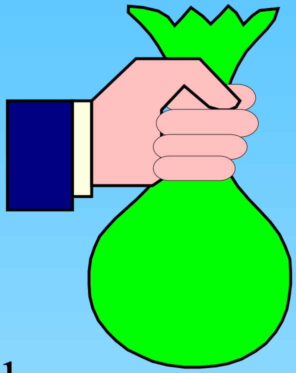
# Time Value of Money

# Time Value of Money



- $100 in hand today is worth more than $100 promised in a year
  - Money earns interest
  - A future amount is worth what has to deposited today to have that amount at that time

Example

Q: *How much would $1,000 promised in one year be worth today if the bank paid 5% interest?*

A: **$952.38. Because one year's interest is**

**$952.38 × .05 = $47.62**

**and**

**$952.38 + $47.62 = $1,000**

# Time Value of Money

- Present Value
  - The amount that must be deposited today to have a future sum at a certain interest rate
- Terminology
  - The ***discounted*** value of a sum present value

# Outline of Approach

Deal with four different types of problems

Amounts

Present value

Future value

Annuities

Present value

Future value

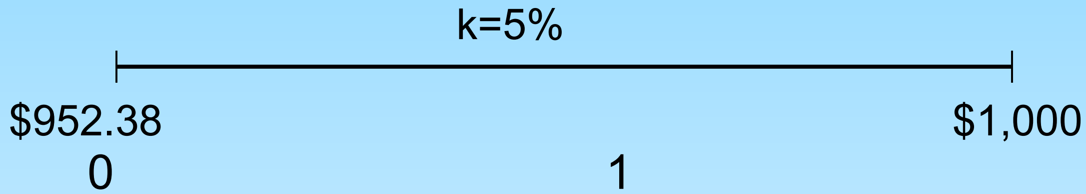
Develop an equation for each

- Time lines - Graphic portrayals

  Place information on line

# Outline of Approach

- Develop an equation for each
- Time lines - Graphic portrayals
  - Place information on line



# The Future Value Factor for k and n

$$FVF_{k,n} = (1+k)^n$$

**Table 6-1**

| | k | | | | | | |
|---|---|---|---|---|---|---|---|
| n | 1% | 2% | 3% | 4% | 5% | 6% | ... |
| 1 | 1.0100 | 1.0200 | 1.0300 | 1.0400 | 1.0500 | 1.0600 | ... |
| 2 | 1.0201 | 1.0404 | 1.0609 | 1.0816 | 1.1025 | 1.1236 | ... |
| 3 | 1.0303 | 1.0612 | 1.0927 | 1.1249 | 1.1576 | 1.1910 | ... |
| 4 | 1.0406 | 1.0824 | 1.1255 | 1.1699 | 1.2155 | 1.2625 | ... |
| 5 | 1.0510 | 1.1041 | 1.1593 | 1.2167 | 1.2763 | 1.3382 | ... |
| 6 | 1.0615 | 1.1262 | 1.1941 | 1.2653 | 1.3401 | 1.4185 | ... |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |

# Example 6-1

Example

How much will $850 be worth if deposited for three years at 5% interest?

**Solution:**

$$FV_n = PV\ [FVF_{k,n}]$$

$$FV3 = \$850\ [FVF_{5,3}]$$

**Look up** $FVF_{5,3} = 1.1576$

$$FV_3 = \$850\ [1.1576].$$

$$= \$983.96$$

**Problem Solving Techniques**

# The Future Value of an Amount

- Problem-Solving Techniques
  - **All time value equations contain four variables**
    - **In this case PV, $FV_n$, k, and n**
- **Every problem will give you three and ask for the fourth.**

# Financial Calculators

- Work directly with equations
- How to use a typical financial calculator in time value
  - Five time value keys
    - Use either four or five keys
  - Some calculators distinguish between inflows and outflows
    - If PV is entered as positive the computed FV is negative

# Financial Calculators

| Basic Calculator Keys | |
|---|---|
| n | Number of time periods |
| I/Y | Interest rate (%) |
| PV | Present Value |
| FV | Future Value |
| PMT | Payment |

# Financial Calculators

Example

Q: What is the present value of $5,000 received in one year if interest rates are 6%?

A: Input the following values on the calculator and compute the PV:

[redacted] 1

[redacted] 6

[redacted] 5000

[redacted] 0

[redacted] **4,716.98** ← **Answer**

# The Expression for the Present Value of an Amount

Example 6.3

Example

Q: What interest rate will grow \$850 into \$983.96 in three years?

A:

# The Expression for the Present Value of an Amount

Example 6.4

Example

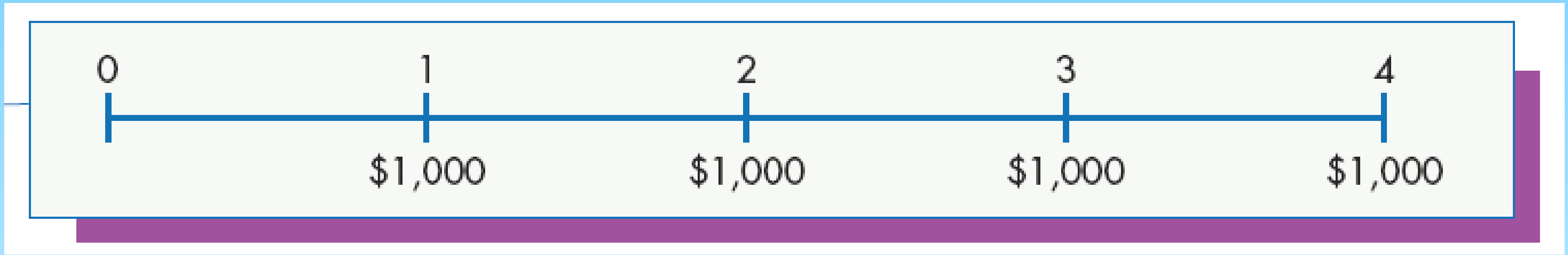
Q: How long does it take money invested at 14% to double?

# Annuity Problems

- Annuities
  - A finite series of equal payments separated by equal time intervals
    - Ordinary annuities
      - Payments occur at the end of the time periods
    - Annuities due
      - Payments occur at the beginning of the time periods

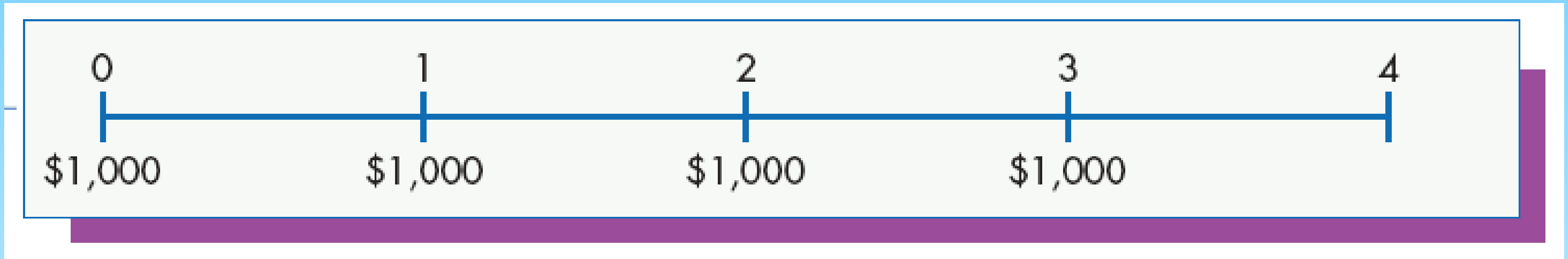
# Ordinary Annuity

Figure 6.1



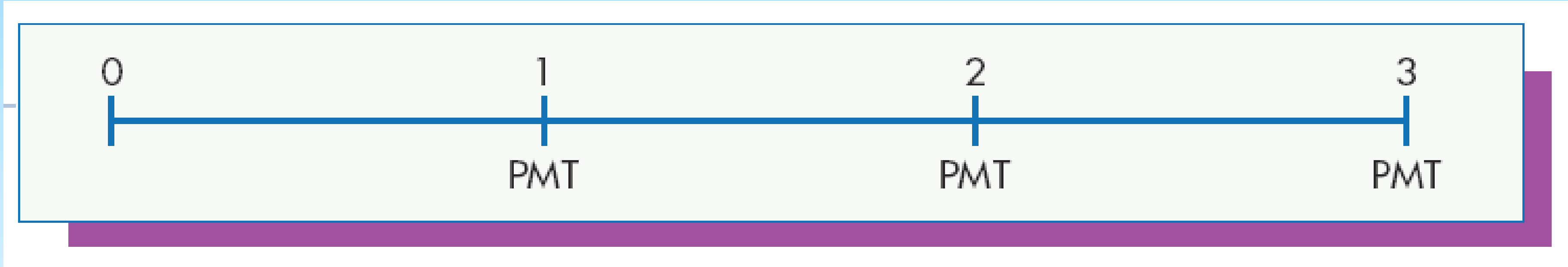
# Annuity Due

Figure 6.2



# The Future Value of an Annuity—Developing a Formula

- Future value of an annuity
  - The sum, at its end, of all payments and all interest if each payment is deposited when received

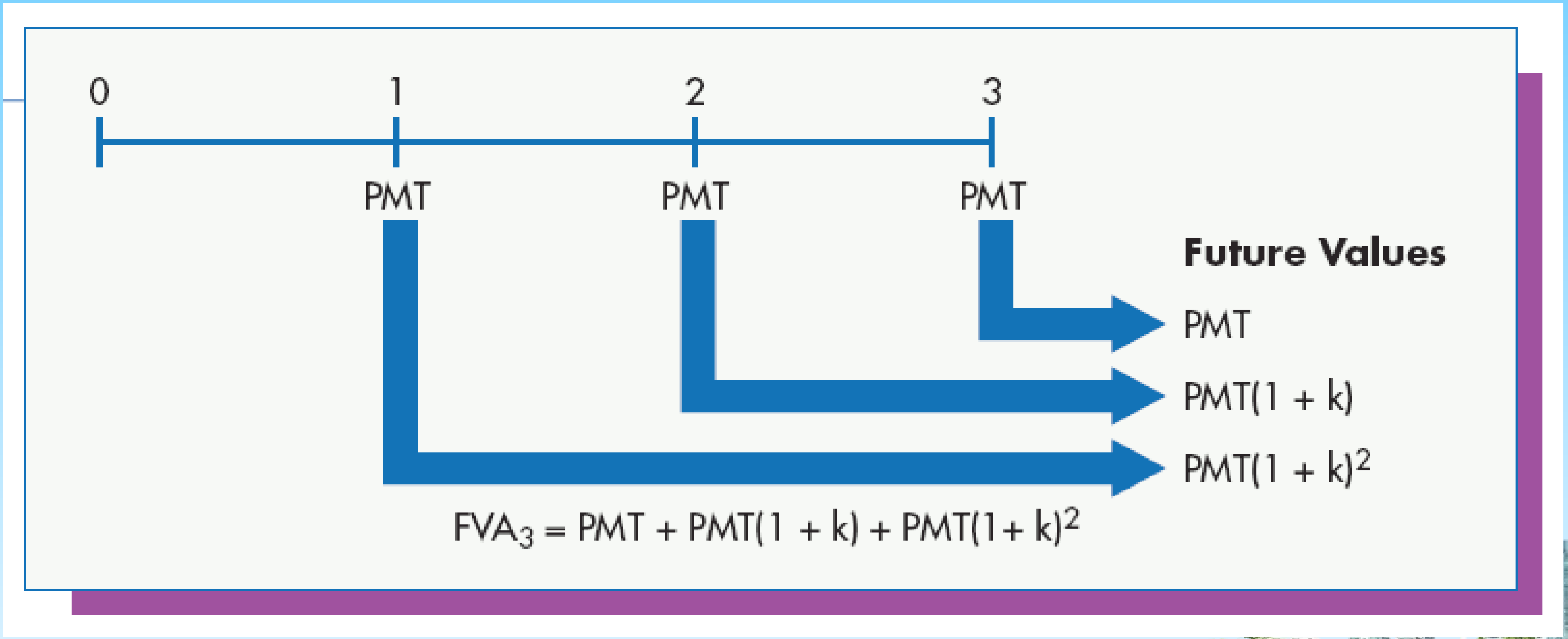


# The Future Value of an Annuity —Solving Problems

- Four variables in the future value of an annuity equation
  1. The future value of the annuity itself
  2. The payment
  3. The interest rate
  4. The number of periods
     - Helps to draw a time line

# Future Value of a Three-Year Ordinary Annuity

Figure 6.4



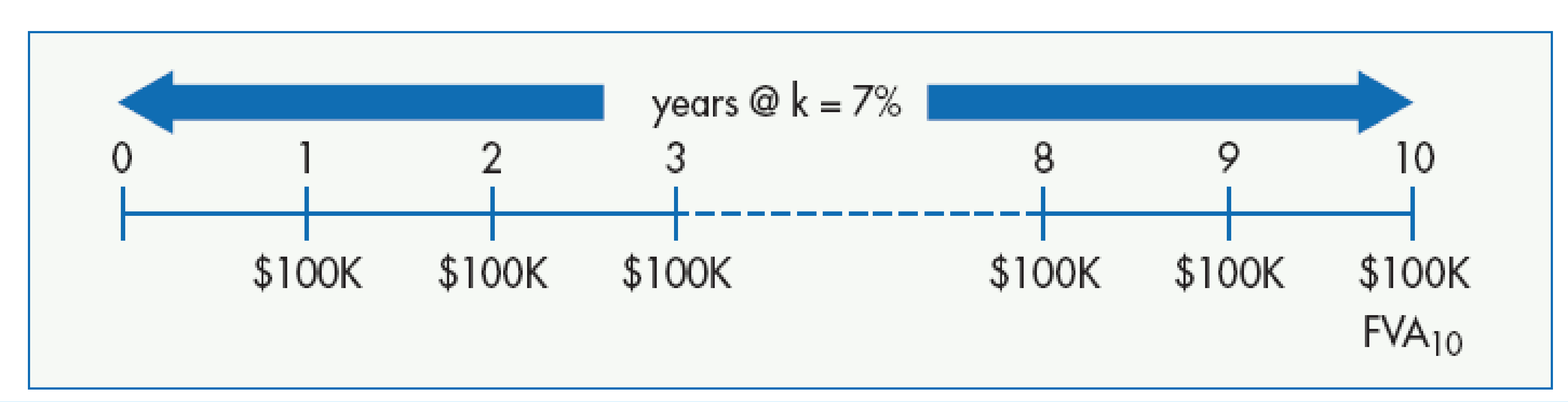
# The Future Value of an Annuity

Example 6.5

Example

Q: The Brock Corporation owns the patent to an industrial process and receives license fees of $100,000 a year on a 10-year contract for its use. Management plans to invest each payment until the end of the contract to provide funds for development of a new process at that time. If the invested money is expected to earn 7%, how much will Brock have after the last payment is received?

# Timeline for Ex. 6.5



# The Sinking Fund Problem

- Companies borrow money by issuing bonds
  - No repayment of principal is made during the bond's life
  - Principal is repaid at maturity in a lump sum
    - A sinking fund provides cash to pay off principal at maturity
      - Must determine the periodic deposit to have the needed amount at the bond's maturity—a future value of an annuity problem

# The Sinking Fund Problem

Example 6.6

Example

Q: The Greenville Company issued bonds totaling $15 million for 30 years. The bond agreement specifies that a sinking fund must be maintained after 10 years, which will retire the bonds at maturity. Although no one can accurately predict interest rates, Greenville's bank has estimated that a yield of 6% on deposited funds is realistic for long-term planning. How much should Greenville plan to deposit each year to be able to retire the bonds with the money put aside?

# Compound Interest and Non-Annual Compounding

- Compounding
  - Earning interest on interest
- Compounding periods
  - Interest is usually compounded annually, semiannually, quarterly or monthly
    - Interest rates are quoted by stating the nominal rate followed by the compounding period

# The Effective Annual Rate

- Effective annual rate (EAR)
  - The annually compounded rate that pays the same interest as a lower rate compounded more frequently

# The Effective Annual Rate

Example

Q: If 12% is compounded monthly, what annually compounded interest rate will get a depositor the same interest?

A: If the initial deposit was $100, you would have $112.68 after one year of 12% interest compounded monthly. Thus, an annually compounded rate of 12.68% [($112.68 ÷ $100) – 1] would have to be earned.

# Year-end Balances at Various Compounding Periods for $100 Initial Dep and $k_{nom}$ = 12%

| Compounding | Final Balance |
|---|---|
| Annual | $112.00 |
| Semiannual | 112.36 |
| Quarterly | 112.55 |
| Monthly | 112.68 |

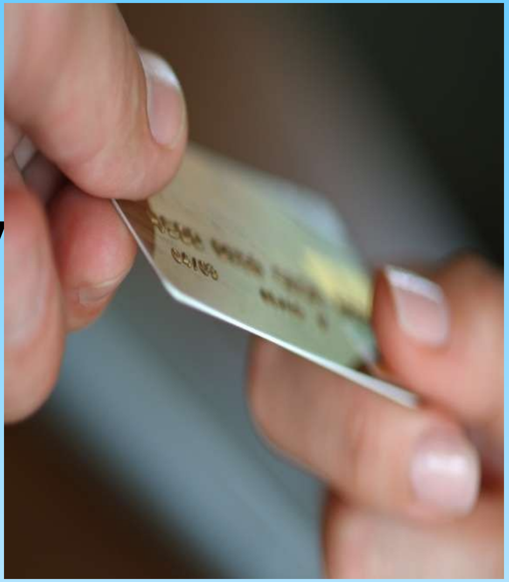
# Changes in the Effect of Compounding at Different Rates

Table 6.3

| Nominal Rate | EAR for Monthly Compounding | Effective Increase | Increase as % of $k_{nom}$ |
|---|---|---|---|
| 6% | 6.17% | .17% | 2.8% |
| 12 | 12.68 | .68 | 5.7 |
| 18 | 19.56 | 1.56 | 8.7 |

# The APR and the EAR

- The annual percentage rate (APR) associated with credit cards is actually the nominal rate and is less than the EAR



# Compounding Periods and the Time Value Formulas

- Time periods must be compounding periods
- Interest rate must be the rate for a single compounding period
  - With a quarterly compounding period
    - $k_{nom}$ is divided by 4, and
    - n is multiplied by 4

# Compounding periods and the Time Value Formulas

Example 6.7

Example

Q: You want to buy a car costing $15,000 in 2½ years. You plan to save the money by making equal monthly deposits in your bank account, which pays 12% compounded monthly. How much must you deposit each month?

# The Present Value of an Annuity—Solving Problems

- There are four variables in the present value of an annuity equation
  1. The present value of the annuity itself
  2. The payment
  3. The interest rate
  4. The number of periods
     - Problems present 3 variables and ask for the fourth

# The Present Value of an Annuity

Example 6.9

Example

Q: The Shipson Company has just sold a large machine to Baltimore Inc. on an installment contract. The contract calls for Baltimore to make payments of $5,000 every six months (semiannually) for 10 years. Shipson would like its cash now and asks its bank to discount the contract and pay it the present (discounted) value. Baltimore is a good credit risk, so the bank is willing to discount the contract at 14% compounded semiannually. How much should Shipson receive?

A: The contract represents an annuity with payments of $5,000. Adjust the interest rate and number of periods for semiannual compounding and solve for the present value of the annuity.

# Amortized Loans

- An amortized loan's principal is paid off regularly over its life

  - A constant payment is made periodically
    - Represents the present ue of an annuity

# Amortized Loans

Example 6.11

Example

Q: Suppose you borrow $10,000 over four years at 18% compounded monthly repayable in monthly installments. How much is your loan payment?

A:

| |
|---|
| N |
| I/Y |
| PV |
| FV |
| PMT |

# Amortized Loans

Example 6.12

Example

Q: Suppose you want to buy a car and can afford to make payments of $500 a month. The bank makes three-year car loans at 12% compounded monthly. How much can you borrow toward a new car?

N

I/Y

FV

PMT

PV

# Loan Amortization Schedules

- Detail the interest and principal in each loan payment
- Show the beginning and ending balances of unpaid principal for each period
- Need to know
  - Loan amount (PVA)
  - Payment (PMT)
  - Periodic interest rate (k)

# Loan Amortization Schedules Example

Example

Q: Develop an amortization schedule for the loan demonstrated in Example 6.12

| Period | Beginning Balance | Payment | Interest @ 1% | Principal Reduction | Ending Balance |
|---|---|---|---|---|---|
| 1 | $15,053.75 | $500.00 | $150.54 | $349.46 | $14,704.29 |
| 2 | 14,704.29 | 500.00 | 147.04 | 352.96 | 14,351.33 |
| 3 | ______ | 500.00 | ______ | ______ | ______ |
| 4 | ______ | 500.00 | ______ | ______ | ______ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

**Note that the Interest portion of the payment is decreasing while the Principal portion is increasing.**

# Mortgage Loans

- Used to buy real estate
- Often the largest single financial transaction in a person's life
  - Typically an amortized loan over 30 years
    - During the early years, nearly all of the payment goes toward paying interest
    - This reverses toward the end of the mortgage

# Mortgage Loans

- Implications of mortgage payment pattern
  - Early mortgage payments provide a large tax savings reducing the effective cost of borrowing
  - Halfway through a mortgage's life half of the loan is not yet paid off
- Long-term loans result in large total interest amounts over the life of the loan

# Mortgage Loans—Example

Example

Q: Calculate the monthly payment for a 30-year 7.175% mortgage of $150,000. Also calculate the total interest paid over the life of the loan.

A: Adjust the n and k for monthly compounding and input the following calculator keystrokes.

| |
|---|
| N |
| I/Y |
| FV |
| PV |
| PMT |

# The Annuity Due

- Payments occur at the beginning of each period
- The future value of an annuity due
  - Since each payment is received one period earlier, it spends one period longer in the bank earning interest

# The Annuity Due

Example 6.13

Example

Q: The Baxter Corporation started making sinking fund deposits of $50,000 per quarter today. Baxter's bank pays 8% compounded quarterly, and the payments will be made for 10 years. What will the fund be worth at the end of that time?

A: Adjust the k and n for quarterly compounding and input the following calculator keystrokes.

| |
|---|
| N |
| I/Y |
| PMT |
| PV |
| FV |

# The Present Value of an Annuity Due

- Recognizing types of annuity problems
  - Always represent a stream of equal payments
  - Always involve some kind of a transaction at one end of the stream of payments
    - End of stream—future value of an annuity
    - Beginning of stream—present value of an annuity

# Recognizing Types of Annuity Problems

- There's always a stream of equal payments with a transaction *either* at the end *or* at the beginning
  - End — future value of an annuity
  - Beginning — present value of an annuity

# Perpetuities

- A stream of regular payments going on forever
  - An infinite annuity
- Future value of a perpetuity
  - Makes no sense because there is no end point
- Present value of a perpetuity
  - A diminishing series of numbers
    - Each payment's present value is smaller than the one before

$$PV_p = \frac{PMT}{k}$$

# Perpetuities

Example 6.14

Example

Q: The Longhorn Corporation issues a security that promises to pay its holder \$5 per quarter indefinitely. Money markets are such that investors can earn about 8% compounded quarterly on their money. How much can Longhorn sell this special security for?

A: Convert the k to a quarterly k and plug the values into the equation.

$$PV_p = \frac{PMT}{k} = \frac{\$5}{0.02} = \$250$$

You may also work this by inputting a large n into your calculator (to simulate infinity), as shown below.

| | |
|---|---|
| N | 999 |
| I/Y | 2 |
| PMT | 5 |
| FV | 0 |
| PV | **250** ← **Answer** |

# Continuous Compounding

- Compounding periods can be shorter than a day
  - As the time periods become infinitesimally short, interest is said to be compounded continuously
- To determine the future value of a continuously compounded value:

$$FV_n = PV\left(e^{kn}\right)$$

# Continuous Compounding

Example 6.16

Example

Q: The First National Bank of Charleston is offering continuously compounded interest on savings deposits. Such an offering is generally more of a promotional feature than anything else. If you deposit \$5,000 at 6½% compounded continuously and leave it in the bank for 3½ years, how much will you have? Also, what is the equivalent annual rate (EAR) of 12% compounded continuously?

A: To determine the future value of \$5,000, plug the appropriate values into the equation.

A: To determine the EAR of 12% compounded continuously, find the future value of \$100 compounded continuously in one year, then calculate the annual return.

# Time Value Formulas

Table 6.5

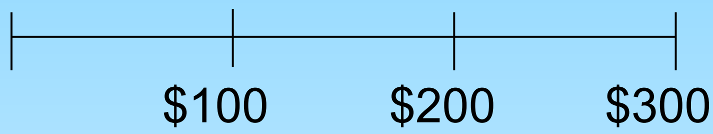
| Equation Number | Formula | Table |
|---|---|---|
| | **Amounts** | |
| 6.4 | $FV_n = PV[FVF_{k,n}]$ | A-1 |
| 6.7 | $PV = FV_n[PVF_{k,n}]$ | A-2 |
| | **Ordinary Annuities** | |
| 6.13 | $FVA_n = PMT[FVFA_{k,n}]$ | A-3 |
| 6.19 | $PVA = PMT[PVFA_{k,n}]$ | A-4 |
| | **Annuities Due** | |
| 6.22 | $FVAd_n = PMT[FVFA_{k,n}](1 + k)$ | A-3 |
| 6.23 | $PVAd = PMT[PVFA_{k,n}](1 + k)$ | A-4 |
| | **Perpetuity** | |
| 6.24 | $PV_P = PMT/k$ | |
| | **Continuous Compounding** | |
| 6.25 | $FV_n = PV(e^{kn})$ | |

# Multipart Problems

- Time value problems are often combined due to complex nature of real situations
  - A time line portrayal can be critical to keeping things straight

# Uneven Streams and Imbedded Annuities

- Many real world problems have sequences of uneven cash flows
  - These are NOT annuities
    - For example, determine the present value of the following stream of cash flows




- Must discount each cash flow individually

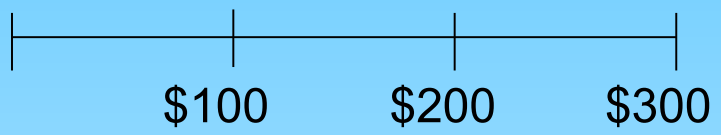
Not really a problem when determining either present or future value

Becomes a problem when determining an interest rate

# Uneven Streams and Imbedded Annuities

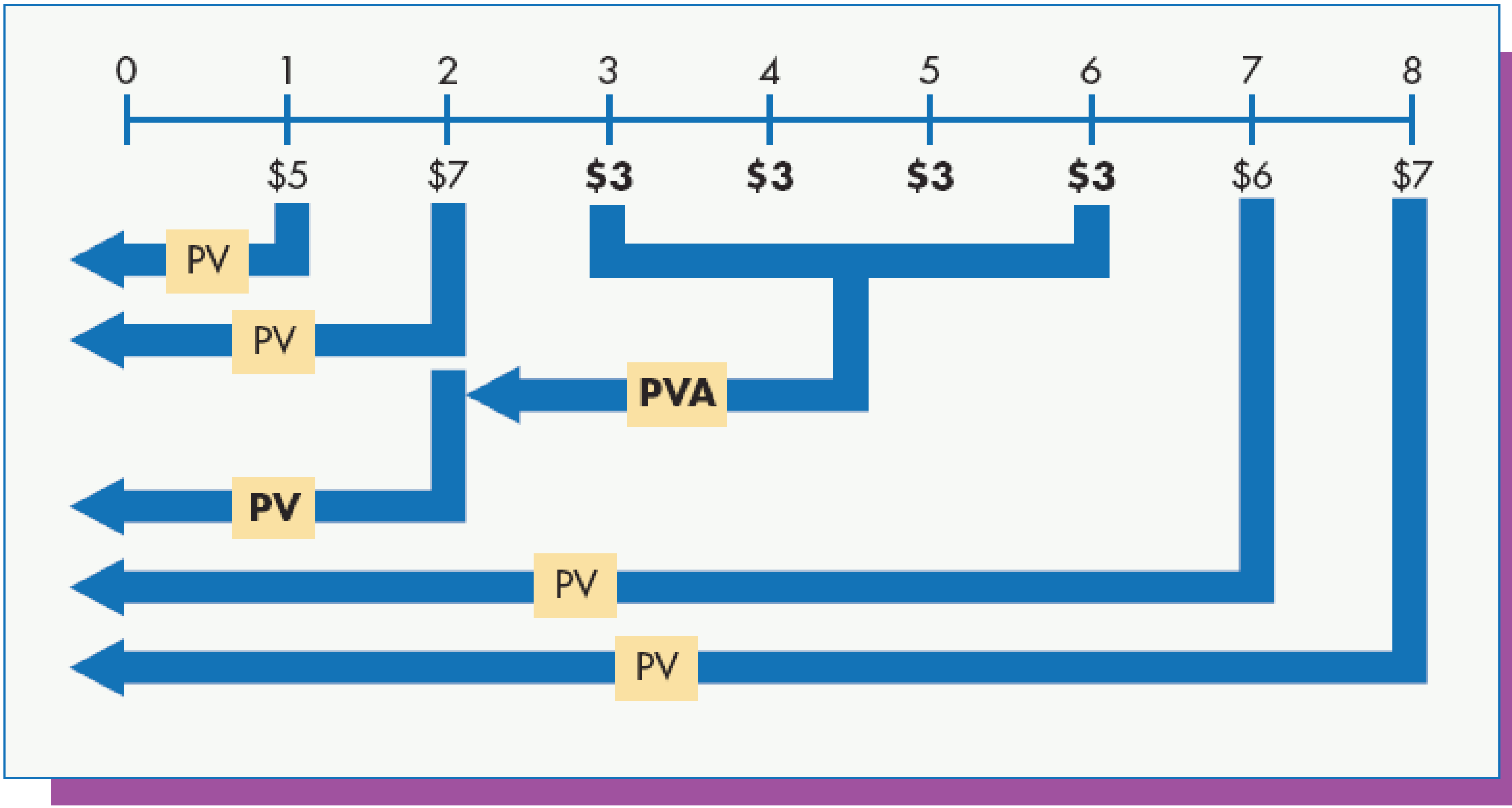
Example

Q: Calculate the interest rate at which the present value of the stream of payments shown below is $500.



# Present Value of an Uneven Stream

Example 6.20



# Calculator Solutions for Uneven Streams

- Financial calculators and spreadsheets have the ability to handle uneven streams with a limited number of payments
- Generally programmed to find the present value of the streams or the k that will make the present value of the stream equal to a given amount