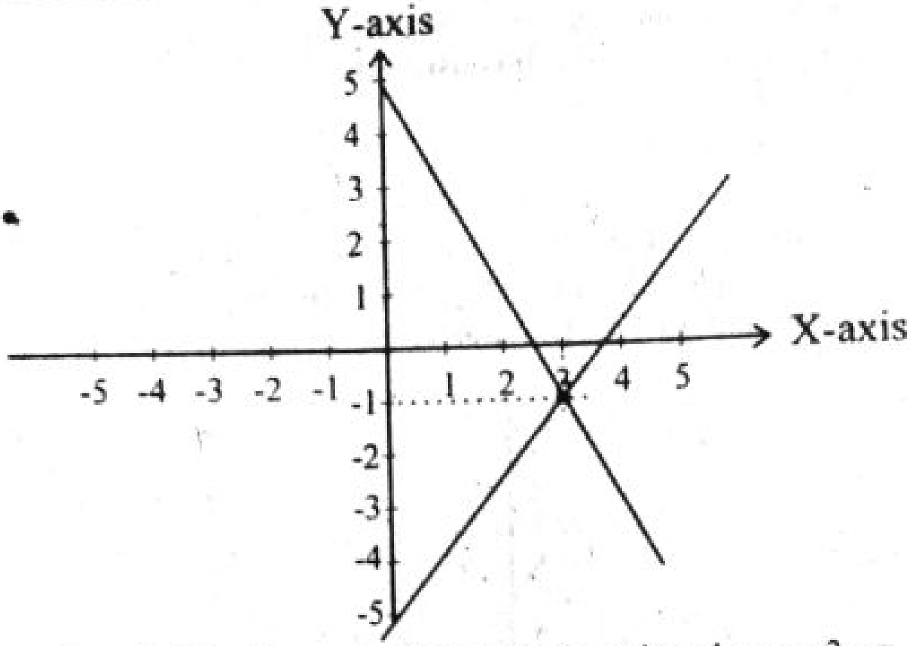


The lines (1) and (2) in the graph intersect at the point where x=3,y= -1

EXAMPLE 2.8

Use graphing to find the solution of the system.

$$4x + 3y = 11$$
$$2x - 5y = -1$$

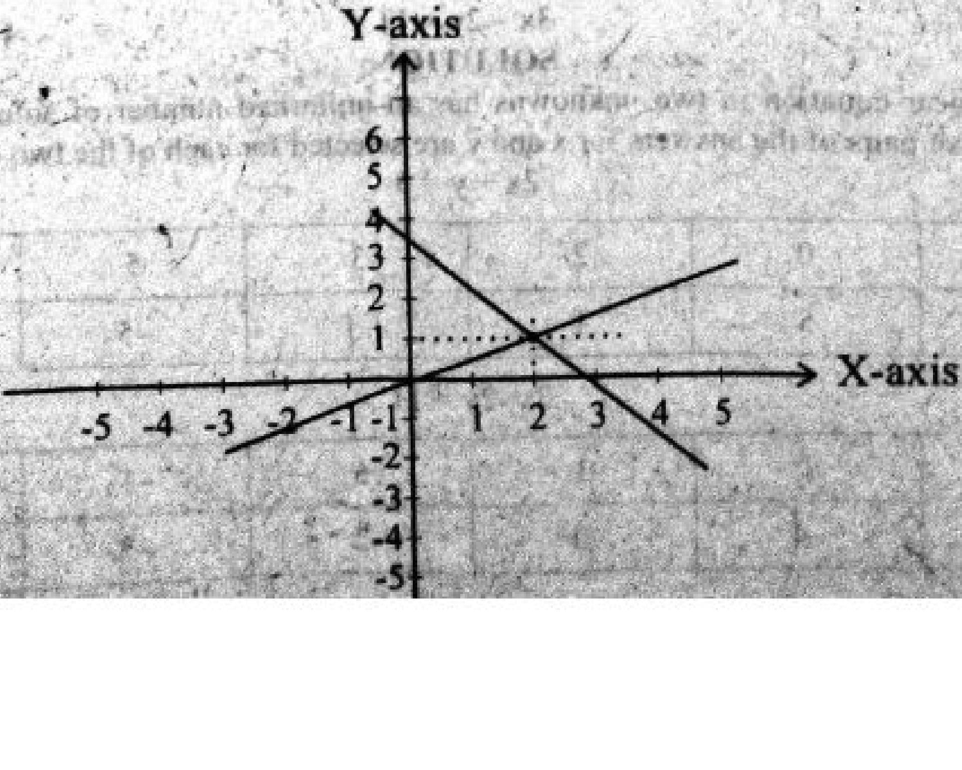
SOLUTION

$$4x + 3y = 11$$

x	−1	0	1	2	3	4
y	5	11/3	7/3	1	−1/3	−5/3

$$2x - 5y = -1$$

x	−1	0	1	2	3	4
y	−1/5	1/5	3/5	1	$1^2/_5$	9/5



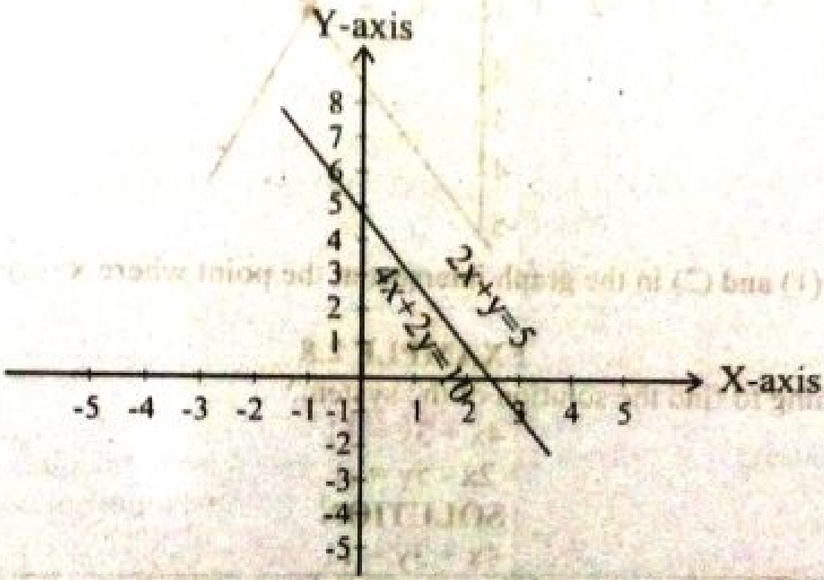
There is no point of intersection. There is no pair of numbers that satisfies both equations. Such equations are said to be **inconsistent.**

Consider the system

$$2x + y = 5$$

$$4x + 2y = 10$$

In this case any pair of numbers that satisfies the first equation also satisfies the second equation. For example the pair x=0, y=5 satisfies both equations, the pair x=1, y=3 satisfies both equations and so on. The graphs of 4x+2y=10 coincides with the graph of 2x+y=5



The intersection consists of all points on this line. In this case the equations are said to be **dependent.**

EXAMPLE 2.7

Use graphing to find the solution of the system.

$$3x + y = 5$$

$$3x - 2y = 11$$

SOLUTION

A linear equation in two unknowns has an unlimited number of solutions in pairs. Here, only five pairs of the answers for x and y are selected for each of the two equations.

$$2x + y = 5$$

X	0	2	3	5	6
Y	5	1	–1	–5	–7

$$3x - 2y = 11$$

X	1	3	5	–1	–3
Y	–4	–1	2	–7	–10

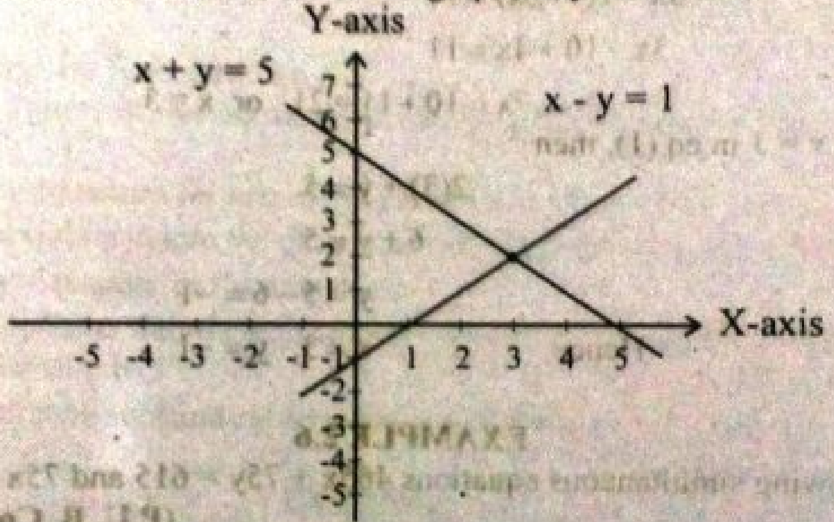
2.1.3 Use of Graph

The equations are referred to as s system of equation, and the ordered pairs (x,y) that satisfy all the equations are the solutions to the system. We can use graphing to find the solution to a system of equations.

The use of a graph to illustrate a solution of simultaneous linear equation is valuable. A pair of equations of the first degree in two variables, such as

$$x - y = 1, \quad x + y = 5$$

The solution set of $x - y = 1$ consists of all pairs of value of x and y that satisfy this equation, such as (0,-1), (1,0), (2,1), (3,2), (4,3).... The solution set of $x + y = 5$ consists of all pairs of numbers that satisfy this equation, such as (0,5), (1,4), (2,3), (3,2), (4,1)...... The solution set of the system consists of the intersection of these two sets, that is, the pair of numbers common to both sets. In this example x=3, y=2 is the single pair of numbers that satisfies both equations. The situation is shown graphically as follows:

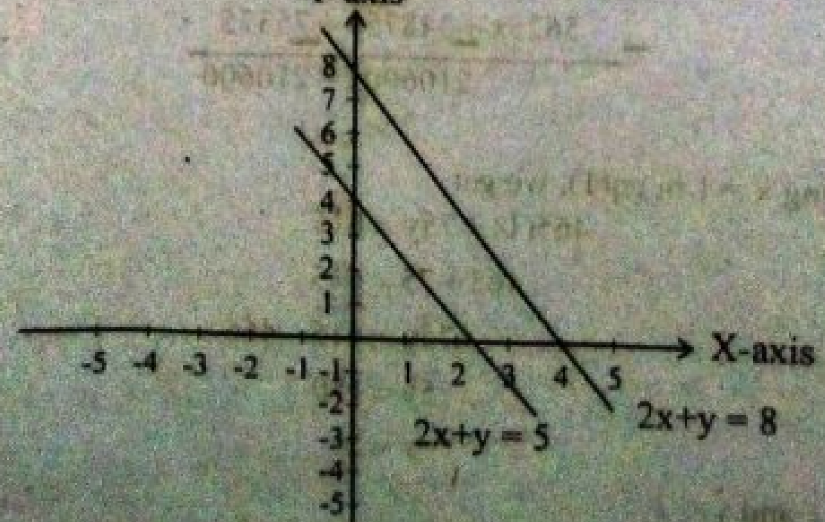


Such equation are said to be consistent.

Consider the system

$$2x + y = 5$$
$$2x + y = 8$$

In this case, the graph consists of a pair of parallel lines as



$$x = \frac{5-2(1)}{3} = \frac{5-2}{3} = \frac{3}{3} = 1$$

Hence $x = 1, y = 1$

EXAMPLE 2.5

Solve the two equations simultaneously $2x + y = 5$ and $3x - 2y = 11$

SOLUTION

$$2x + y = 5 \quad \ldots\ldots\ldots(1)$$

$$3x - 2y = 11 \quad \ldots\ldots\ldots(2)$$

Solve eq (1) for y in terms of x, i.e.

$$y = 5 - 2x \quad \ldots\ldots\ldots\ldots (3)$$

Substituting the value of y from eq (3) in eq (2), we get

$$3x - 2(5 - 2x) = 11$$

$$3x - 10 + 4x = 11$$

$$7x = 10 + 11 = 21 \quad \text{or} \quad x = 3$$

Substitute $x = 3$ in eq (1), then

$$2(3) + y = 5$$

$$6 + y = 5$$

$$y = 5 - 6 = -1$$

Hence $\quad x = 3, \; y = -1$

Multiplying eq (2) by -2, and adding, we get

$$\begin{array}{r} 2x - 5y = 4 \\ -2x - 2y = -6 \\ \hline -9y = -2 \\ y = \dfrac{2}{9} \end{array}$$

For x, putting y = 2/9 in eq (1), we get

$$2x - 5\left(\frac{2}{9}\right) = 4 \quad \text{or} \quad 2x - \frac{10}{9} = 4$$

$$2x = 4 + \frac{10}{9} = \frac{36+10}{9} \quad \text{or} \quad 2x = \frac{46}{9}$$

Thus $\quad x = \dfrac{23}{9}$ and $y = \dfrac{2}{9}$

2.1.2 Elimination by Substitution

To solve a system of two equations in two variables by substitution, following are the steps:

1. Solve one of the equations for one of the variables in terms of the other.
2. Substitute this expression into the other equation to give one equation in one unknown.
3. Solve this linear equation for the unknown.
4. Substitute this solution into the equation in step (1) or into one of the original equations to solve for other variable.
5. Check the solution by substituting for x and y in both original equations.

EXAMPLE 2.4

Solve the system containing $3x + 2y = 5$ and $9x - 8y = 1$

(B. Z. U. – B. Com 1999-S)

SOLUTION

$$3x + 2y = 5 \quad \text{..........(1)}$$

$$9x - 8y = 1 \quad \text{..........(2)}$$

Solve eq (1) for x in terms of y, i.e.

$$3x = 5 - 2y \quad \text{or} \quad x = \frac{5-2y}{3}$$

Substituting the value of x in eq (2), we get

$$9\left(\frac{5-2y}{3}\right) - 8y = 1 \quad \text{or} \quad 3(5-2y) - 8y = 1$$

$$15 - 6y - 8y = 1 \quad \text{or} \quad -14y = 1 - 15 = -14$$

$$y = 1$$

Substitute $y = 1$ in $x = \dfrac{5-2y}{3}$

Hence the required solution is given by $x = 3, y = -1$

ii) **Subtraction (Eliminate x by subtracting)**
Multiplying eq (1) by 3 and eq (2) by 2 and subtracting

$$\begin{array}{r} 6x + 3y = 15 \\ -\underline{\;6x \mp 4y = \underset{-}{2}2\;} \\ 7y = -7 \\ y = -1 \end{array}$$

Substituting $y = -1$ in eq (1), we get

$$2x + (-1) = 5 \text{ or } 2x - 1 = 5$$

$$2x = 5 + 1 \text{ or } 2x = 6 \text{ or } x = 3$$

Hence $x = 3, y = -1$

EXAMPLE 2.2

Find the common solution of the equations; $2x + 4y = 7$ and $2x - y = 1$

(B. Z. U. – B. Com 1998-

SOLUTION

$$2x + 4y = 7 \quad ....(1)$$

$$2x - y = 1 \quad ....(2)$$

Subtracting eq (2) from (1), we get

$$\begin{array}{r} 2x + 4y = 7 \\ \underline{-\;2x \mp y = \underset{-}{1}} \\ 5y = 6 \end{array}$$

$$y = \frac{6}{5}$$

To find the value of x, putting $y = {}^{6}/_{5}$ in eq(1), we get

$$2x + 4\left(\frac{6}{5}\right) = 7 \Rightarrow 2x + \frac{24}{5} = 7$$

$$2x = 7 - \frac{24}{5} = \frac{35 - 24}{5} = \frac{11}{5}$$

$$x = \frac{11}{10}$$

Thus $x = \frac{11}{10}$ **and** $y = \frac{6}{5}$

EXAMPLE 2.3

Solve the system $2x - 5y = 4$ and $x + 2y = 3$

(I. U. – B. Com 2000-

SOLUTION

$$2x - 5y = 4 \quad ....(1)$$

$$x + 2y = 3 \quad ....(2)$$

2.1.1 Elimination by Addition or Subtraction

To solve a system of two equations in two variables by addition or subtraction, following are the steps.

1. If necessary, multiply one or both equations by a non zero number that will make the coefficients of one of the variables identical, except perhaps for signs.
2. Add or subtract the equations to eliminate one of the variables.
3. Solve for the variable in the resulting equation.
4. Substitute the solution into one of the original equations and solve for the remaining variable.
5. Check the solutions in both original equations.

EXAMPLE 2.1

Find the common solution of the equations; $2x + y = 5$ and $3x - 2y = 11$

SOLUTION

i) **Addition (Eliminate by addition)**

$$2x + y = 5 \quad \text{..........(1)}$$

$$3x - 2y = 11 \quad \text{.........(2)}$$

Multiplying equation (1) by 2 and adding in equation (2), we get

$$\begin{array}{r} 4x + 2y = 10 \\ \underline{3x - 2y = 11} \\ 7x \quad = 21 \end{array}$$

$$x = 3$$

Substituting $x = 3$ in eq (1), we get

$$2(3) + y = 5 \text{ or } 6 + y = 5 \text{ or } y = -1$$