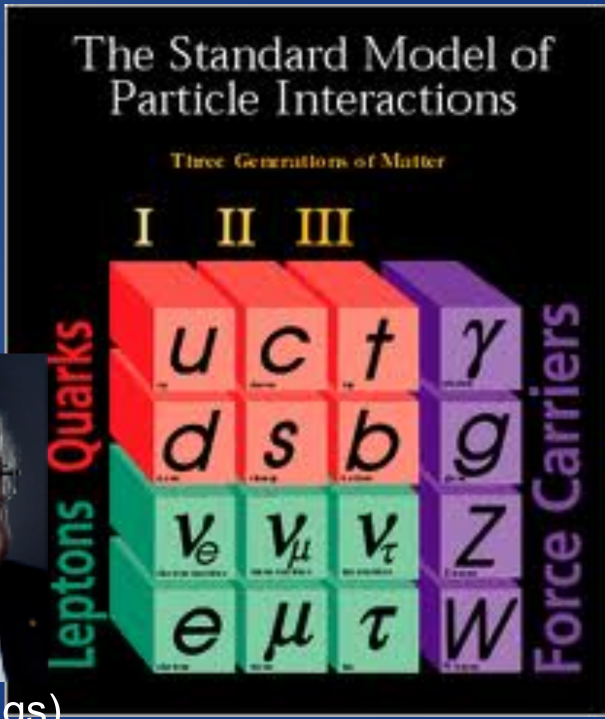
- This led to the development of the ***Standard Model of elementary particles***
- In 2012 the last piece of the puzzle completes the pictures!
- **And now? What's next?**

**Is it possible to go even further?**

Sorry I'm late!

(Peter Higgs)



# Single loop magnetic field

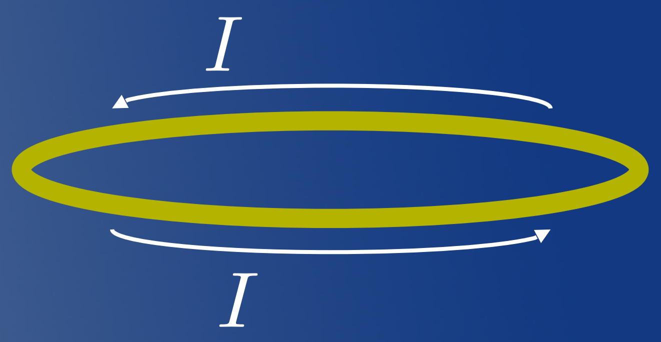
- Recall the ***<u>Biot-Savart Law:</u>***

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

$$\Downarrow$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \oint \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

Line integral over the wire



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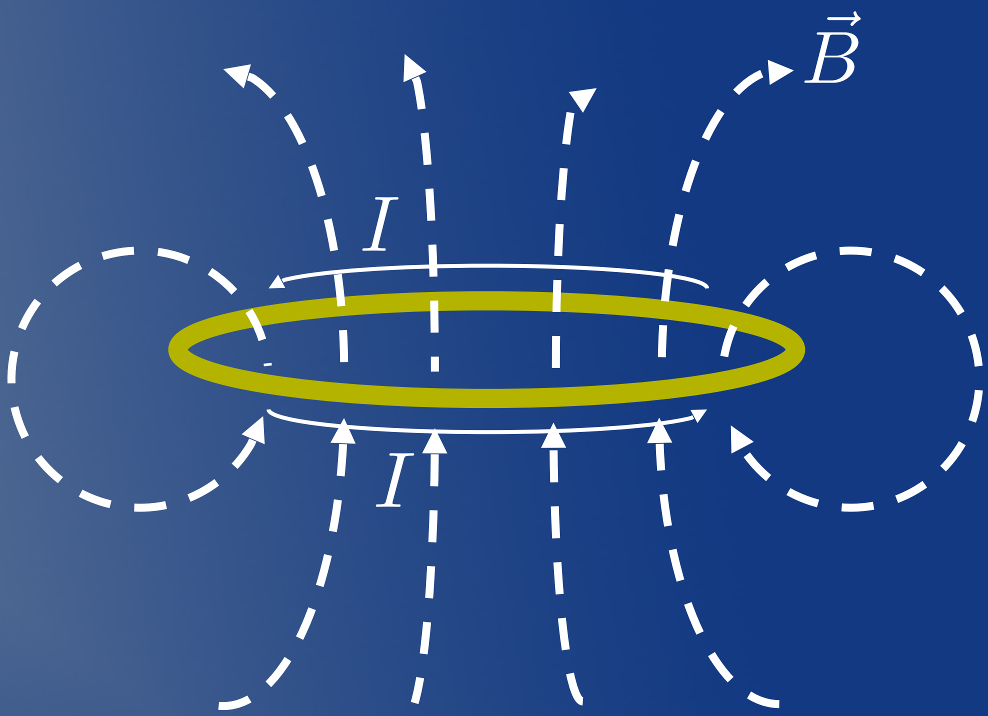


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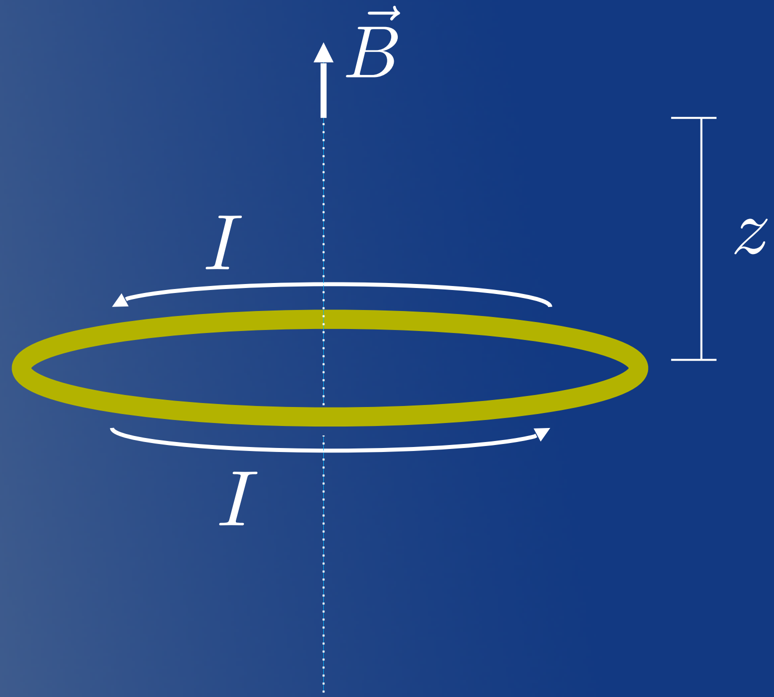



- A single ring of wire will generate a field following the right hand rule and with magnitude **on the ring axis**:

$$B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(R^2 + z^2\right)^{3/2}}$$

*R* = radius of the loop
*z* = height from the plane of the loop

# Circular motion of charge in B field

- The second ingredient we are going to use is the motion of a charge experiencing a constant magnetic field
- Last time (exp. 3) we saw that a particle of charge *q*, entering a region of space with constant *B* field at a velocity *v* feels a Lorentz force:

$$F = qvB$$

- Since velocity and *force are perpendicular* to each other the motion will be circular
- In particular, the centripetal force must be provided by the Lorentz force itself and hence:

$$qvB = m\frac{v^2}{r} \quad \Rightarrow \quad \frac{q}{m} = \frac{v}{rB}$$

# Circular motion of charge in B field

- However, measuring the velocity of an elementary particle is a very non-trivial business
- A simple solution to this problem is to speed the electrons up thanks to a ***known*** potential difference (*e.g.* generated by two plates with opposite charge)
- In this case conservation of energy tells us that:

$$qV = \frac{1}{2}mv^2 \quad \Rightarrow \quad \frac{q}{m} = \frac{v^2}{2V}$$

- Combining it with the previous result, we can eliminate the velocity from the equation:

$$\boxed{\frac{q}{m} = \frac{2V}{r^2 B^2}}$$

# *The Experiment*

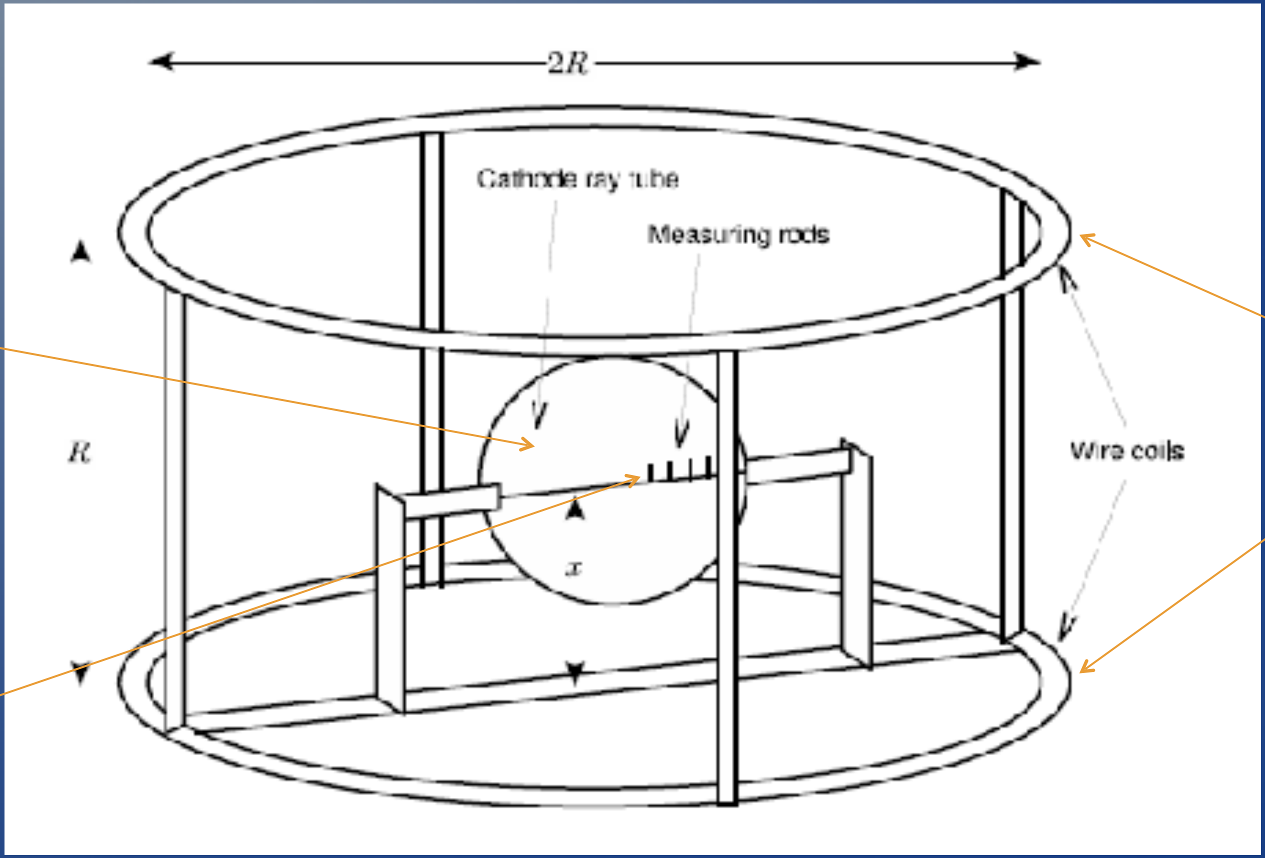
# Main goals

- ***The equipment:***
  - ***Helmholtz coils:*** provide magnetic field
  - ***Cathode ray tube:*** provides electron beam
  - ***Measuring rods:*** allow to measure the radius of the circular motion

- ***The experiment:***
  - Alignment of experimental apparatus with the ambient magnetic field $B_E$
  - Preliminary measure of $B_E$
  - Bend electron beam with magnetic field and measure *e/m*

# Apparatus

- For this experiment:
  - Use two coils with current $I$
  - Cathode ray tube centered in between Helmholtz coils



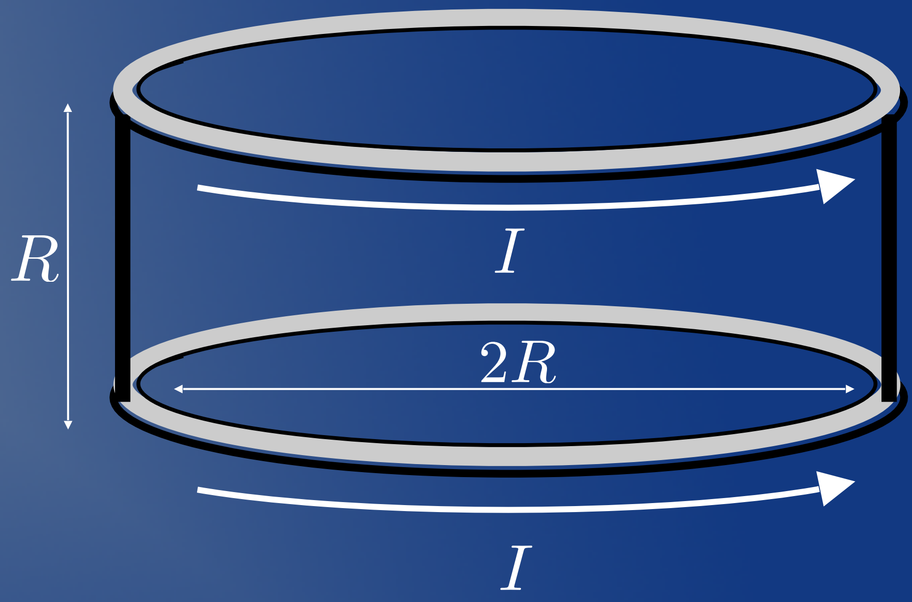

Electrons will appear in here

Metallic "comb" used to measure *r*

Generate the magnetic field

# Helmholtz coils (uniform magnetic field)

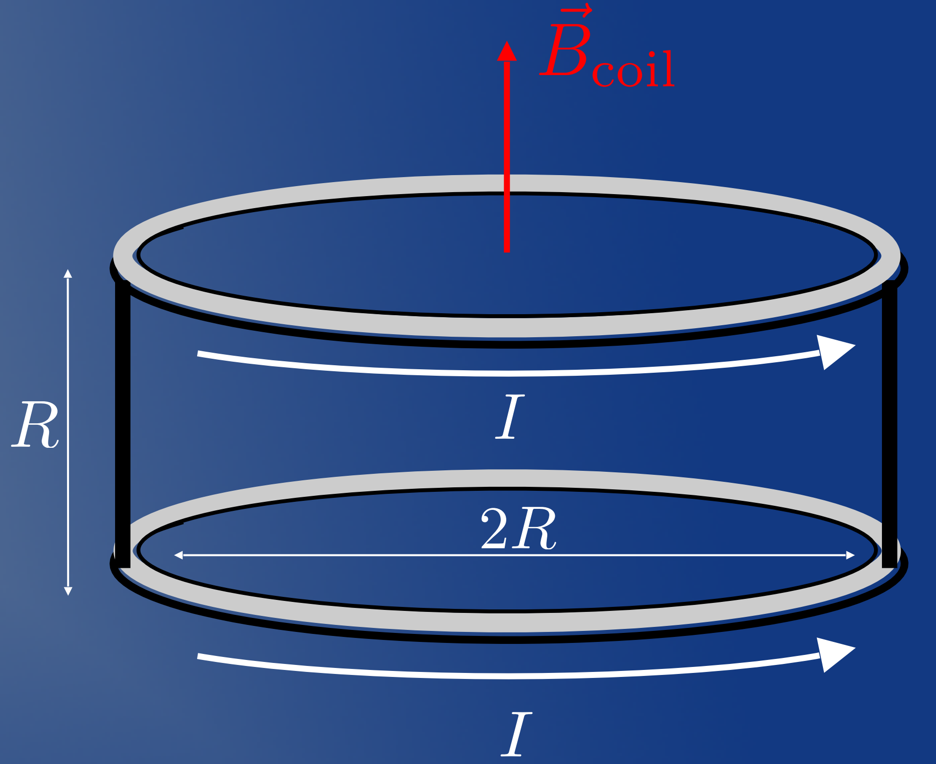
- For this experiment:
  - Use two coils with current $I$



# Helmholtz coils (uniform magnetic field)

- For this experiment:
  - Use two coils with current $I$
- For this setup we need the magnetic field at the center of the Helmotz coils (*z* = *R/2*):

$$B_I = \frac{\mu_0}{4\pi} \frac{2\pi R^2 N I}{(R^2 + (R/2)^2)^{3/2}}$$
$$\equiv C\, I$$




**This represents magnetic field at *center* of experimental apparatus. (Location where cathode ray tube will sit)**

- Calculate constant *C* from *R* and *N!*

# How to manage the magnetic field

- The power supply for coils and filament looks like:

Ammeter

Coil current adjust knob (changes the current flowing in the coils)

Coil current reverse (changes the direction of the current and hence of the *B* field)

# Preliminary alignment

- ***The equipment:***
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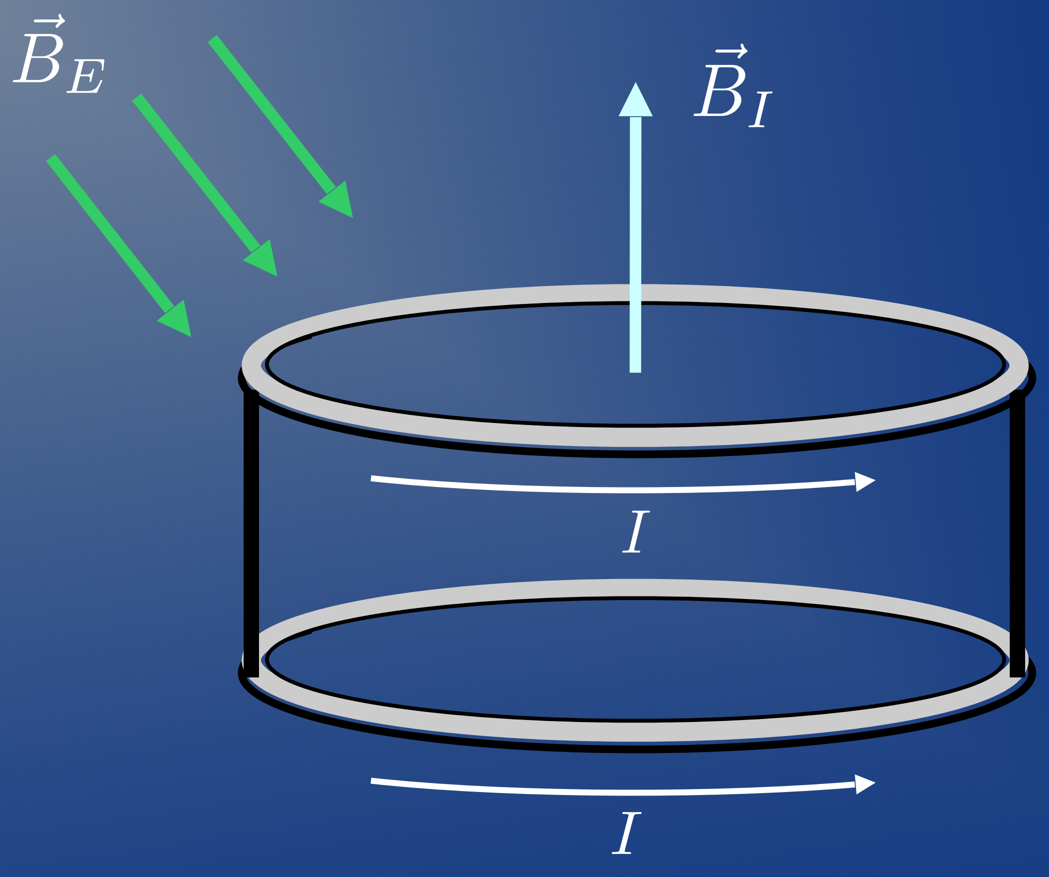
# Aligning to environment magnetic field

- Even without any current the electron will still experience the ambient magnetic field $B_E$ (Earth, nearby magnets, etc.)
- We want to minimize and estimate this effect



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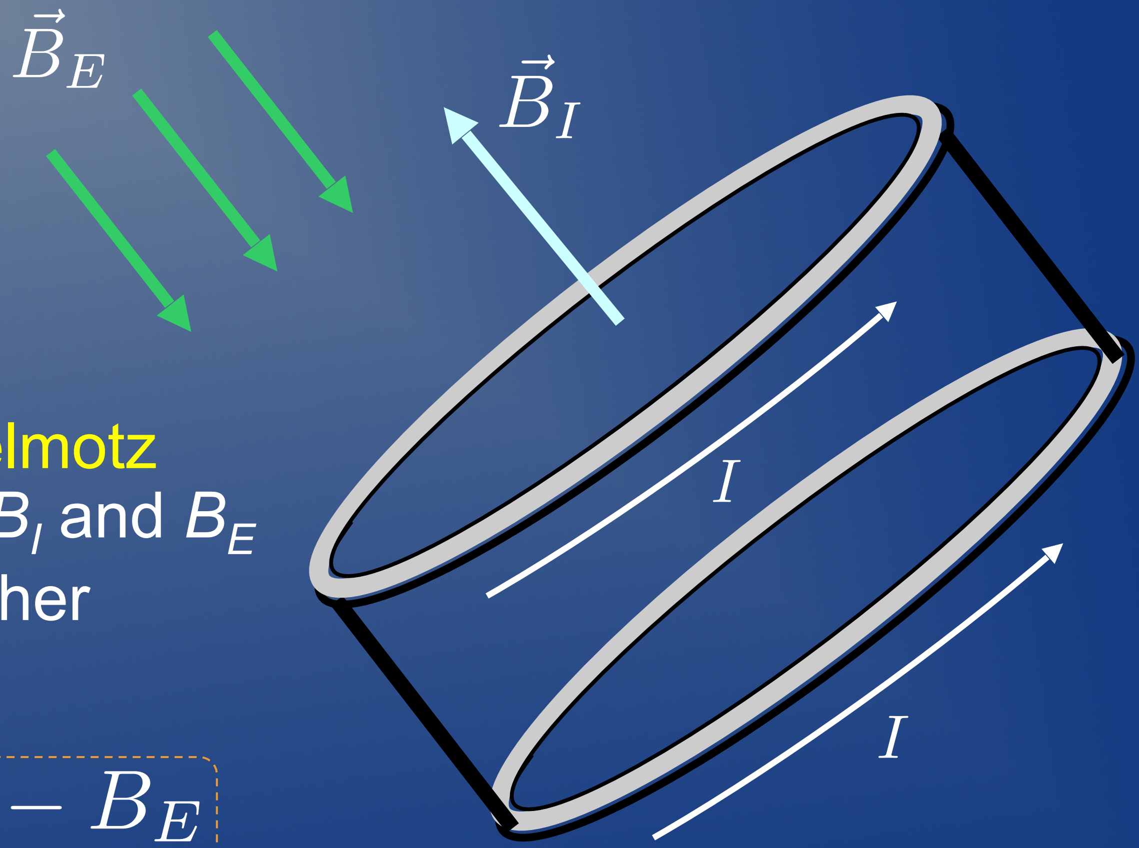
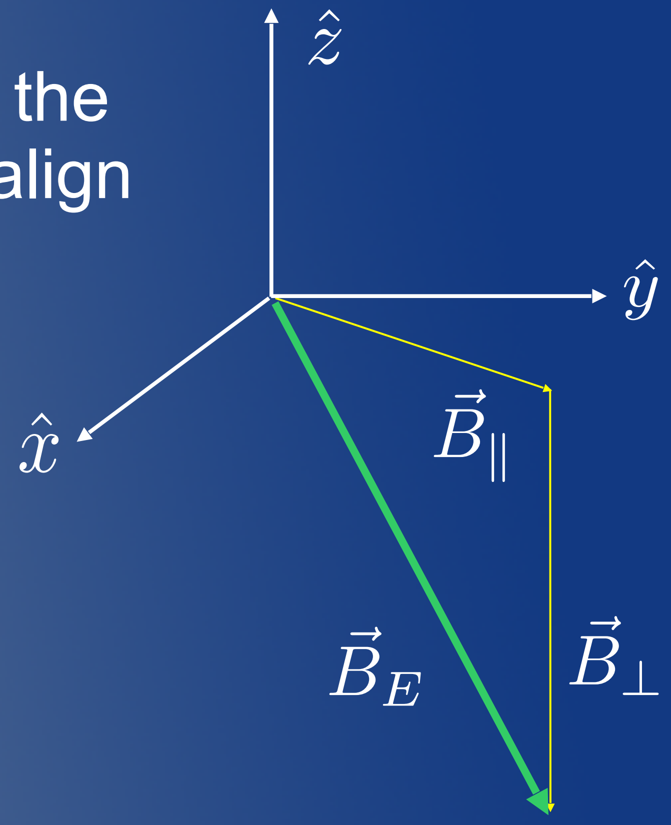



- By rotating the Helmotz coil we can bring $B_I$ and $B_E$ parallel to each other
- Therefore:

$$B_{\text{net}} = B_I - B_E$$

# Aligning to environment field

- To fully determine the magnitude of the ambient magnetic field we need to align the experiment to $B_E$



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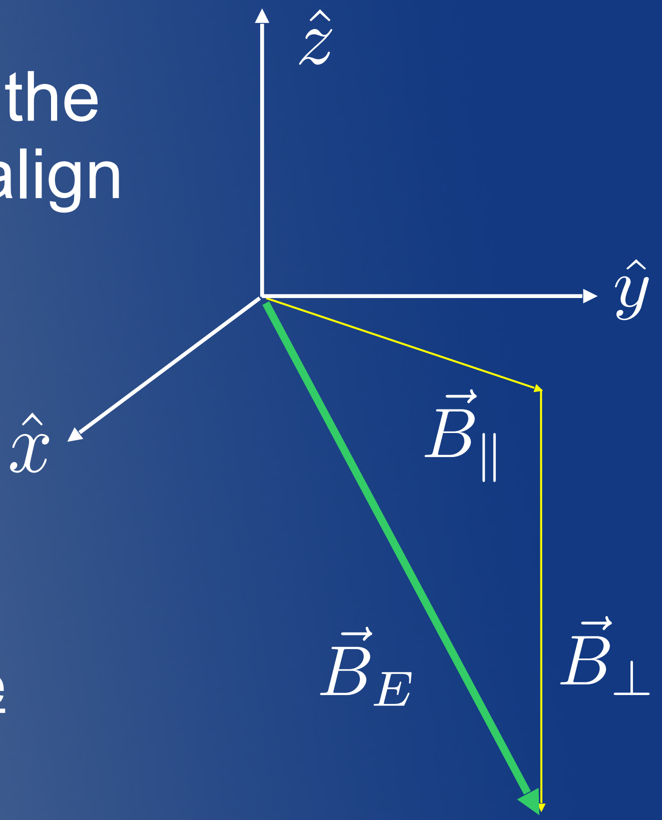
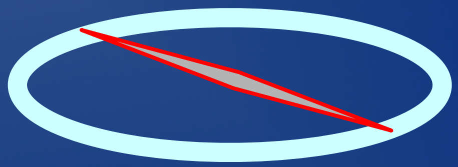



- **Procedure:** Use compass
  - Place compass flat horizontally
  - Rotate coils to align with $\vec{B}_{\|}$ . Needle must be on 90 or 270 degrees



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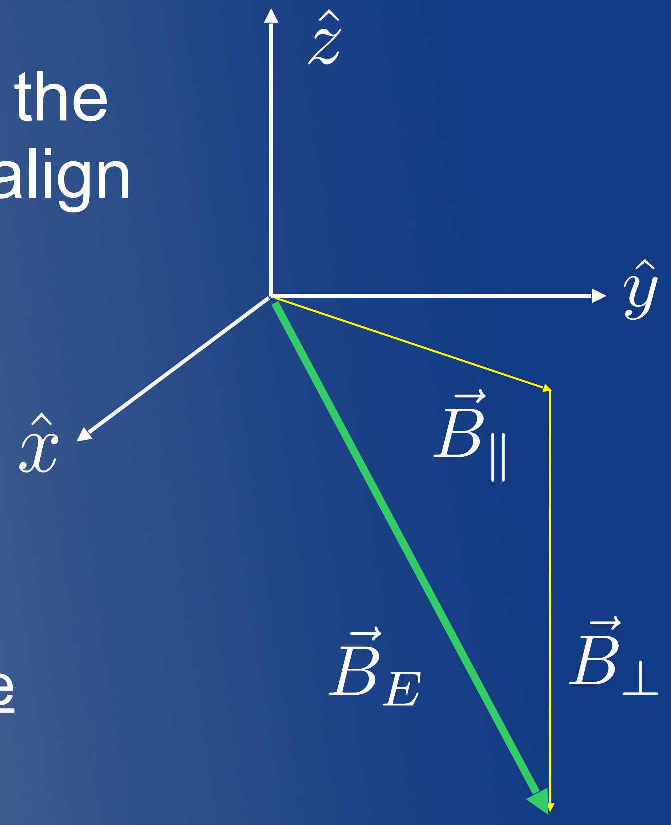
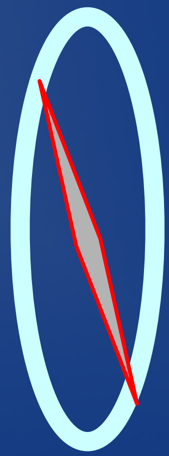
- **Procedure:** Use compass
  - Place compass flat horizontally
  - Rotate coils to align with $\vec{B}_{\|}$. <u>Needle must be on 90 or 270 degrees</u>
  - Rotate compass vertically
  - Rotate experiment to align with $\vec{B}_{\perp}$. <u>Needle must be on 0 or 180 degrees</u>
- Now your Helmotz coil is nicely aligned with the ambient field!



# Aligning to environment field: tips

- Make sure to align horizontally ***before*** you align vertically.
- Reading compass needle:
  - When aligning, needle will oscillate!
  - Wait until compass needle has stopped oscillating (maybe help reduce the oscillations with your hand) to determine if experimental apparatus is aligned
  - **Tip:** Wait about 30 seconds before determining if it's aligned.
- Parallax errors:
  - Once again, try to be careful and consistent in eye alignment with compass needle
- This preliminary set up is really important. Once you're done ask your TA to check that everything is fine!

# Aligning to environment field: tips

- After you completed your alignment procedure the whole apparatus should look like:

# Preliminary measure of $B_E$

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# Adjusting voltage on heated filament

- Same power supply as Helmholtz coils

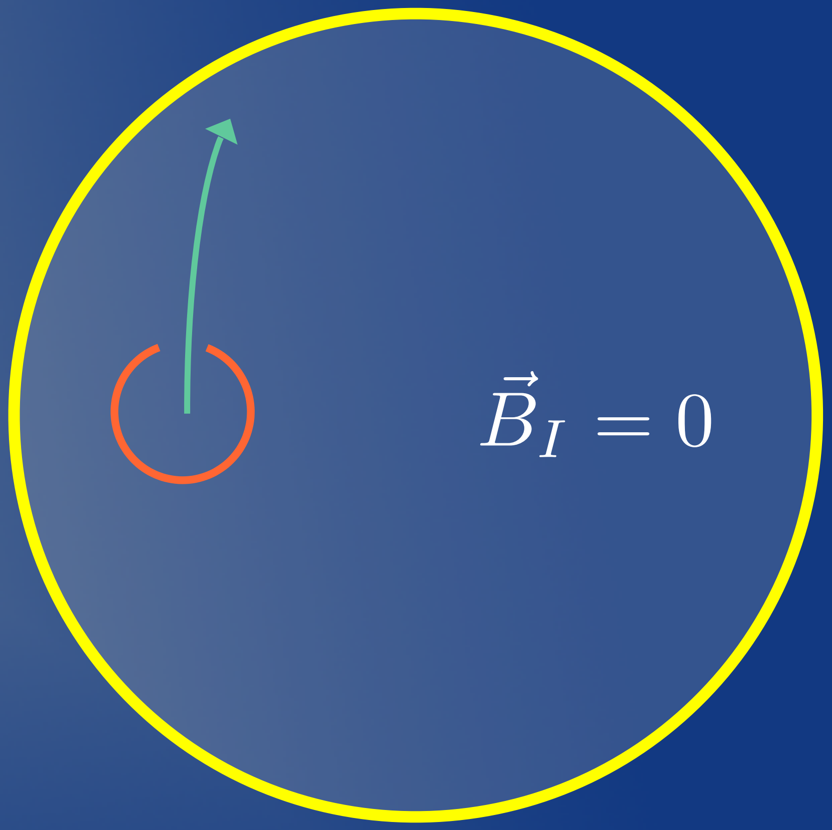

<u>Voltmeter</u> – measure voltage potential on the filament that ejects the electrons

Voltage adjust knob

Filament potential reversal

# Measuring the Ambient field

- Turn Helmholtz coils off : $\vec{B}_I = 0$
- Note that the beam will be curved anyway
- Set the Helmholtz coil current gain setting to 200mA
- Using the "coil reverse" switch and "current adjustment knob", determine the current needed to make the beam trajectory straight
- When the beam is straight it must be $B_I = B_E$
- Determine $B_E$ (with errors!)



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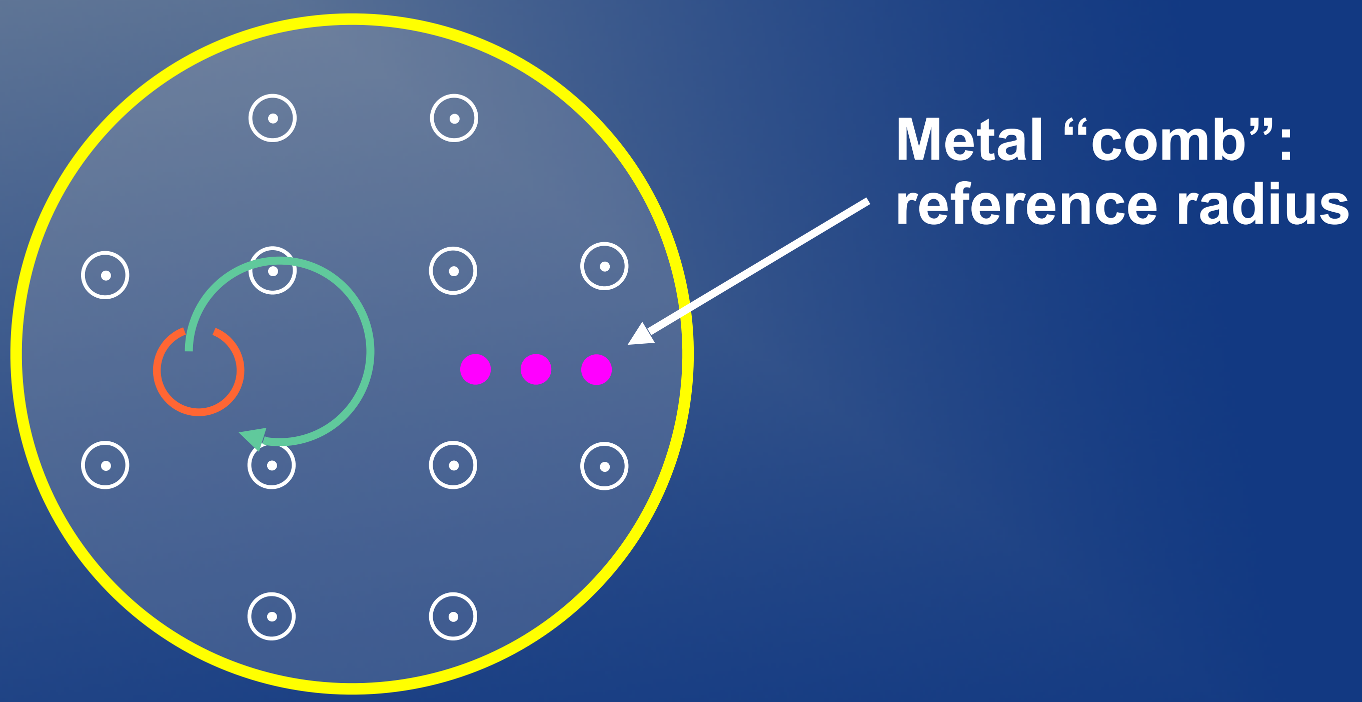
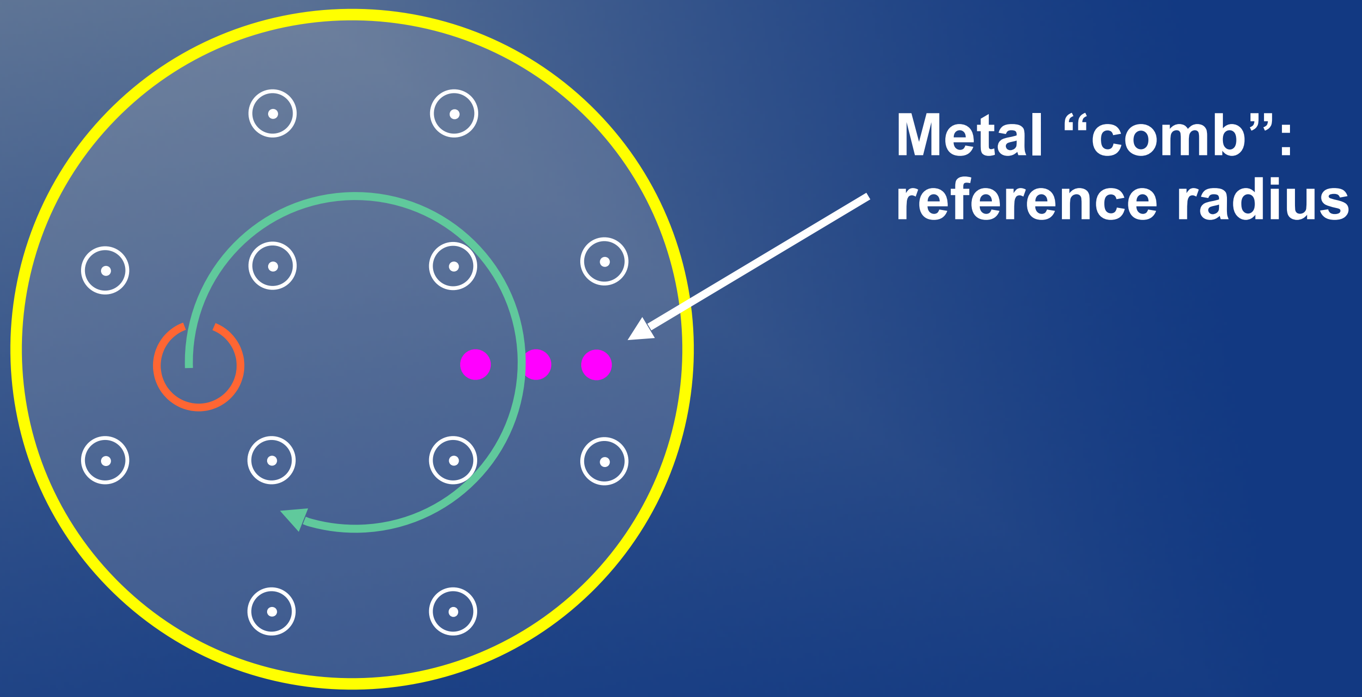
# Measuring circular orbits

- Turn Helmholtz coils on and set the gain switch to 10A sensitivity
- Set the filament voltage to 40V (*i.e.* set the energy of the electrons)



# Measuring circular orbits

- Turn Helmholtz coils on and set the gain switch to 10A setting
- Set the filament voltage to 40V (*i.e.* set the energy of the electrons)
- Adjust the coil current to change radius of curvature



# Measuring circular orbits

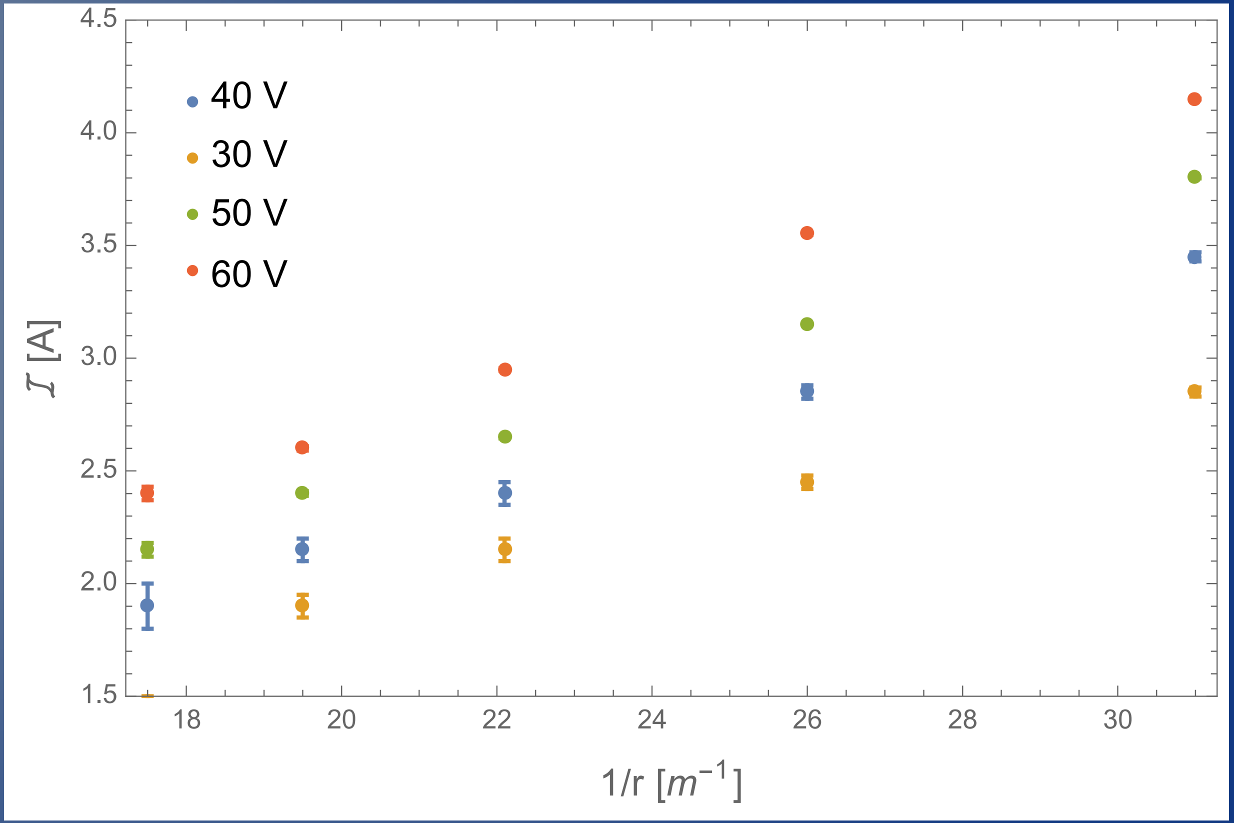
- Measure the coil current ( *I* ) needed to reach each bar with a fixed filament voltage V = 40 V
- The radius for each bar is given in the lab manual
- **Recording uncertainties for coil current:**
  - Try to reduce parallax: Keep eye alignment consistent when reading ammeter and voltmeter.
  - ***Electron current has a considerable width***. Uncertainty in coil current should be done by finding maximum and minimum current as you scan the width of the beam at a given radius.
  - ***Be consistent*** with where you call reference radius of the bar. (*i.e.* let inner radius of bar be reference radius)
- Repeat measurements for different filament voltages: 20V, 60V, 70V, 80V, 100V

# Analysis

- Linearize the data by plotting *I vs. 1/r*
- According to what explained in the previous slides:

$$I = \left(\frac{1}{C}\sqrt{\frac{2V}{e/m}}\right)\frac{1}{r} + \left(\frac{B_E}{C}\right)$$

- Perform a linear (<u>weighted!</u>) fit and find slope and intercept




- Compare the value obtained for <u>*e/m*</u> and the <u>ambient field</u> with:
  - The accepted: $(e/m)_{acc} = 1.759 \times 10^{11}$ C/kg
  - The value of $B_E$ measured in the preliminary part

# Final tips

- The hardest part of the experiment is definitely to determined $B_E$ with the method explained in part 2: try to determine what uncertainty you have on this part accurately! It probably will be fairly large
- Most of the procedures are two-people jobs. The best solution is probably to have someone turning the knobs and someone else looking at the apparatus
- Be very careful when handling the structure! Always move the instrument using the wooden part, not the coils neither the central bulb