Experiment No. 1: The Simple Pendulum

1. Objective

The objective of this experiment is to determine the time period (T) of simple pendulum.

2. Apparatus

- Supporting rod
- String
- Table clamp
- Stop watch
- Bob
- Vernier caliper
- Scale

3. Theory

A simple pendulum may be described ideally as a point mass suspended by a massless string from some point about which it is allowed to swing back and forth in a place. A simple pendulum can be approximated by a small metal sphere which has a small radius and a large mass when compared relatively to the length and mass of the light string from which it is suspended. If a pendulum is set in motion so that is swings back and forth, its motion will be periodic. The time that it takes to make one complete oscillation is defined as the period T. Another useful quantity used to describe periodic motion is the frequency of oscillation. The frequency f of the oscillations is the number of oscillations that occur per unit time and is the inverse of the period, f = 1/T. Similarly, the period is the inverse of the frequency, T = l/f. The maximum distance that the mass is displaced from its equilibrium position is defined as the amplitude of the oscillation.

When a simple pendulum is displaced from its equilibrium position, there will be a restoring force that moves the pendulum back towards its equilibrium position. As the motion of the pendulum carries it past the equilibrium position, the restoring force changes its direction so that it is still directed towards the equilibrium position. If the restoring force $\vec{F}$ is opposite and directly proportional to the displacement x from the equilibrium position, so that it satisfies the relationship,

$$\vec{F} = -K\vec{x} \qquad (1.1)$$

Then the motion of the pendulum will be simple harmonic motion and its period can be calculated using the equation for the period of simple harmonic motion

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (1.2)$$

It can be shown that if the amplitude of the motion is kept small, Equation (1.2) will be satisfied and the motion of a simple pendulum will be simple harmonic motion, and Equation (1.2) can be used.

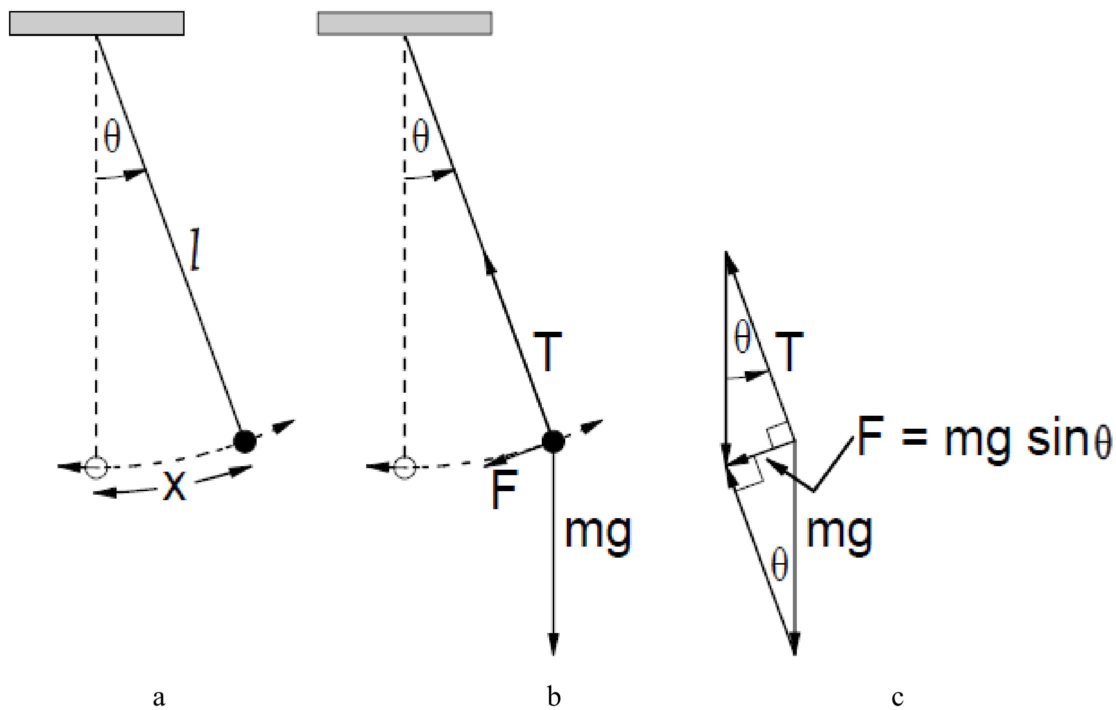

Figure 1.1: Illustration of the restoring force for simple pendulum

The restoring force for a simple pendulum is supplied by the vector sum of the gravitational force on the mass, mg and the tension in the string, T. The magnitude of the restoring force depends on the gravitational force and the displacement of the mass from the equilibrium position. Consider Figure 1.1 where a mass m is suspended by a string of length *l* and is displaced from its equilibrium position by an angle θ and a distance x along the arc through which the mass moves. The gravitational force can be resolved into two components, one along the radial direction, away from the point of suspension, and one along the arc in the direction that the mass moves. The component of the gravitational force along the arc provides the restoring force F and is given by

$$F = -mg\sin\theta \qquad (1.3)$$

Where g is the acceleration of gravity, θ is the angle the pendulum is displaced, and the minus sign indicates that the force is opposite to the displacement. For small amplitudes where θ is small, sinθ can be approximated by θ measured in radians so that Equation (1.3) can be written as

$$F = -mg\,\theta \qquad (1.4)$$

The angle θ in radians is $\frac{x}{l}$, the arc length divided by the length of the pendulum or the radius of the circle in which the mass moves. The restoring force is then given by

$$F = -mg\,\frac{x}{l} \qquad (1.5)$$

and is directly proportional to the displacement x and is in the form of Equation (1.1) where $k = \frac{mg}{l}$. Substituting this value of k into Equation (1.2), the period of a simple pendulum can be found by

$$T = 2\pi\sqrt{\frac{m}{mg/l}} \qquad (1.6)$$

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (1.7)$$

Therefore, for small amplitudes the period of a simple pendulum depends only on its length and the value of the acceleration due to gravity.

4. Procedure

1. Use a vernier caliper to measure the diameter d of the spherical ball and from this calculate its radius r. Record the values of the diameter and radius in meters in the table 1.1.
2. Adjust the length of the pendulum to 400 mm. The length of the simple pendulum is the distance from the point of suspension to the center of the ball. Measure the length of the string *ls* from the point of suspension to the top of the ball using a meter stick. Record this value for the length of the string in the table 1.1. Add the radius of the ball to the string length *ls* and record that value as the length *l* of the pendulum in the table 1.1.

   $$l = ls + r \qquad (1.8)$$

3. Displace the pendulum about 5º from its equilibrium position and let it swing back and forth. Measure the total time that it takes to make 20 complete oscillations. Record that time in the table 1.1.
4. Increase the length of the pendulum by 100mm and repeat the measurements made in the previous steps until the length increases to approximately 800 mm.
5. Calculate the period of the oscillations for each length by dividing the total time by the number of oscillations, 20. Record the values in the appropriate column of your data table 1.1.
6. Graph the period of the pendulum as a function of its length and place in Figure 1.2. The length of the pendulum is the independent variable and should be plotted on the horizontal axis or abscissa (x axis). The period is the dependent variable and should be plotted on the vertical axis or ordinate (y axis).
7. Compare your measured time period T with the calculated value of T by using Equation 1.7 and record the %Difference in the table. The value of g on earth is 9.8 m/s2.

Table 1.1

S#	Length of String l_S (mm)	Length of Pendulum $l = l_S + r$ (mm)	Time for 20 Oscillations (S)	Time Period (S) $\frac{Time\ for\ 20\ Cycles}{Total\ No.\ of\ Cycles}$	Time Period (Theoretical values) (S)	%Difference $\left(\frac{Measured\ - Theoratical}{Theoratical}\right) * 100$
1	400					
2	500					
3	600					
4	700					
5	800					

Figure 1.2: Time Period Vs Length

5. Questions

1. How would the period of a simple pendulum be affected if it were located on the moon instead of the earth?

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2. What effect does the mass of the ball have on the period of a simple pendulum? What would be the effect of replacing the steel ball with a wooden ball, a lead ball, and a ping pong ball of the same size?

3. Examine your graph of Figure 1.2 and describe the change in the period per unit length.

6. Conclusions