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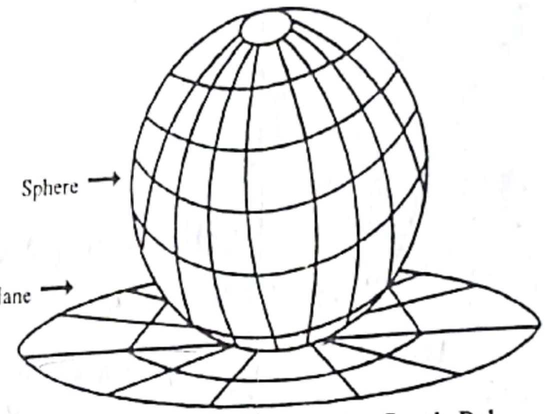
CHAPTER 8

Zenithal Map Projections

THE zenithal map projections are constructed by projecting the parallels and the meridians of a globe on a plane placed tangentially (Fig. 57) to it at one of the poles or at its any other point. When the plane is placed tangentially at a point on the equator, the projection is called a zenithal projection (equatorial



Plane is tangent to the sphere at the South Pole.
Fig. 57

case). When placed tangentially at a point between a pole and the equator it is called a zenithal projection (oblique case) and when the plane is placed tangentially at one of the poles, it is called a zenithal projection (polar case). In this book we shall study only the *polar cases*. The properties common to the zenithal projections (polar cases) are given below :

1. The pole is the centre of a projection.
2. The parallels are concentric circles.
3. The meridians on a globe are great circles. They are projected as straight lines on these projections. Therefore, all great circles passing through the centre of these projections are represented as straight lines.
4. The meridians are straight lines radiating from the centre of the projection. They are spaced uniformly at the correct angular interval.
5. The plane on which the globe is projected is placed tangentially at one of the poles and the centre of the projection coincides with the pole. Therefore, the meridians on these projections represent true azimuths* or bearings. The bearings from the centre of all zenithal projections are thus true. The

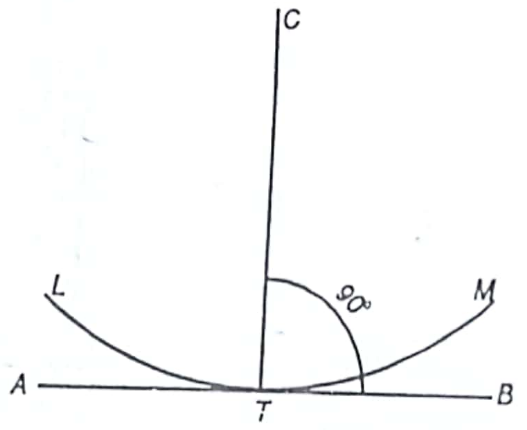
*An azimuth is a bearing measured clockwise from the north. When measured from the true north, it is a true azimuth and when measured from the magnetic north, it is a magnetic azimuth.

azimuths being true in these projections, the latter are also called *azimuthal projections*.

6. The outlines of maps on these projections are circular.

7. The meridians intersect the parallels at right angles. The right angle is formed between the mer and the tangent placed at the point of intersection as shown in Fig. 58.

8. The projections are more suitable for showing polar areas than other areas.



CT	*meridian of longitude*
AB	*tangent*
T	*point of intersection*
LM	*parallel of latitude*

Fig. 58

We shall study the following zenithal map projections :

(i) The Polar Zenithal Equal-Area Projection.

(ii) The Polar Zenithal Equidistant Projection.

(iii) The Polar Gnomonic Projection.

(iv) The Polar Stereographic Projection.

(v) The Polar Orthographic Projection.

The Gnomonic, Stereographic and Orthographic Projections are *perspective projections*. In the ca these perspective projections, the plane is placed tangentially at the North (or South) Pole but the pos of the point from which the rays are projected to meet the plane is different for different projections example, it is at the centre for the Gnomonic Projection, antipodal for the Stereographic Projection, infinity for the Orthographic Projection.

I. THE POLAR ZENITHAL EQUAL-AREA PROJECTION

This projection is also called Lambert's Equal-Area Projection after its inventor J.H. Lambert designed it in the year 1772.

Let us draw the Polar Zenithal Equal-Area Projection for the Northern Hemisphere on the s 1 : 200,000,000 spacing parallels at 15° interval and meridians at 30° interval.

Construction

Radius of the earth=250,000,000 inches.

∴ Radius of the globe on the scale of 1 : 200,000,000

$$= \frac{250,000,000}{200,000,000} = \frac{5}{4} = 1.25 \text{ inches}$$

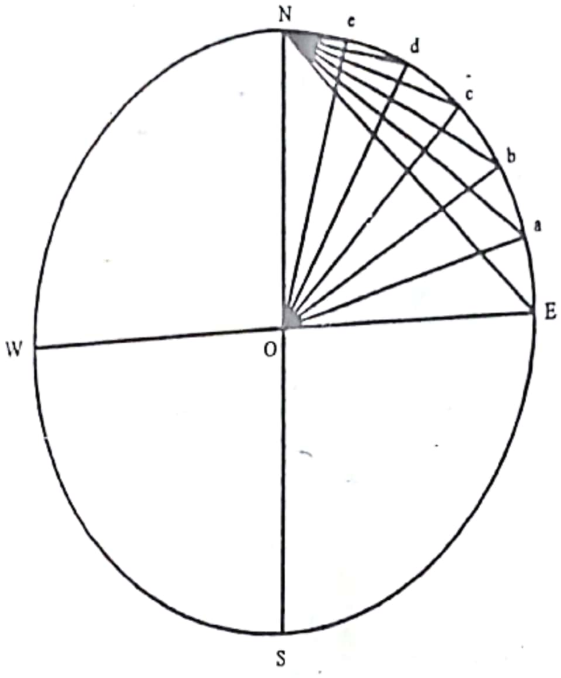


Fig. 59

Draw a circle with radius equal to the radius of the globe, i.e. 1.25 inches. *This circle represents the globe.* Let NS its polar diameter and WE its equatorial diameter, intersect each other at right angles at O, the centre of the circle (Fig. 59).

Draw radii Oa, Ob, Oc, Od and Oe making angles of 15°, 30°, 45°, 60° and 75° respectively with OE. Join Ne, Nd, Nc, Nb, Na and NE by straight lines. These straight lines are chords. With radius equal to chord Ne and N' as centre draw a circle (Fig. 60). This circle represents 75°N parallel. Similarly with centre N' and radii equal to the chords Nd, Nc, Nb, Na and NE draw circles to represent the parallels of 60°, 45°, 30°, 15° and the equator (0°) respectively.

Draw straight lines AB and CD intersecting each other at right angles at the centre i.e. point N' (Fig. 60). Radius N'B represents the 0° meridian, N'A 180° meridian, N'D 90°E meridian and N'C 90°W meridian. Draw with the help of a protractor more radii at 30° interval to represent the other meridians as shown in Fig. 60.

The Polar Zenithal Equal-Area Projection for the *Southern Hemisphere* is also drawn as explained above. The degrees of the meridians are, however, written differently. The 0° meridian of the projection showing the Southern Hemisphere is placed where 180° meridian is placed in the projection showing the Northern Hemisphere (Fig. 61).

It should also be noted that in the case of the projection showing the Northern Hemisphere, the 90° meridian on the right-hand side is the eastern meridian and that on the left-hand side is the western meridian. If, however, the 90° meridian on the left-hand side is marked as 90° East and 90° meridian on the

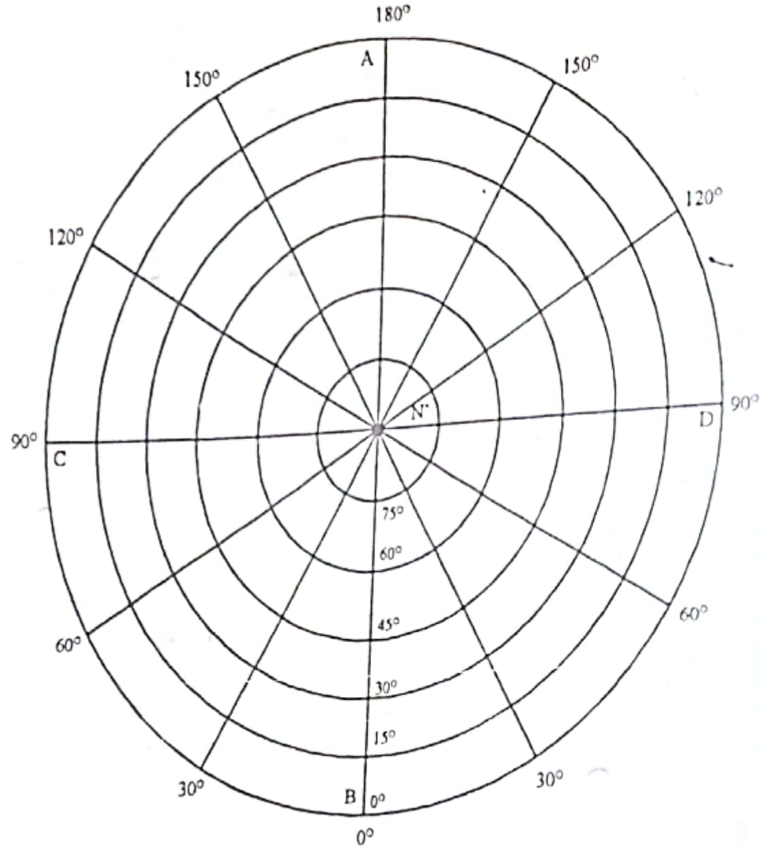


Fig. 60

right-hand side marked as 90° West without bringing any change in the positions of 0° and 180° mer it will represent the Southern Hemisphere. This also applies to all other zenithal projections. See a map of Antarctica and that of the Arctic Ocean in an atlas.

Properties

1. The parallels are concentric circles. The pole is a point forming the centre of the project

2. The meridians are straight lines radiating from the pole. They are spaced correctly at true a interval i.e. the azimuths are true in this projection.

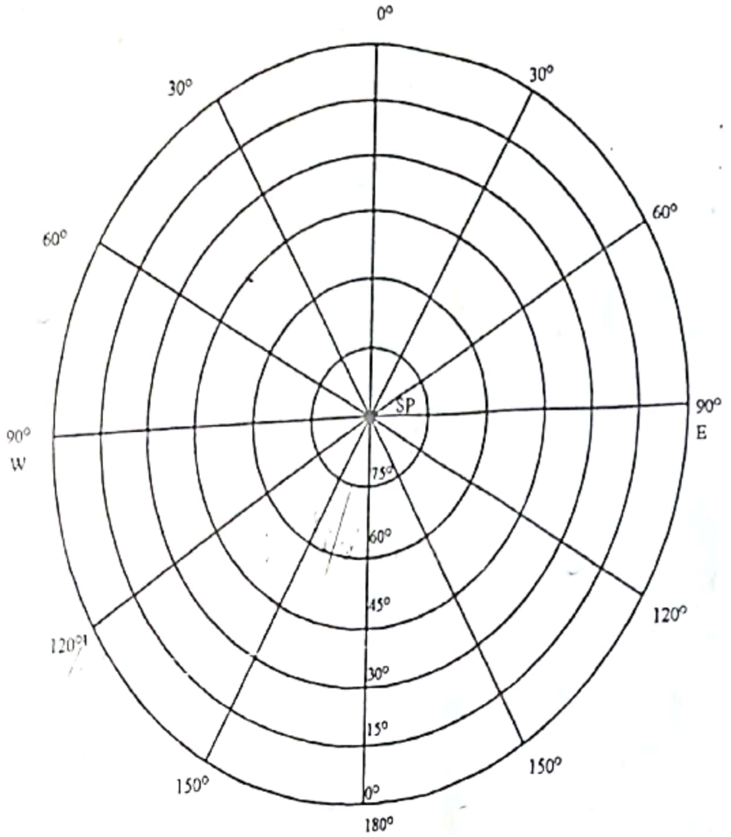
3. The meridians intersect the parallels at right angles.

4. The scale along the parallels increases away from the centre of the projection.

5. The distances between the parallels go on decreasing away from the centre of the project means there is a decrease in the scale along the meridians towards the margin of the projection. The de in the scale along the meridians is in the same proportion in which there is increase in the scale alo parallels away from the centre of the projection with the result that the projection is equal-area. This is further elaborated as under :

In this projection a parallel is a circle drawn with the radius equal to the chord connecting th with that parallel. For example, the parallel of 75°N has been drawn with radius equal to chord N parallel of 60°N with the radius equal to chord Nd and so on (Fig. 59).

To illustrate how the area of the circle described by 75°N parallel is equal to the area of the sp cap of the globe extending from the pole to the 75°N parallel and the area of the circle describin



SP=South Pole

Fig. 61

parallel is equal to the area of the spherical cap of the globe extending from the pole to the 60°N parallel and so on, we proceed as follows:

In Fig. 62, r is the radius of the globe, c the chord connecting N (pole) with a parallel and L is the radius of the parallel.

$$c^2 = h^2 + L^2$$
$$= h^2 + r^2 - (r - h)^2$$
$$= h^2 + r^2 - r^2 + 2rh - h^2 = 2rh$$

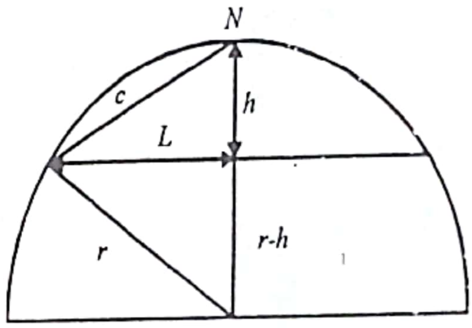


Fig. 62

But the area of a circle $=\pi R^2$ where R is the radius of the circle.

$\therefore$ area of the circle with radius equal to chord $c = \pi c^2$ or $2\pi rh$. Now area of a *zone* (area between two parallels) $=2\pi rh$ where r is the radius of the globe and h is the height of the zone. Thus, the area enclosed by a parallel (a circle in this projection) is equal to πc^2. So area contained within a parallel on this projection is equal to the area of the spherical cap of the globe extending from the pole to that parallel. Hence, it is an equal-area projection.

6 Shapes are more and more distorted away from the centre of the projection because the scale along the meridians is too small and that along the parallels too large. The shapes are compressed along the meridians but stretched along the parallels.

7. The shapes of the central areas of the projection are represented in a satisfactory way for the sh... are distorted here but very slightly.

Limitations

Shapes being distorted away from the centre of the projection, it is only the central part of the projec... that can be represented in a satisfactory way.

Generally, the area lying between a pole and 45° parallel can be shown on this projection satisfacto...

Uses

(1) Since it is an equal-area projection and the shapes of the countries in the central part are presen... in a fairly satisfactory way this projection is used for preparing political and distribution maps of p... regions.

(2) It is also used for making general-purpose maps of large areas in the Northern Hemisphere

II. THE POLAR ZENITHAL EQUIDISTANT PROJECTION

Let us draw a network of the Polar Zenithal Equidistant Projection for the Northern Hemisphere the scale of 1 : 200,000,000 with parallels at 15° interval and meridians at 30° interval.

Construction

Radius of the earth = 250,000,000 inches

∴ Radius of the globe on the scale of 1 : 200,000,000

$$= \frac{250,000,000}{200,000,000} = 1.25''$$

We are required to draw parallels at an interval of 15°

∴ Length of the arc subtended by 15°

$$= 2\pi r \times \frac{15}{360} = 2 \times \frac{22}{7} \times 1.25 \times \frac{15}{360}$$

$$= 0.327''$$

With N as centre and radius equal to 0.327″ draw a circle. This circle represents 75°N parallel. ... draw a circle representing 60°N parallel, double the length of the arc subtended by 15° i.e. take a rad... of 0.327 × 2 or 0.654″ and with N as centre draw the circle. Similarly circles with the same centre a... radii equal to 0.327 × 3 or 0.981″, 0.327 × 4 or 1.308″, 0.327 × 5 or 1.635″ and 0.327 × 6 or 1.962″ w... represent parallels of 45°N, 30°N, 15°N and the equator (0°).

Draw straight lines AB and CD intersecting each other at right angles at the centre ∴ point N. Rad... NB represents the 0° meridian, NA 180° meridian, ND 90°E meridian and NC 90°W meridian. Draw w... the help of a protractor more radii at 30° interval to represent other meridians as shown in Fig. 63.

Properties

1. The parallels are concentric circles. The pole is a point forming the centre of the projection
2. The meridians are straight lines radiating from the pole and spaced correctly at true angular inter... i.e. the azimuths are true in this projection.
3. The meridians intersect the parallels at right angles.

4. Since the spacings between the parallels represent true distances, the scale along the meridians is correct.

5. The scale along the parallels increases away from the centre of this projection.

6. The areas are exaggerated and shapes distorted, the exaggeration and distortion increasing away from the centre of the projection.

7. The projection is neither equal-area nor orthomorphic.

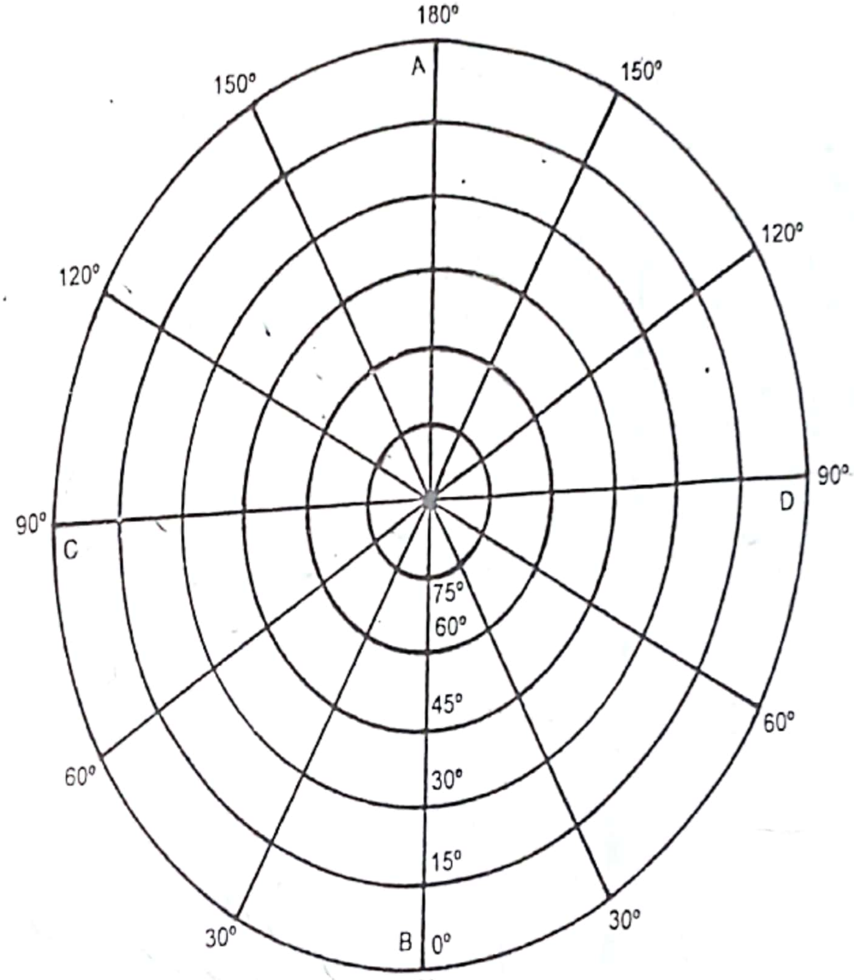


Fig. 63

Limitations

Shapes being greatly distorted away from the centre of the projection, it is only a small area in the central part of the projection that can be represented in a satisfactory way. The area lying between the pole and 60° parallel is shown satisfactorily.

Uses

1. This projection is commonly used for preparing maps of polar areas used for general purposes.

2. Since the scale along the meridians is correct, narrow strips running along the meridians are shown fairly correctly.

III. GNOMONIC PROJECTION (POLAR CASE)

It is also called a great-circle sailing chart. Its name has been derived from the gnomon of a ...

Let us draw a network of the Polar Gnomonic Projection on the scale of 1 : 350,000,000 with ... at 30° interval and showing the parallels of 30°N, 45°N, 60°N and 75°N.

Construction

Radius of the earth =250,000,000 inches

∴ Radius of the globe on the scale of 1 : 350,000,000

$$= \frac{250,000,000}{350,000,000} = 0.714''$$

Draw a circle with radius equal to the radius of the globe, i.e., 0.714". This circle represents the ... Let NS, its polar diameter and WE, its equatorial diameter, intersect each other at right angles at ... centre of the circle (Fig. 64).

Let LM represent a plane. It is placed at the north pole and it is perpendicular to the polar di... NS. Draw radii Oa, Ob, Oc, and Od making angles of 30°, 45°, 60° and 75° respectively with OE. P... these radii to meet LM as shown in Fig. 64. With N' as centre and Nd', Nc', Nb' and Na' as radii ... circles (Fig. 65). These circles represent 75°N, 60°N, 45°N and 30°N parallels respectively. It should be noted that OE is parallel to LM. Therefore, when radius OE is produced, it does not meet LM. Owing to this reason, the equator cannot be shown on the Polar Gnomonic Projection.

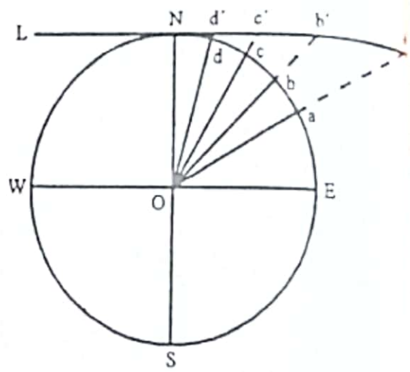


Fig. 64

Draw AB and CD as straight lines intersecting each other at right angles at the centre i.e. point N' of the projection (Fig. 65). Radius N'B represents the 0° meridian, N'A 180° meridian, N'D 90°E meridian and N'C 90°W meridian. Draw with a protractor radii at 30° interval to represent other meridians as shown in Fig. 65.

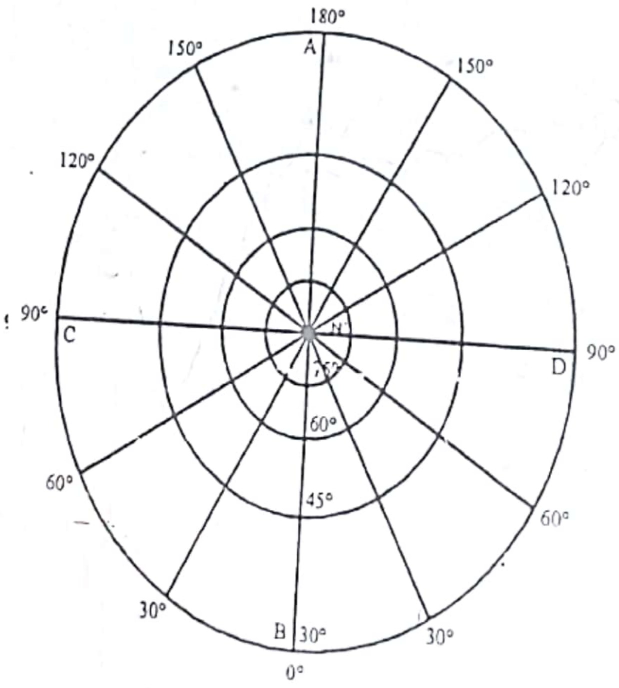


Fig. 65

Properties

1. The parallels are concentric circles. The pole is a point forming the centre of the projection. The distance of the equator from the centre of the projection cannot be determined because the radius passing through the equator (Fig. 64) becomes parallel to the plane placed on the pole. Accordingly the equator cannot be shown on this projection.

2. The meridians are straight lines radiating from the pole and spaced uniformly at their correct angular interval, thus, making the direction from the centre as true.

3. The meridians intersect the parallels at right angles.

4. The parallels are not spaced at equal distances. The distances between the parallels increase rapidly towards the margin of the projection. This has resulted in the exaggeration of the scale along the meridians.

5. The scale along the parallels also increases rapidly away from the centre of the projection.

6. The areas are exaggerated and shapes distorted on this projection, the exaggeration and distortion increasing away from the centre of the projection. The exaggeration in the meridian scale is greater than that in any other zenithal projection.

7. It is neither equal-area nor orthomorphic.

8. A part of a great circle which is an arc on the globe and a curve on Mercator's Projection is represented as a straight line on this projection. This is because the radii from the centre of the globe are produced to meet the plane (on which the projection is drawn) placed tangentially at the pole. To be more explicit, the intersection of the plane passing through the centre of the globe and the plane placed tangentially at the pole makes a straight line. Thus, any straight line on this projection is a part of a great circle. This property is useful because it is with the help of this projection that we can mark the positions of great circles on Mercator's Projection.

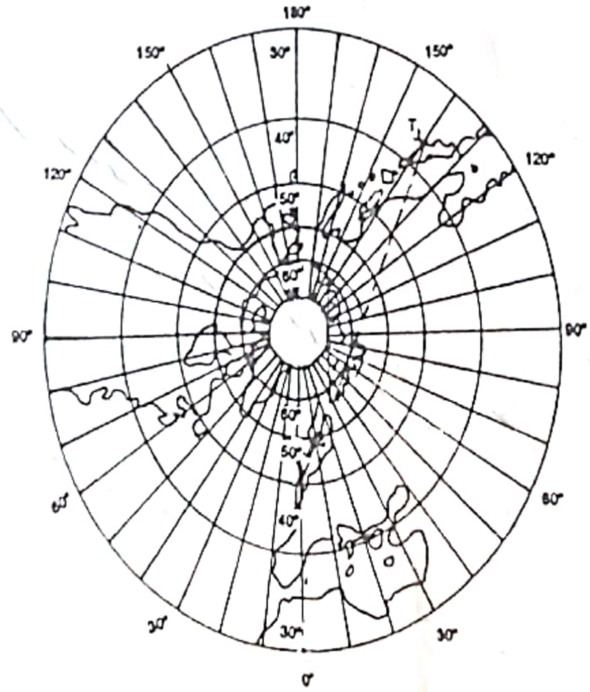


Fig. 66

9. A straight line which serves as a part of a great circle on Gnomonic Projection, does not ... as a loxodrome. Therefore, the direction that we maintain with the help of a loxodrome on Merc... Projection is not possible to maintain on Gnomonic Projection.

Limitations

1. As already stated, the equator cannot be shown on this projection.

2. Shapes are greatly distorted and areas greatly enlarged away from the centre of the projectio... is only a small area in the central part of the projection that can be represented in a satisfactory way. How... it is constructed generally for the area lying between one of the poles and 30° parallel.

Uses

1. A straight line connecting any two points on this projection is a part of a great circle. There... when we need to mark a great circle on Mercator's Projection, we make use of Gnomonic Projection... Fig. 66, a broken straight line connecting L (London) with T (Tokyo) is a part of a great circle. This... has been transferred to Mercator's Projection (Fig. 42) where it appears as a curve.

2. The direction which can be marked on Mercator's Projection by drawing loxodromes on it, ca... be marked on Gnomonic Projection. It is, therefore, Mercator's Projection which is used mostly for prep... navigation charts. It is used for preparing air navigation charts of arctic regions because Merca... Projection is not drawn to show these regions.

IV. THE STEREOGRAPHIC PROJECTION (POLAR CASE)

Let us draw a network of the Polar Stereographic Projection for the Northern Hemisphere on the s... of 1 : 330,000,000 with parallels at 15° interval and meridians at 30° interval.

Construction

Radius of the earth =250,000,000 inches

∴ Radius of the globe on the scale of 1 : 330,000,000

$$= \frac{250,000,000}{330,000,000} = 0.757''$$

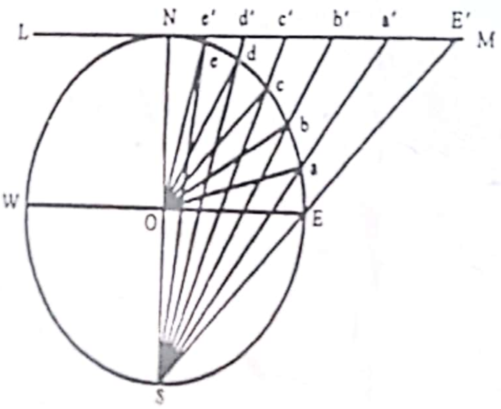


Fig. 67

Draw a circle with radius equal to the radius of the g... i.e. 0.757". The circle represents the globe. Let NS its p... diameter and WE its equatorial diameter intersect each o... at right angles at O, the centre of the circle.

Let LM represent a plane. It is placed at the north... i.e. at N and it is perpendicular to the diameter NS. ... radii Oa, Ob, Oc, Od and Oe making angles of 15°, 30°... 60° and 75° respectively with OE.

Draw lines joining S (south pole) with E, a, b, c, ... e. Produce these lines to meet LM as shown in Fig. 67... N' as centre and Ne', Nd', Nc', Nb', Na' and NE as radii... concentric circles (Fig. 68). These circles represent ... 60°N, 45°N, 30°N and 15°N parallels and the equator (0°) respectively.

Draw straight lines AB and CD intersecting each other at right angles at the centre point N'. Radius N'B represents the 0° meridian, N'A 180° meridian, N'D 90°E meridian

N'C 90°W meridian. Draw with a protractor radii at 30° interval to represent other meridians as shown in Fig. 68.

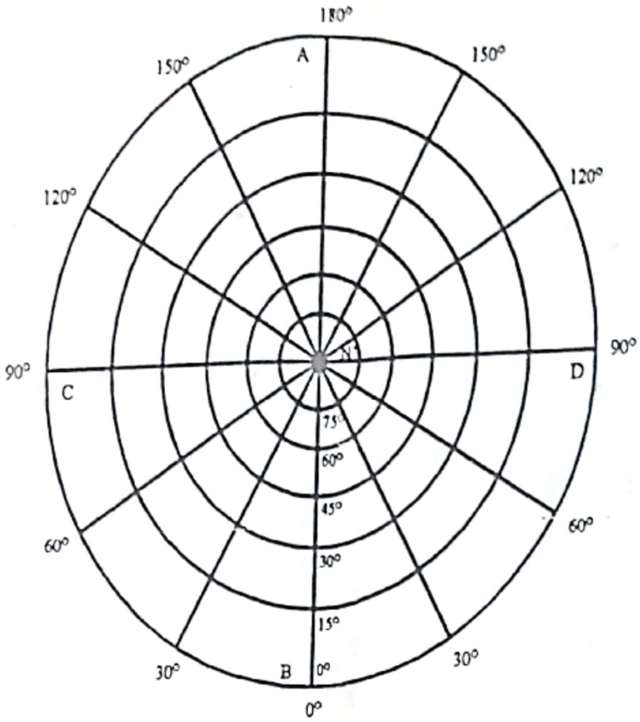


Fig. 68

Properties

1. The parallels are concentric circles. The pole is a point forming the centre of the projection.

2. The meridians are straight lines radiating from the pole i.e. the centre of the projection and spaced uniformly at their true angular interval, thus, making the direction from the centre as true.

3. The meridians intersect the parallels at right angles.

4. The parallels are not spaced at equal distances. The distances between them increase towards the margin of the projection. Exaggeration in the meridian scale is, however, less than that in the case of Gnomonic Projection.

5. The scale along the parallels also increases away from the centre of the projection. The scale along the parallels, however, increases in the same proportion in which it increases along the meridians with the result that at any point the scale along the parallel is equal to the scale along the meridian.

6. This is, therefore, an orthomorphic projection and the shapes of small areas are preserved on it.

7. The areas are exaggerated on this projection, the exaggeration increases away from the centre of this projection. The shapes of large areas are, therefore, not preserved. The exaggeration in areas is, however, smaller than that in the Gnomonic Projection.

8. A circle drawn on a globe is represented by a circle on this projection. No other projection possesses this property. This projection since can be centred anywhere on a globe, this property of the projection is very useful for plotting the ranges of aeroplanes, missiles, radio waves, etc., from the centre of the projection.

Limitations

Areas are enlarged away from the centre of the projection. It is only a small area in the central the projection that is represented in a satisfactory way. The representation of area is more satisfactory projection than in the Gnomonic Projection.

Uses

1. The entire Northern or Southern Hemisphere can be shown on this projection; and it is used to show the world in hemispheres as being orthomorphic it gives good visual look of the

2. Since the shapes of small countries is preserved to some extent on this projection, it commonly for preparing general-purpose maps of polar areas. In the U.S.A. it is used for pre aeronautical charts for the polar areas. (See also pp. 106 & 107).

3. This projection (polar case) is used for preparing sea-navigation routes of arctic region

4. It is also used for making daily weather map of the polar areas.

V. ORTHOGRAPHIC PROJECTION (POLAR CASE)

Let us draw a network of the Polar Orthographic Projection for the Northern Hemisphere on the of 1 : 350,000,000 with parallels at 15° interval and meridians at 30° interval.

Construction

Radius of the earth = 250,000,000 inches.

∴ radius of the globe on the scale of 1 : 350,000,000

$$= \frac{250,000,000}{350,000,000} = 0.714''$$

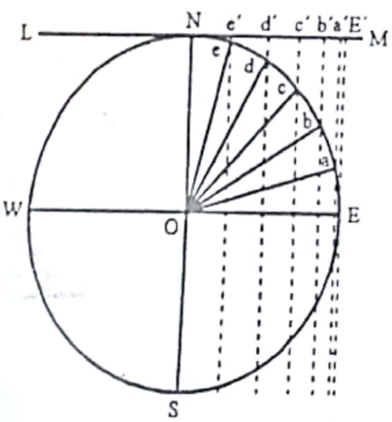


Fig. 69

Draw a circle with radius equal to the radius of the globe i.e. 0 This circle represents the globe. Let NS its polar diameter and W equatorial diameter intersect each other at right angles at O, the of the circle (Fig. 69).

Let LM represent a plane. It is placed at the north pole perpendicular to the polar diameter NS. Draw radii Oa, Ob, Oc, O Oe making angles of 15°, 30°, 45°, 60° and 75° respectively wi Draw lines parallel to NS and passing through the points E, a, b and e. Produce these lines to meet LM at points E', a', b', and e'.

With N' as centre and Ne', Nd', Nc', Nb', Na' and NE' as radi concentric circles. These circles represent 75°N, 60°N, 45°N, 30°N parallels and the equator (0°) respectively (Fig. 70).

Draw straight lines AB and CD intersecting each other at right angles at the centre i.e. po (Fig. 70). Radius N'B represents the 0° meridian, N'A 180° meridian, N'D 90°E meridian N'C meridian. Draw with a protractor radii at 30° interval to represent the other meridians as shown in Fi

Properties

1. The parallels are concentric circles. The pole is a point forming the centre of the projection

2. The meridians are straight lines radiating from the pole and spaced uniformly at their correct

interval, thus, making the direction from the centre as true.

3. The meridians intersect the parallels at right angles.

4. The parallels are not spaced at equal distances. The distances between them decrease rapidly towards the margin of the projection. As a result the scale along the meridians decreases rapidly away from the centre of the projection.

5. The scale along the parallels is, however, correct.

6. Since the scale along the meridians decreases rapidly away from the centre, the shapes are much distorted, the distortion increasing away from the centre of the projection.

7. It is neither equal-area nor orthomorphic.

8. The projection presents a view which appears when a pole of a globe is seen from a distant point lying above it (the pole).

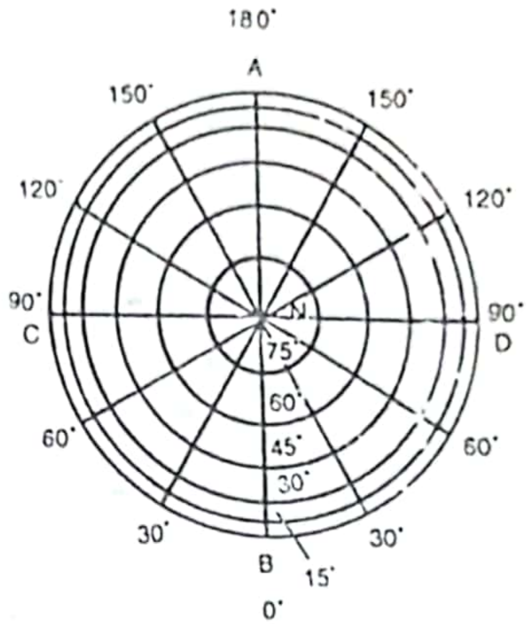


Fig. 70

Limitations

The shapes are much distorted near the margin of the projection. The sizes of the areas are diminished away from the centre of the projection. It is only a small area in the central part of the projection that can be represented in a satisfactory way.

Uses

This projection is used to prepare charts for showing heavenly bodies.

QUESTIONS