

## VARIABLES

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Simply all those items which go on changing or which have the tendency to change are known as **variables**. As the case of temperature, light, or speed of some aircraft, etc. The mathematicians denote the variables by symbols like  $x$ ,  $y$ ,  $z$ , etc. Accordingly the mathematical economists define variables as, " **A variable is a symbol which during a discussion may assume different values**". As the speed of a motor cycle is represented by '  $S$  ' and such  $S$  can assume different values. Again, the temperature which is denoted by '  $T$  ' can go for different values during the 24 hours. How many values a variable can assume is given the name of **range** of the variable. For example, if the speed of a motor cycle can adopt the values in between zero and 120 kilometers per hour — such will be the **range** of the speed of the motorcycle, where the zero is the **lower limit** of the range and 120 will be the **upper limit** of the range.

## ECONOMIC VARIABLES

In economics we have a lot of variables like quantity demanded, quantity supplied, utility, costs, revenues, savings, investment, consumption, exports and imports etc. To represent economic variables we use the first alphabet of their names, as : Qd for the quantity demanded; Qs for the quantity supplied; U for utility; Q for quantity; C for costs; R for revenues; S for savings, C for consumption and ; I for investment. But in some cases we do not follow this principle. As we use X for exports; M for imports; R for transfer payments and; Y for national income.

## **ECONOMIC VARIABLES AND THEIR FUNCTIONAL RELATIONSHIPS**

The economic variables are divided into micro and macro economic variables. Accordingly, first we discuss microeconomic variables and their relationships. Afterwards, we will analyse macroeconomic variables and their relationships.

### **MICROECONOMIC VARIABLES AND THEIR FUNCTIONAL RELATIONSHIPS**

Price (  $P$  ), quantity demanded (  $Q_d$  ), quantity supplied (  $Q_s$  ), utility (  $U$  ), costs (  $C$  ) and revenues (  $R$  ) etc. are the major microeconomic variables. The following functional relationships exist between these variables:



### 1. DEMAND FUNCTION

The quantity demanded ( $Q_d$ ) depends upon price ( $P$ ). Such functional relationship between demand and price is given the name of demand function. It is as:

$$Q_d = f(P)$$

Regarding demand function, a law is presented known as **Law of Demand** which states that "*Other things remaining the same the quantity demanded varies inversely with changes in price*". Thus to represent such relationship and behaviour between  $Q$  and  $P$  in the presence of certain assumptions a **standard demand function** is presented as:  $Q = a - bP$  where  $P$  and  $Q$  are variables,  $a$  and  $b$  represent parameters (assumptions of the law of demand) and the negative sign ( $-$ ) depicts inverse relationship between price and demand. Thus the demand function is a decreasing function. It is as:  $P \uparrow, Q_d \downarrow$  and  $P \downarrow, Q_d \uparrow$

The standard demand function is linear. Hence its graph is a straight line.

The **average demand function** =  $\frac{Q}{P}$ . While the **marginal demand function** = slope

of the demand curve = derivative of demand function =  $\frac{dQ}{dP}$ .

### 2. SUPPLY FUNCTION

The quantity supplied ( $Q_s$ ) depends on price ( $P$ ). Such functional relationship between supply and price is given the name of supply function. It is as:  $Q_s = f(P)$ . Regarding supply function, a law is presented known as **Law of Supply** which states that "*Other things remaining the same the quantity supplied varies directly with changes in price*". Thus to present such relationship and behaviour between  $Q$  and  $P$  in the presence of certain assumptions a **standard supply function** is presented as:

$Q = a + bP$ , where  $P$  and  $Q$  are variables,  $a$  and  $b$  represent parameters (assumptions of the law of supply) and positive sign ( $+$ ) depicts positive or direct relationship between price and supply. Thus the standard supply function is linear. Hence its graph is a straight line. The supply function is an increasing function. It is as:

$$P \uparrow, Q_s \uparrow \quad \text{and} \quad P \downarrow, Q_s \downarrow$$

The **average supply function** =  $\frac{Q}{P}$ . While the **marginal supply function** = slope

of the supply curve = derivative of supply function =  $\frac{dQ}{dP}$ .

### 3. UTILITY FUNCTION

The utility ( $U$ ) derived from any good depends upon the units ( $Q$ ) of the good consumed. Such functional relationship between  $U$  and  $Q$  is given the name of utility function. It is as:  $U = f(Q)$ . Regarding utility function, a law is presented which is known as **Law of Diminishing Marginal Utility** which states that "*Other things remaining the same, alongwith increase in the units of the commodity consumed the total utility ( $U$ ) increases at a decreasing rate. Hence marginal utility ( $MU$ ) decreases*". Thus to represent such relationship between  $U$  and  $Q$  in the presence of certain assumptions, a standard utility function is presented as:  $U = aQ - bQ^2$  where



U and Q are variables,  $a$  and  $b$  represent parameters (assumptions of the law) and negative sign alongwith the square power on Q shows that utility increases at a decreasing rate. The rate of change of utility function or slope of utility function is called **marginal utility**. It is as :  $MU = \frac{dU}{dQ}$ . The standard utility function is quadratic. Hence its graph is a parabola.

#### 4. PRODUCTION FUNCTION

According to classical economists the production of any good ( Q ) depends upon the units of labor ( L ). Such functional relationship between Q and L is known as Production Function. It is as :  $Q = f( L )$ . Regarding production function, a law is presented which is known as **Law of Variable Proportions** which states that " *Other things remaining the same, alongwith increase in the units of the labor the total production increases at different rates, i.e., it increases at a constant rate, at a decreasing rate and at an increasing rate* ". Thus to represent relationship and behaviour between Q and L in the presence of certain assumptions, a **standard production function** is presented as :  $Q = aL - bL^2 + cL^3$ , where Q and L are variables,  $a$ ,  $b$  and  $c$  represent parameters. The average production function is called **Average Product of Labor**. It is as :

$AP_L = \frac{Q}{L}$ . While the marginal production function while slope of production function or derivative of production function is given the name of **Marginal Product of Labor**. It is as:  $MP_L = \frac{dQ}{dL}$ . The standard production function is cubic. Hence its graph is a cubic curve.

#### 5. COST FUNCTION

The costs of production ( C ) of any firm depend upon the quantity ( Q ) produced by the firm. Such functional relationship between C and Q is known as cost function. It is as :  $C = f( Q )$ . Regarding cost function we have a behaviour which states that alongwith increase in the units of a good produced the costs or total costs increase at different rates, i.e., the costs increase at a decreasing rate, the costs increase at a constant rate and the costs increase at an increasing rate. Thus to represent relationship and behaviour between C and Q in the presence of certain assumptions a **standard cost function** is presented as :  $C = aQ - bQ^2 + cQ^3$  where C and Q are variables and  $a$ ,  $b$  and  $c$  represent parameters. The average cost function is called **Average Costs**.

It is as :  $AC = \frac{C}{Q}$ . While the **marginal cost function** = slope of cost function = derivative of cost function is given the name of **Marginal Costs**. It is as :  $MC = \frac{dC}{dQ}$ .

The **standard cost function** is cubic. Hence its graph is a cubic curve.

#### 6. REVENUE FUNCTION

The revenues ( R ) of any firm depend upon the quantity ( Q ) sold by the firm. Such functional relationship between R and Q is known as **revenue function**. It is as:



$R = f(Q)$ . Regarding revenue function we have a behaviour which states that revenues of the monopolist increase at a decreasing rate. Thus to represent relationship and behaviour between  $R$  and  $Q$  in the presence of certain assumptions a **standard revenue function** is presented as :  $R = aQ - bQ^2$  where  $R$  and  $Q$  are variables and  $a$  and  $b$  represent parameters. The average revenue function is called **Average Revenue**. It is as :  $AR = \frac{R}{Q}$ . While the **marginal revenue function** = slope of revenue function

= derivative of revenue function is called **Marginal Revenue**. It is as:  $MR = \frac{dR}{dQ}$ . The **standard revenue function** is quadratic. Hence its graph is a parabola.

### MACRO ECONOMIC VARIABLES AND THEIR FUNCTIONAL RELATIONSHIPS

Consumption (  $C$  ), Savings (  $S$  ), National Income (  $Y$  ), Investment (  $I$  ), Rate of Interest (  $i$  ), Exports (  $X$  ), Imports (  $M$  ), Government Expenditure (  $G$  ), Taxes (  $T$  ) and Transfer payments (  $R$  ) are major macroeconomic variables. The following functional relationships exist between these variables.

#### 1. CONSUMPTION FUNCTION

According to Keynes the consumption expenditures (  $C$  ) of an economy depend upon income (  $Y$  ) of the economy. Such functional relationship between  $C$  and  $Y$  is known as consumption function. It is as :  $C = f(Y)$ . It is an increasing function which states that " *alongwith increase in income, consumption expenditures increase and vice versa*". It is as :  $Y \uparrow, C \uparrow$  and  $Y \downarrow, C \downarrow$ .

Regarding consumption function we have Keynes **Law of Consumption** which states that there exists a non-proportional relationship between consumption and income. Accordingly, the **standard short run consumption function** is presented as:

$C = C_0 + cY$  where  $C_0$  = autonomous consumption and  $cY$  = induced consumption. The average consumption function is called average propensity to consume =  $APC = \frac{C}{Y}$ . While the slope of consumption function or derivative of

consumption function is called **Marginal Propensity to Consume** =  $MPC = \frac{dC}{dY}$ .

The standard consumption function is linear. Hence its graph is a straight line. It is told that in case of longrun there exists a proportional relationship between  $C$  and  $Y$ . Hence its standard form is as :  $C = cY$ .

#### 2. SAVING FUNCTION

According to Keynes the savings (  $S$  ) of the economy depend upon income (  $Y$  ) of the economy. Such functional relationship between  $S$  and  $Y$  is known as saving function. It is as :  $S = f(Y)$ . It is also an increasing function which states that " *savings increase alongwith increase in income and vice versa*". It is as :

$$Y \uparrow, S \uparrow \quad \text{and} \quad Y \downarrow, S \downarrow$$

The **standard short run saving function** is presented as  $S = -S_0 + sY$  where  $-S_0$  = autonomous savings and  $sY$  = induced savings. The average saving function is called



$APS = \frac{S}{Y}$ . While the slope of saving function or derivative of saving function is called **Marginal Propensity to Save** =  $MPS = \frac{dS}{dY}$ . The standard saving function

is linear. Hence its graph is a straight line. It is told that in case of longrun there exists a proportional relationship between S and Y. Hence its standard form is as :  $S = sY$ .

### 3. INCOME FUNCTION

According to Keynes the national income ( Y ) of a country depends upon its investment ( I ). Such functional relationship between Y and I is known as income function. It is as :  $Y = f(I)$ . It is an increasing function which states that "*alongwith increase in investment, national income goes on to increase and vice versa*". It is as :

$$I \uparrow, Y \uparrow \quad \text{and} \quad I \downarrow, Y \downarrow.$$

The derivative of income function is called 'Multiplier' ( K ) or investment multiplier which shows the ratio of change in income (  $\Delta Y$  ) to change in investment (  $\Delta I$  ). It is

as : 
$$K = \frac{\Delta Y}{\Delta I} = \frac{dY}{dI}$$

### 4. INVESTMENT FUNCTION

According to Keynes investment ( I ) depends upon rate of interest ( i ). Such functional relationship between I and i is known as investment function. It is as :  $I = f(i)$ . It is a decreasing function which states the "*alongwith increase in the rate of interest, investment decreases and vice versa*". It is as :  $i \uparrow, I \downarrow$  and  $i \downarrow, I \uparrow$ .

### 5. INVESTMENT FUNCTION

According to Clark the investment ( I ) depends upon the level of income ( Y ). Such functional relationship between I and Y is known as investment function. It is as :  $I = f(Y)$ . It is an increasing function which states that "*alongwith increase in the level of income, the investment also increases and vice versa*". It is as :  $Y \uparrow, I \uparrow$  and  $Y \downarrow, I \downarrow$ .

The derivative of investment function is called accelerator coefficient ( w ) which shows the ratio of change in investment (  $\Delta I$  ) to change in income (  $\Delta Y$  ). It is as:

$$w = \frac{\Delta I}{\Delta Y} = \frac{dI}{dY}$$

### 6. IMPORT FUNCTION

The imports ( M ) of a country depend upon income ( Y ) of the country. Such functional relationship between M and Y is called Import Function. It is as :  $M = f(Y)$ . It is an increasing function which states that "*alongwith increase in income ( Y ) the imports ( M ) also increase and vice versa*". It is as :  $Y \uparrow, M \uparrow$  and  $Y \downarrow, M \downarrow$ .

The **standard import function** is presented as :  $M = M_0 + mY$ , where  $M_0$  = autonomous imports and  $mY$  = induced imports. The average import function is called

**average propensity to import** =  $APM = \frac{M}{Y}$  while the slope of the import function or

derivative of import function is called **marginal propensity to import** =  $MPM = \frac{dM}{dY}$ .

The standard import function is linear. Hence, its graph is a straight line.