

### Chapter 4

# The Valuation of Long-Term Securities



### The Valuation of Long-Term Securities

Distinctions Among Valuation Concepts

**Bond Valuation** 

**Preferred Stock Valuation** 

**Common Stock Valuation** 

Rates of Return (or Yields)



#### What is Value?

Liquidation value represents the amount of money that could be realized if an asset or group of assets is sold separately from its operating organization.

Going-concern value represents the amount a firm could be sold for as a continuing operating business.



#### What is Value?

#### **Book value** represents either

- (1) <u>an asset</u>: the accounting value of an asset -- the asset's cost minus its accumulated depreciation;
- (2) <u>a firm</u>: total assets minus liabilities and preferred stock as listed on the balance sheet.



#### What is Value?

Market value represents the market price at which an asset trades.

Intrinsic value represents the price a security "ought to have" based on all factors bearing on valuation.



#### **Bond Valuation**

Important Terms
Types of Bonds
Valuation of Bonds
Handling Semiannual
Compounding



#### Important Bond Terms

A bond is a long-term debt instrument issued by a corporation or government.

The maturity value (MV) [or face value] of a bond is the stated value. In the case of a U.S. bond, the face value is usually \$1,000.



#### Important Bond Terms

The bond's <u>coupon rate</u> is the stated rate of interest; the annual interest payment divided by the bond's face value.

The <u>discount rate</u> (capitalization rate) is dependent on the risk of the bond and is composed of the risk-free rate plus a premium for risk.



#### Different Types of Bonds

A <u>perpetual bond</u> is a bond that *never* matures. It has an infinite life.

$$V = \frac{1}{(1 + k_d)^1} + \frac{1}{(1 + k_d)^2} + \dots + \frac{1}{(1 + k_d)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{1}{(1+k_d)^t} \quad \text{or} \quad (PVIFA_{k_d, \infty})$$

$$V = I / k_d$$

[Reduced Form]

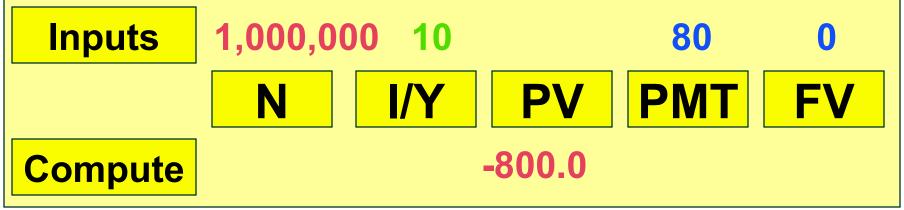


#### Perpetual Bond Example

Bond P has a \$1,000 face value and provides an 8% coupon. The appropriate discount rate is 10%. What is the value of the perpetual bond?



### "Tricking" the Calculator to Solve



N: "Trick" by using <u>huge</u> N like 1,000,000!

I/Y: 10% interest rate per period (enter as 10 NOT .10)

PV: Compute (resulting answer is cost to purchase)

PMT: \$80 annual interest forever (8% x \$1,000 face)

FV: \$0 (investor never receives the face value)



#### Different Types of Bonds

A <u>non-zero coupon-paying bond</u> is a coupon-paying bond with a finite life.

$$V = \frac{1}{(1 + k_d)^1} + \frac{1}{(1 + k_d)^2} + ... + \frac{1 + MV}{(1 + k_d)^n}$$

$$= \sum_{t=1}^{n} \frac{1}{(1 + k_d)^t} + \frac{MV}{(1 + k_d)^n}$$

$$V = I(PVIFA_{k_d, n}) + MV(PVIF_{k_d, n})$$



#### Coupon Bond Example

Bond C has a \$1,000 face value and provides an 8% annual coupon for 30 years. The appropriate discount rate is 10%. What is the value of the *coupon bond*?

```
V = $80 (PVIFA<sub>10%, 30</sub>) + $1,000 (PVIF<sub>10%, 30</sub>)

= $80 (9.427) + $1,000 (.057)

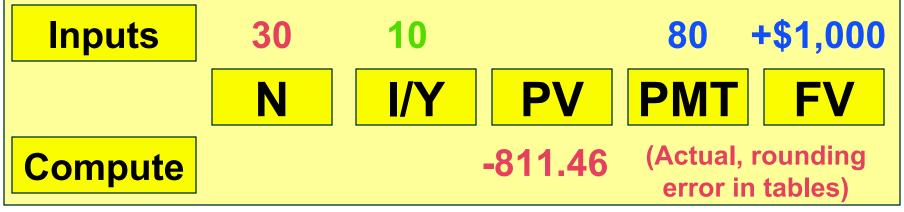
[Table IV] [Table II]

= $754.16 + $57.00

= $811.16.
```



### Solving the Coupon Bond on the Calculator



N: 30-year annual bond

I/Y: 10% interest rate per period (enter as 10 NOT .10)

PV: Compute (resulting answer is cost to purchase)

PMT: \$80 annual interest (8% x \$1,000 face value)

FV: \$1,000 (investor receives face value in 30 years)



#### Different Types of Bonds

A zero-coupon bond is a bond that pays no interest but sells at a deep discount from its face value; it provides compensation to investors in the form of price appreciation.

$$V = \frac{MV}{(1 + k_d)^n} = MV (PVIF_{k_d, n})$$



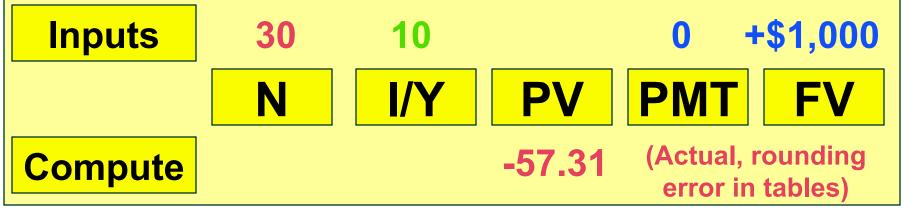
### Zero-Coupon Bond Example

Bond Z has a \$1,000 face value and a 30-year life. The appropriate discount rate is 10%. What is the value of the zero-coupon bond?

```
V = \$1,000 (PVIF_{10\%,30})
= \$1,000 (.057)
= \$57.00
```



### Solving the Zero-Coupon Bond on the Calculator



N: 30-year zero-coupon bond

I/Y: 10% interest rate per period (enter as 10 NOT .10)

PV: Compute (resulting answer is cost to purchase)

PMT: \$0 coupon interest since it pays no coupon

FV: \$1,000 (investor receives only face in 30 years)



#### Semiannual Compounding

## Most bonds in the U.S. pay interest twice a year (1/2 of the annual coupon).

#### Adjustments needed:

- (1) Divide **k** by **2**
- (2) Multiply n by 2
- (3) Divide | by 2



#### Semiannual Compounding

### A <u>non-zero coupon bond</u> adjusted for semiannual compounding.

$$V = \frac{1/2}{(1 + \frac{1}{2})^{1}} + \frac{1/2}{(1 + \frac{1}{2})^{2}} + \dots + \frac{1/2 + \frac{1}{2} + \frac{1}{2}}{(1 + \frac{1}{2})^{2}}$$

$$= \sum_{t=1}^{2^{*}n} \frac{1/2}{(1 + \frac{1}{2})^{t}} + \frac{\frac{1}{2} + \frac{1}{2}}{(1 + \frac{1}{2})^{2}} + \frac{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}}{(1 + \frac{1}{2})^{2}}$$

= 
$$\frac{1}{2} (PVIFA_{k_d/2,2*n}) + \frac{MV}{(PVIF_{k_d/2,2*n})}$$



#### Semiannual Coupon Bond Example

Bond C has a \$1,000 face value and provides an 8% semiannual coupon for 15 years. The appropriate discount rate is 10% (annual rate). What is the value of the *coupon bond*?



#### The Semiannual Coupon Bond on the Calculator

Inputs	30	5		40 -	+\$1,000
	N	I/Y	PV	<b>PMT</b>	FV
Compute	-846.28 (Actual, rounding error in tables)				

N: 15-year semiannual coupon bond (15 x 2 = 30)

I/Y: 5% interest rate per semiannual period (10 / 2 = 5)

PV: Compute (resulting answer is cost to purchase)

PMT: \$40 semiannual coupon (\$80 / 2 = \$40)

FV: \$1,000 (investor receives face value in 15 years)



### Semiannual Coupon Bond Example

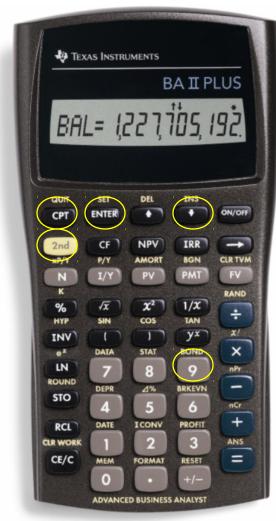
Let us use another worksheet on your calculator to solve this problem.

Assume that Bond C was purchased (settlement date) on 12-31-2000 and will be redeemed on 12-31-2015. This is identical to the 15-year period we discussed for Bond C.

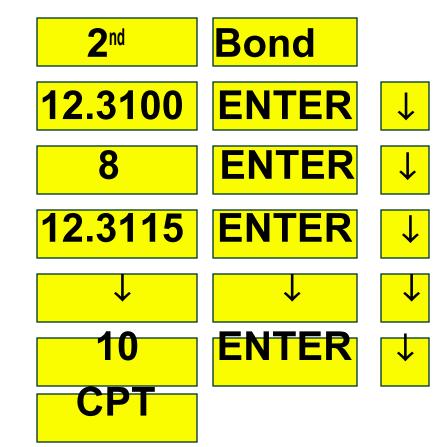
What is its percent of par? What is the value of the bond?



#### Solving the Bond Problem



#### Press:





#### Semiannual Coupon Bond Example

1. What is its percent of par?

84.628% of par (as quoted in financial papers)

2. What is the value of the bond?

84.628% x \$1,000 face value = <u>\$846.28</u>



#### Preferred Stock Valuation

Preferred Stock is a type of stock that promises a (usually) fixed dividend, but at the discretion of the board of directors.

Preferred Stock has preference over common stock in the payment of dividends and claims on assets.



#### Preferred Stock Valuation

$$V = \frac{Div_{P}}{(1 + k_{P})^{1}} + \frac{Div_{P}}{(1 + k_{P})^{2}} + ... + \frac{Div_{P}}{(1 + k_{P})^{\infty}}$$

$$= \sum_{t=1}^{\infty} \frac{Div_{P}}{(1+k_{P})^{t}} \quad \text{or } Div_{P}(PVIFA_{k_{P}, \infty})$$

#### This reduces to a perpetuity!

$$V = Div_p / k_p$$



#### Preferred Stock Example

Stock PS has an 8%, \$100 par value issue outstanding. The appropriate discount rate is 10%. What is the value of the preferred stock?

```
Div_p = $100 (8\%) = $8.00.
= 10\%.
= Div_p / k_p = $8.00 / 10\%
= $80
```



#### Common Stock Valuation

Common stock represents a residual ownership position in the corporation.

Pro rata share of future earnings after all other obligations of the firm (if any remain).

Dividends <u>may</u> be paid out of the pro rata share of earnings.



#### Common Stock Valuation

# What cash flows will a shareholder receive when owning shares of common stock?

- (1) Future dividends
- (2) Future sale of the common stock shares



#### Dividend Valuation Model

#### Basic dividend valuation model accounts for the PV of all future dividends.

$$V = \frac{Div_1}{(1 + k_e)^1} + \frac{Div_2}{(1 + k_e)^2} + ... + \frac{Div_\infty}{(1 + k_e)^\infty}$$

$$= \sum_{t=1}^{\infty} \frac{\text{Div}_t}{(1 + k_e)^t}$$

Div<sub>t</sub>: Cash dividend at time t

k<sub>e</sub>: Equity investor's required return



### Adjusted Dividend Valuation Model

The basic dividend valuation model adjusted for the future stock sale.

$$V = \frac{Div_1}{(1 + k_e)^1} + \frac{Div_2}{(1 + k_e)^2} + ... + \frac{Div_n + Price_n}{(1 + k_e)^n}$$

n: The year in which the firm's

shares are expected to be sold.

**Price**<sub>n</sub>: The expected share price in year n.



### Dividend Growth Pattern Assumptions

The dividend valuation model requires the forecast of <u>all</u> future dividends. The following dividend growth rate assumptions simplify the valuation process.

**Constant Growth** 

No Growth

**Growth Phases** 



#### Constant Growth Model

The constant growth model assumes that dividends will grow forever at the rate g.

$$V = \frac{D_0(1+g)}{(1+k_e)^1} + \frac{D_0(1+g)^2}{(1+k_e)^2} + ... + \frac{D_0(1+g)^\infty}{(1+k_e)^\infty}$$

D: Dividend paid at time 1.

The constant growth rate.

k<sub>e</sub>: Investor's required return.



### Constant Growth Model Example

Stock CG has an expected growth rate of 8%. Each share of stock just received an annual \$3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock?

$$D_1 = \$3.24 (1 + .08) = \$3.50$$

$$V_{cg} = D_1 / (k_e - g) = $3.50 / (.15 - .08) = $50$$



#### Zero Growth Model

The zero growth model assumes that dividends will grow forever at the rate g = 0.

$$V_{ZG} = \frac{D_1}{(1 + k_e)^1} + \frac{D_2}{(1 + k_e)^2} + ... + \frac{D_{\infty}}{(1 + k_e)^{\infty}}$$

D: Dividend paid at time 1.

k<sub>e</sub>: Investor's required return.



### Zero Growth Model Example

Stock ZG has an expected growth rate of 0%. Each share of stock just received an annual \$3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock?

$$D_{1} = \$3.24 (1 + 0) = \$3.24$$

$$V_{ZG} = D_{1} / (k_{e} - 0) = \$3.24 / (.15 - 0)$$

$$= \$21.60$$



#### **Growth Phases Model**

The growth phases model assumes that dividends for each share will grow at two or more *different* growth rates.

$$V = \sum_{t=1}^{n} \frac{D_0(1+Q_1)^t}{(1+k_e)^t} + \sum_{t=n+1}^{\infty} \frac{D_n(1+Q_1)^t}{(1+k_e)^t}$$



#### Growth Phases Model

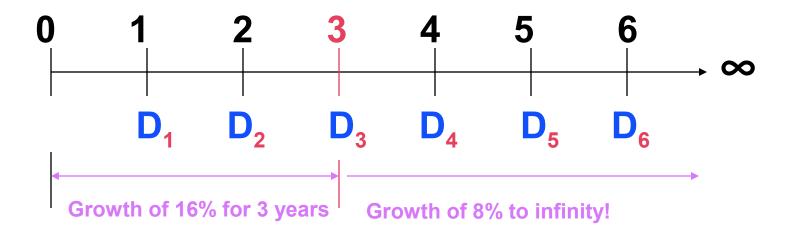
Note that the second phase of the growth phases model assumes that dividends will grow at a constant rate g<sub>2</sub>. We can rewrite the formula as:

$$V = \sum_{t=1}^{n} \frac{D_0(1+g_1)^t}{(1+k_e)^t} + \left[\frac{1}{(1+k_e)^n}\right] \frac{D_{n+1}}{(k_e-g_2)}$$



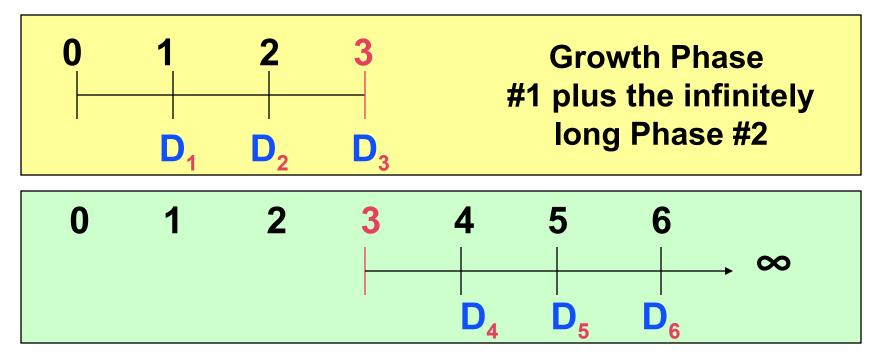
Stock GP has an expected growth rate of 16% for the first 3 years and 8% thereafter. Each share of stock just received an annual \$3.24 dividend per share. The appropriate discount rate is 15%. What is the value of the common stock under this scenario?





Stock GP has two phases of growth. The first, 16%, starts at time t=0 for 3 years and is followed by 8% thereafter starting at time t=3. We should view the time line as two separate time lines in the valuation.



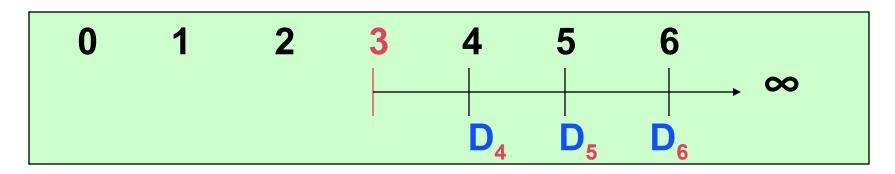


Note that we can value Phase #2 using the Constant Growth Model



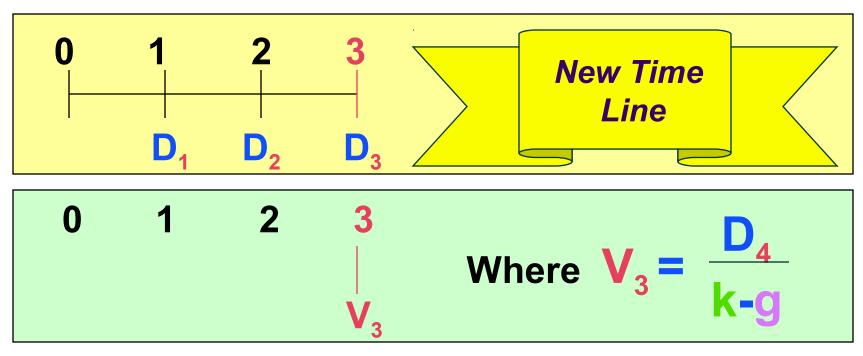
$$V_3 = \frac{D_4}{k-g}$$

We can use this model because dividends grow at a constant 8% rate beginning at the end of Year 3.



Note that we can now replace <u>all</u> dividends from Year 4 to infinity with the *value* at time t=3, V<sub>3</sub>! Simpler!!





Now we only need to find the first four dividends to calculate the necessary cash flows.



#### Determine the annual dividends.

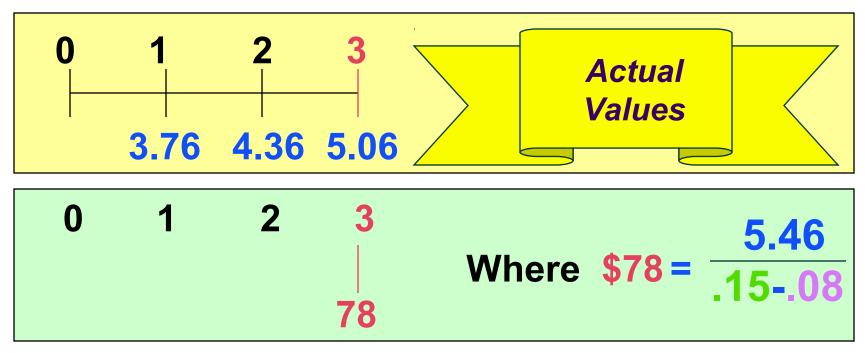
$$D_1 = D_0(1+g_1)^1 = \$3.24(1.16)^1 = \$3.76$$

$$D_2 = D_0(1+g_1)^2 = $3.24(1.16)^2 = $4.36$$

$$D_3 = D_0(1+g_1)^3 = \$3.24(1.16)^3 = \$5.06$$

$$D_4 = D_3(1+g_2)^1 = $5.06(1.08)^1 = $5.46$$





Now we need to find the present value of the cash flows.



We determine the PV of cash flows.

$$PV(D_1) = D_1(PVIF_{15\%,1}) = $3.76 (.870) = $3.27$$

$$PV(D_2) = D_2(PVIF_{15\%,2}) = \$4.36 (.756) = \$3.30$$

$$PV(D_3) = D_3(PVIF_{15\%,3}) = $5.06 (.658) = $3.33$$

$$PV(P_3) = P_3(PVIF_{15\%,3}) = $78 (.658) = $51.32$$



Finally, we calculate the *intrinsic value* by summing all the cash flow present values.

$$V = \frac{\$3.27 + \$3.30 + \$3.33}{V = \$61.22}$$

$$V = \frac{3}{100} \frac{1}{(1 + 15)^{1}} + \frac{1}{(1 + 15)^{1}} \frac{1}{(1 + 15)^{1}}$$



#### Solving the Intrinsic Value Problem using CF Registry

#### **Steps in the Process (Page 1)**

CF Step 1: **Press** key **CLR Work** 2<sup>nd</sup> keys Step 2: **Press** Step 3: For *CF0* Press **Enter** keys 3.76 Step 4: For C01 Press Enter keys **Enter** Step 5: For F01 Press keys 4.36 Enter Step 6: For C02 Press keys Step 7: For F02 Press Enter keys



#### Solving the Intrinsic Value Problem using CF Registry

#### **Steps in the Process (Page 2)**

Step 8: For C03 Press 83.06 Enter ↓ keys

Step 9: For *F03* Press 1 Enter ↓ keys

Step 10: Press ↓ ↓ keys

Step 11: Press NPV

Step 12: Press 15 Enter ↓ keys

Step 13: Press CPT

**RESULT**: Value = \$61.18!

(Actual, rounding error in tables)



### Calculating Rates of Return (or Yields)

#### Steps to calculate the rate of return (or yield).

- 1. Determine the expected cash flows.
- 2. Replace the intrinsic value (V) with the market price (P<sub>1</sub>).
- 3. Solve for the *market required rate of* return that equates the discounted cash flows to the market price.



#### **Determining Bond YTM**

# Determine the Yield-to-Maturity (YTM) for the coupon-paying bond with a finite life.

$$P_{0} = \sum_{t=1}^{n} \frac{1}{(1 + k_{d})^{t}} + \frac{MV}{(1 + k_{d})^{n}}$$

$$= I \left(PVIFA_{k_{d}}, n\right) + MV \left(PVIF_{k_{d}}, n\right)$$

$$k_{d} = YTM$$



#### Determining the YTM

Julie Miller want to determine the YTM for an issue of outstanding bonds at Basket Wonders (BW). BW has an issue of 10% annual coupon bonds with 15 years left to maturity. The bonds have a current market value of *\$1,250.* 

What is the YTM?



#### YTM Solution (Try 9%)

```
$100(PVIFA<sub>9%.15</sub>) +
$1,250 =
          $1,000(PVIF<sub>9%.15</sub>)
               $100(8.061) +
$1,250
               $1,000(.275)
$1,250
               $806.10 + $275.00
               $1,081.10
               [Rate is too high!]
```



#### YTM Solution (Try 7%)

```
$100(PVIFA<sub>7%,15</sub>) +
$1,250 =
          $1,000(PVIF<sub>7%.15</sub>)
$1,250
               $100(9.108) +
               $1,000(.362)
$1,250
               $910.80 + $362.00
               $1,272.80
                [Rate is too low!]
```



#### YTM Solution (Interpolate)



#### YTM Solution (Interpolate)

$$\frac{X}{.02} = \frac{\$23}{\$192}$$



#### YTM Solution (Interpolate)

$$X = \frac{(\$23)(0.02)}{\$192}$$
  $X = .0024$ 

YTM = .07 + .0024 = .0724 or 7.24%



### YTM Solution on the Calculator

Inputs 15 -1,250 100 +\$1,000

N I/Y PV PMT FV

Compute 7.22% (actual YTM)

N: 15-year annual bond

I/Y: Compute -- Solving for the annual YTM

PV: Cost to purchase is \$1,250

PMT: \$100 annual interest (10% x \$1,000 face value)

FV: \$1,000 (investor receives face value in 15 years)



## Determining Semiannual Coupon Bond YTM

Determine the Yield-to-Maturity (YTM) for the semiannual coupon-paying bond with a finite life.

$$P_{0} = \sum_{t=1}^{2n} \frac{1/2}{(1+|k_{d}|/2)^{t}} + \frac{MV}{(1+|k_{d}|/2)^{2n}}$$

$$= (|l/2|)(PVIFA_{k_{d}|/2,2n}) + MV(PVIF_{k_{d}|/2,2n})$$

$$[1+(|k_{d}|/2)]^{2} - 1 = YTM$$



#### Determining the Semiannual Coupon Bond YTM

Julie Miller want to determine the YTM for another issue of outstanding bonds. *The firm* has an issue of 8% semiannual coupon bonds with 20 years left to maturity. The bonds have a current market value of \$950.

What is the YTM?



#### YTM Solution on the Calculator

Inputs 40 -950 40 +\$1,000

N I/Y PV PMT FV

Compute 4.2626% = (k<sub>d</sub> / 2)

N: 20-year semiannual bond (20 x 2 = 40)

I/Y: Compute -- Solving for the semiannual yield now

PV: Cost to purchase is \$950 today

PMT: \$40 annual interest (8% x \$1,000 face value / 2)

FV: \$1,000 (investor receives face value in 15 years)



### Determining Semiannual Coupon Bond YTM

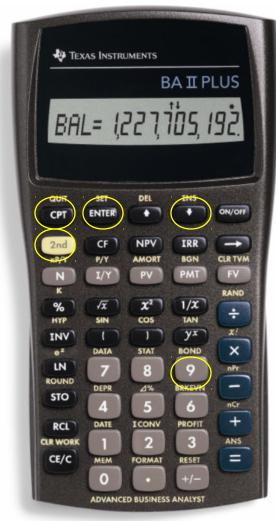
Determine the Yield-to-Maturity (YTM) for the semiannual coupon-paying bond with a finite life.

$$[1 + (k_d/2)]^2 - 1 = YTM$$

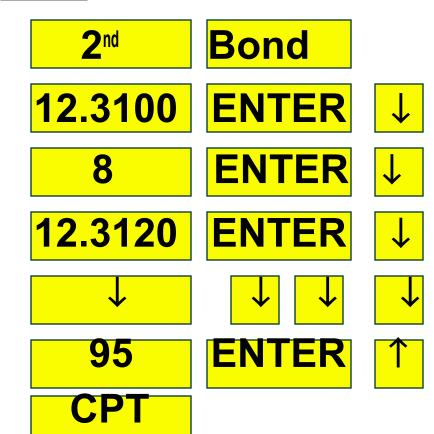
$$[1 + (.042626)]^2 - 1 = .0871$$
  
or 8.71%



#### Solving the Bond Problem



#### Press:





#### Determining Semiannual Coupon Bond YTM

This technique will calculate k<sub>d</sub>.

You must then substitute it into the following formula.

$$[1 + (k_d/2)]^2 - 1 = YTM$$

 $[1 + (.0852514/2)]^2 - 1 = .0871$ or 8.71% (same result!)

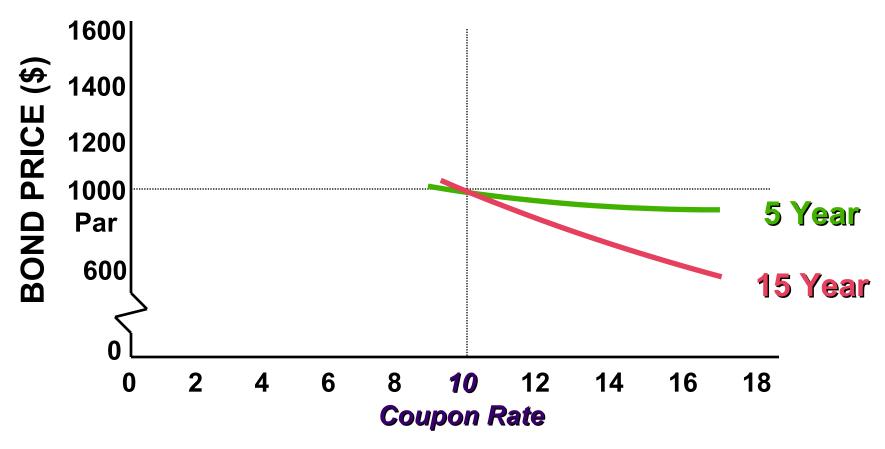


Discount Bond -- The market required rate of return exceeds the coupon rate  $(Par > P_0)$ .

Premium Bond -- The coupon rate exceeds the market required rate of return (P, > Par).

Par Bond -- The coupon rate equals the market required rate of return (P<sub>0</sub> = Par).





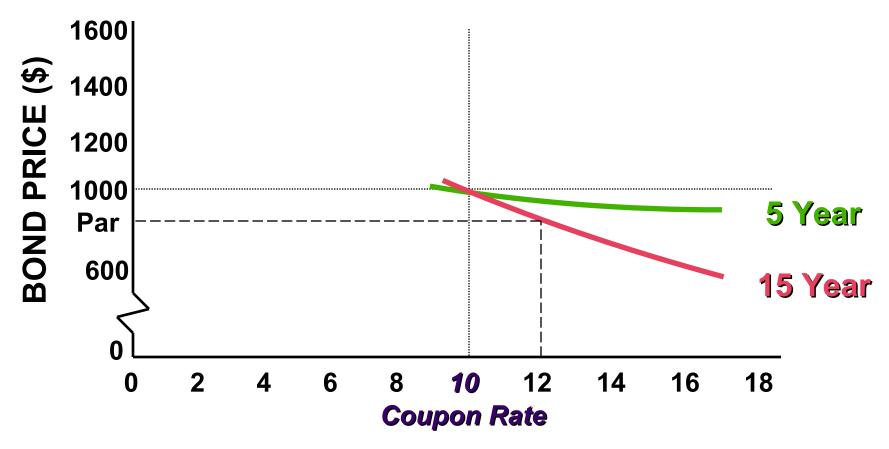
MARKET REQUIRED RATE OF RETURN (%)



When interest rates rise, then the market required rates of return rise and bond prices will fall.

Assume that the required rate of return on a 15-year, 10% coupon-paying bond rises from 10% to 12%. What happens to the bond price?





MARKET REQUIRED RATE OF RETURN (%)



#### Bond Price-Yield Relationship (Rising Rates)

The required rate of return on a 15-year, 10% coupon-paying bond has risen from 10% to 12%.

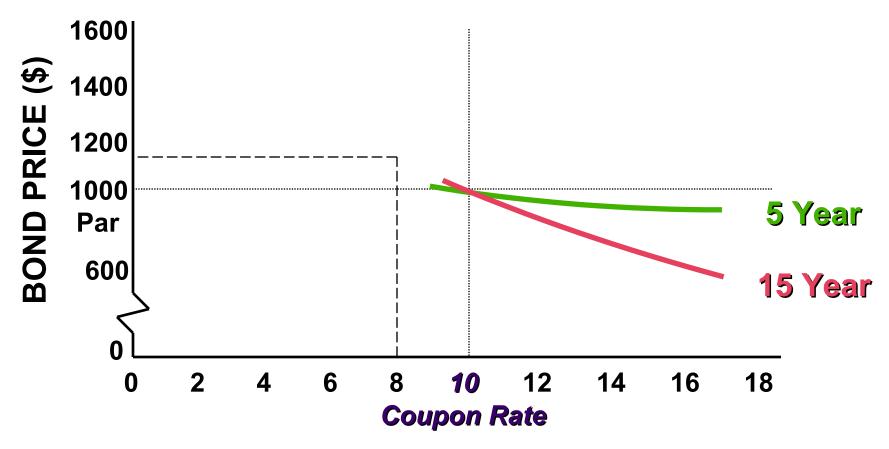
Therefore, the bond price has fallen from \$1,000 to \$864.



When interest rates fall, then the market required rates of return fall and bond prices will rise.

Assume that the required rate of return on a 15-year, 10% coupon-paying bond falls from 10% to 8%. What happens to the bond price?





MARKET REQUIRED RATE OF RETURN (%)



#### **Bond Price-Yield Relationship** (Declining Rates)

The required rate of return on a 15-year, 10% coupon-paying bond has fallen from 10% to 8%.

Therefore, the bond price has risen from \$1,000 to \$1,171.

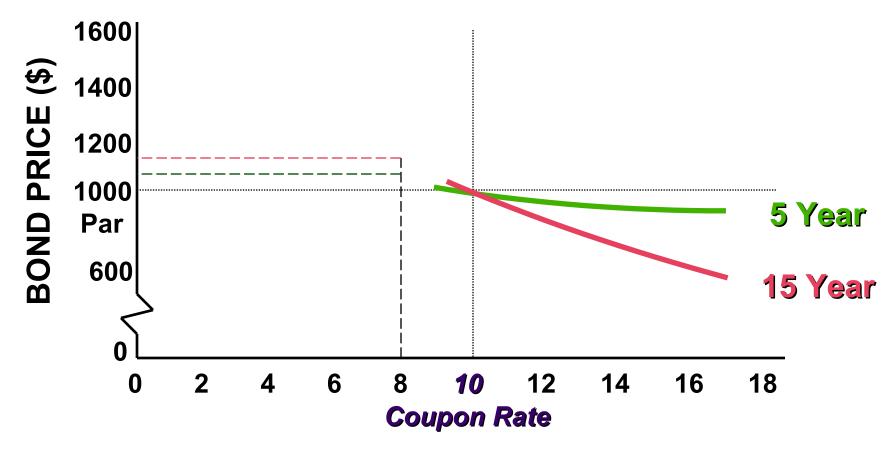


#### The Role of Bond Maturity

The longer the bond maturity, the greater the change in bond price for a given change in the market required rate of return.

Assume that the required rate of return on both the 5- and 15-year, 10% coupon-paying bonds [all] from 10% to 8%. What happens to the changes in bond prices?





MARKET REQUIRED RATE OF RETURN (%)



#### The Role of Bond Maturity

The required rate of return on both the 5- and 15-year, 10% coupon-paying bonds has *fallen* from 10% to 8%.

The 5-year bond price has *risen* from \$1,000 to \$1,080 for the 5-year bond (+8.0%).

The 15-year bond price has *risen* from \$1,000 to \$1,171 (+17.1%). *Twice as fast!* 



### The Role of the Coupon Rate

For a given change in the market required rate of return, the price of a bond will change by proportionally more, the <u>lower</u> the coupon rate.



### Example of the Role of the Coupon Rate

Assume that the market required rate of return on two equally risky 15-year bonds is 10%. The coupon rate for Bond H is 10% and Bond L is 8%.

What is the rate of change in each of the bond prices if market required rates fall to 8%?



#### **Example of the Role of the Coupon Rate**

The price on Bonds H and L prior to the change in the market required rate of return is \$1,000 and \$848, respectively.

The price for **Bond H** will rise from \$1,000 to \$1,171 (+17.1%).

The price for Bond L will rise from \$848 to \$1,000 (+17.9%). It rises faster!



#### Determining the Yield on Preferred Stock

#### Determine the yield for preferred stock with an infinite life.

$$P_0 = Div_p / k_p$$

Solving for k, such that

$$k_p = Div_p / P_0$$



#### Preferred Stock Yield Example

Assume that the annual dividend on each share of preferred stock is \$10. Each share of preferred stock is currently trading at \$100. What is the *yield* on preferred stock?

$$k_p = $10 / $100.$$

$$k_p = 10\%$$
.



#### Determining the Yield on Common Stock

Assume the constant growth model is appropriate. Determine the yield on the common stock.

$$P_0 = D_1 / (k_e - g)$$

Solving for k such that

$$k_e = (D_1 / P_0) + g$$



### Common Stock Yield Example

Assume that the expected dividend (D<sub>1</sub>) on each share of common stock is \$3. Each share of common stock is currently trading at \$30 and has an expected growth rate of 5%. What is the *yield* on common stock?

$$k_e = (\$3 / \$30) + 5\%$$

$$k_{e} = 15\%$$