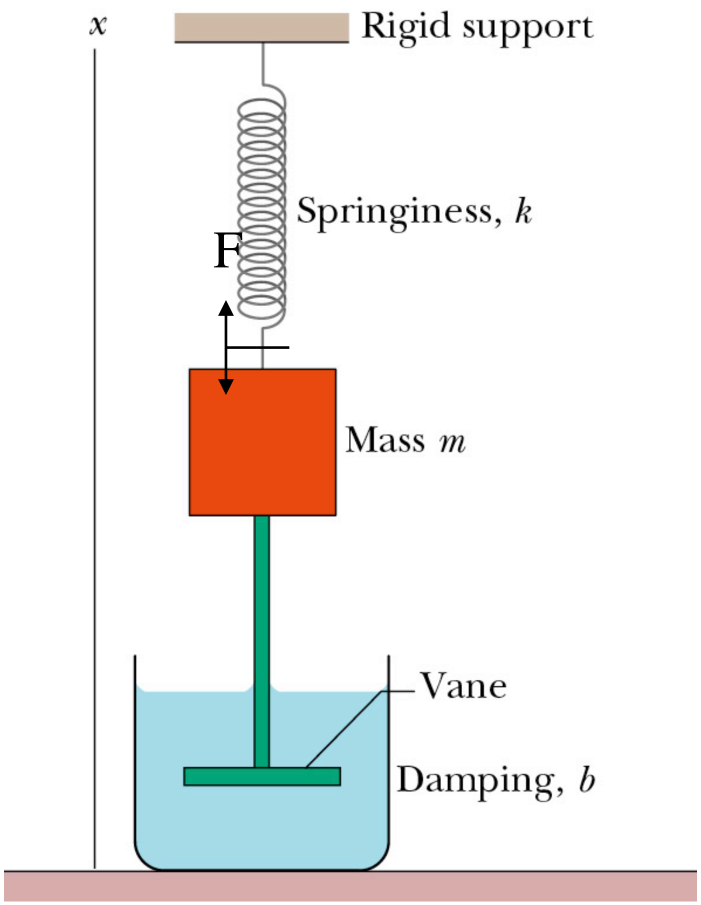
Lecture 11
Chapter 16
Waves I

Forced oscillator from last time
Slinky example

Coiled wire
Rope
Transverse Waves demonstrator
Longitudinal Waves magnetic balls
Standing Waves machine

Damped simple harmonic oscillator with applied force and Resonance



Attach the mass on the left to a motor that moves in a circle

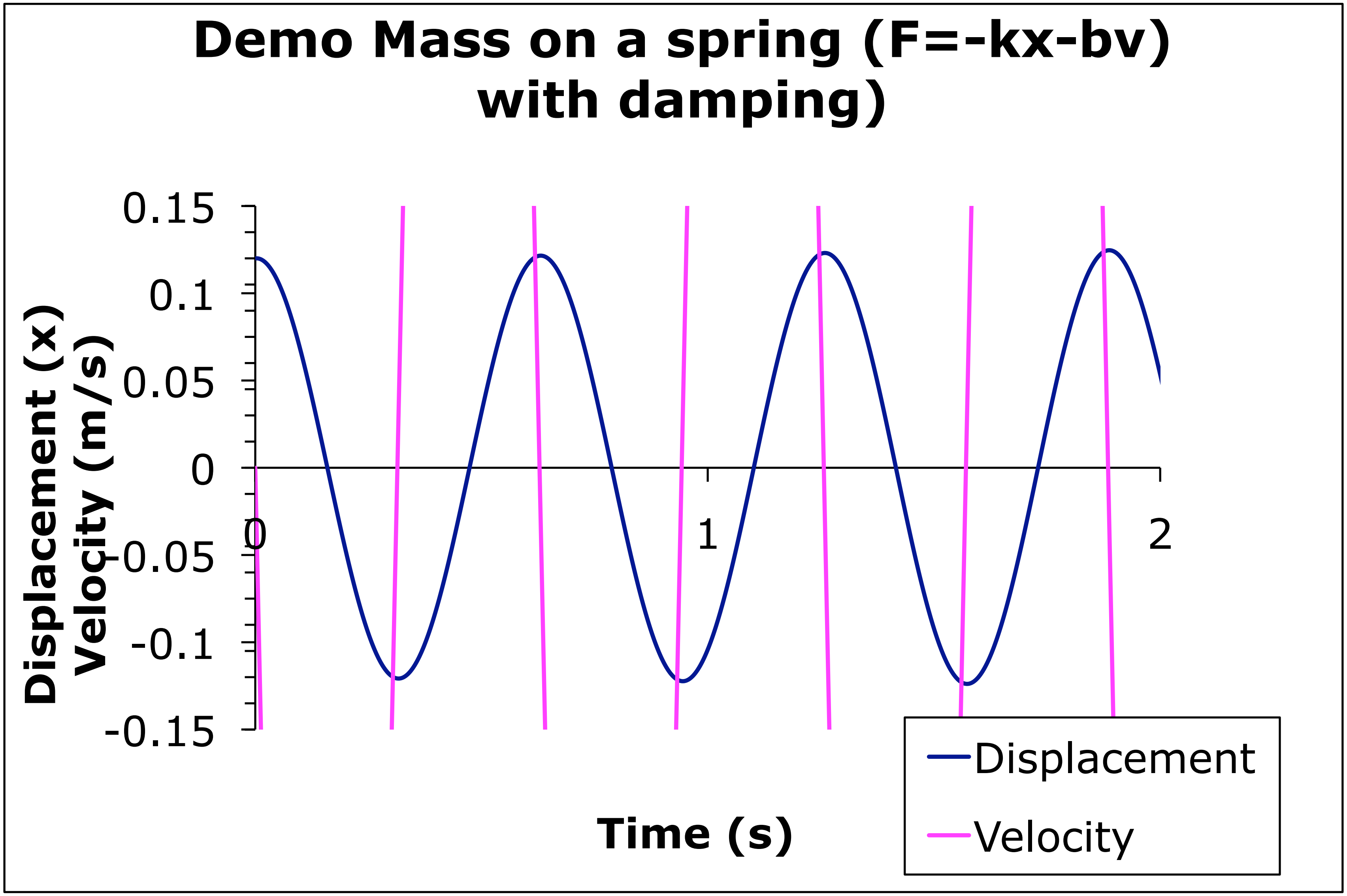
Demo example with applied force and a dampening force. Air acts as the dampening force. The motor is the applied force.

$$F_0 \sin(\omega t) - by - kx = ma$$

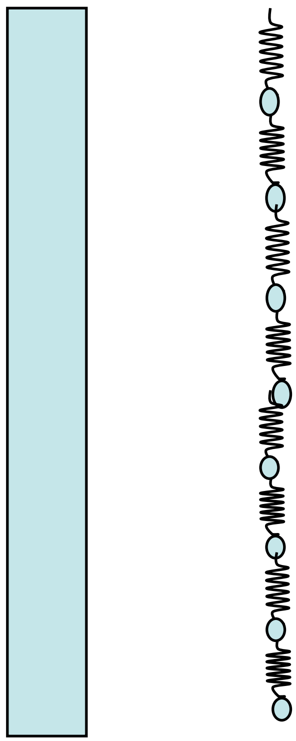
$$m\frac{dp}{dt} = F_0 \sin(\omega t) - by - kx$$

$$v_i = v_{i-1} + (\frac{F_0}{m}\sin(\omega t_{i-1}) - \frac{b}{m}v_{i-1} - \frac{k}{m}x_{i-1})\Delta t$$

http://www.physics.purdue.edu/class/applets/phe/resonance.htm


Demo Mass on a spring (F=-kx-bv) with damping)
Displacement (x)
Velocity (m/s)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
1
2
Time (s)
Displacement
Velocity

When you drop a metal rod, does the top begin to accelerate before the bottom?

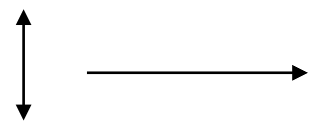


Falling elastic bars and springs,
American Journal of Physics
75, no. 7, July 2007,
J.M. Aguirregabbiria, etc.

Waves in general

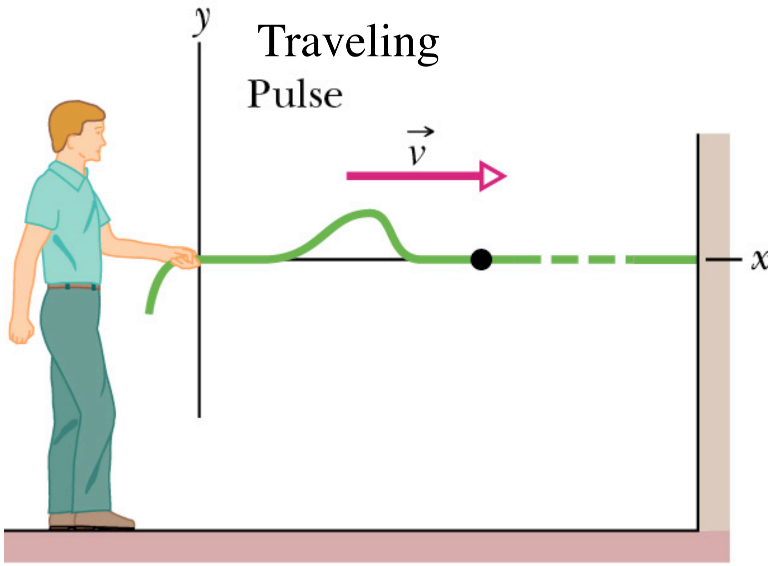
- Two Types of waves
  Transverse and longitudinal waves
- Characteristics
  - Wavelength and frequency
  - Speed of the wave
- Examples
- Properties of waves
  - Energy and power
  - Superposition,
  - Interference,
  - standing waves,
  - resonance

Kinds of waves

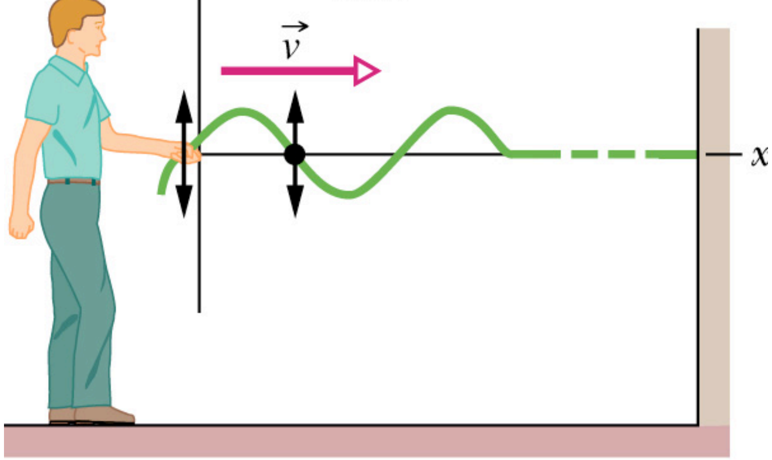
- Transverse

  - Waves on a string
  - Electromagnetic waves
- Longitudinal waves

  - Sound
- Transverse and Longitudinal
  - Water
  - seismic
- Probability waves
  - Electron, proton, and matter waves - quantum waves

Examples of Traveling Transverse Waves



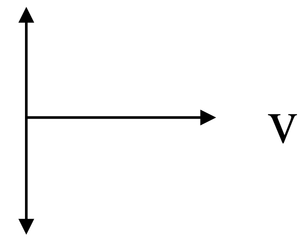
(a)



(b)

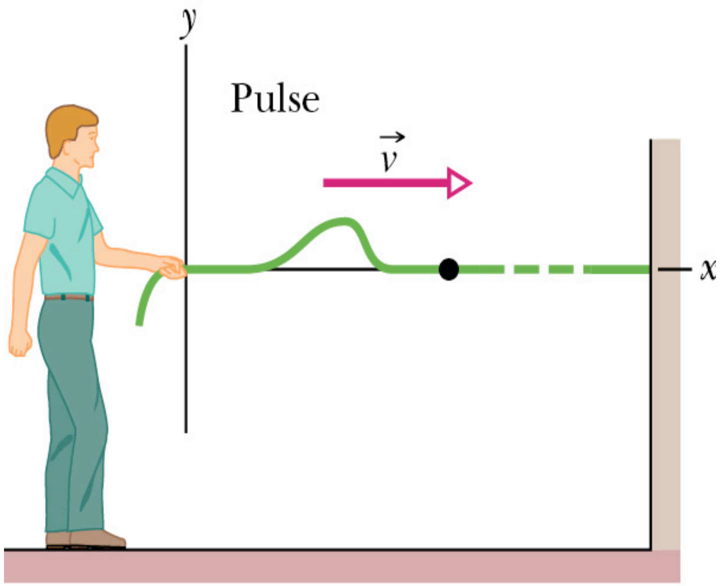
1) Use rope. Vibrations are up and down or perpendicular to the velocity of the wave or direction of propagation. This is called transverse.

Note reflected wave is under the rope. It is negative in amplitude while the incident wave is positive.



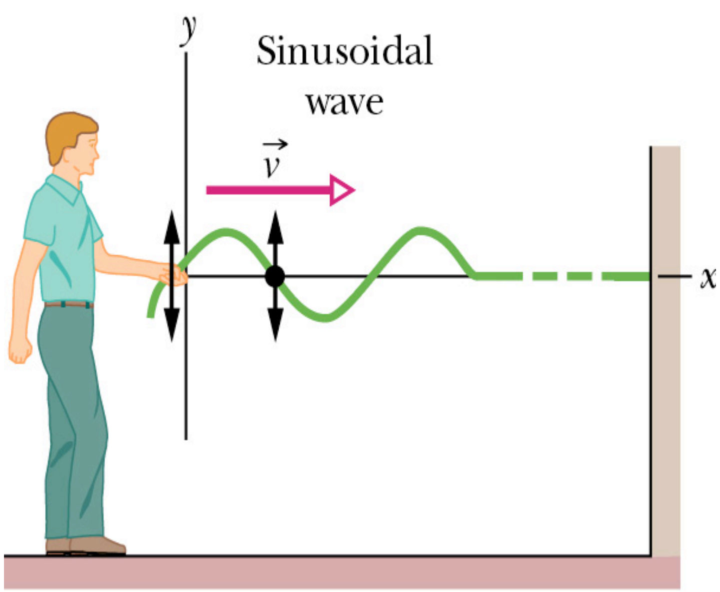
$$v = \sqrt{\frac{T}{\mu}}$$

Examples of Traveling Transverse Waves



(a)

2) Now use transverse wave demonstrator. Here the reflected wave is on top. Reflected wave amplitude is positive as is the incident wave.

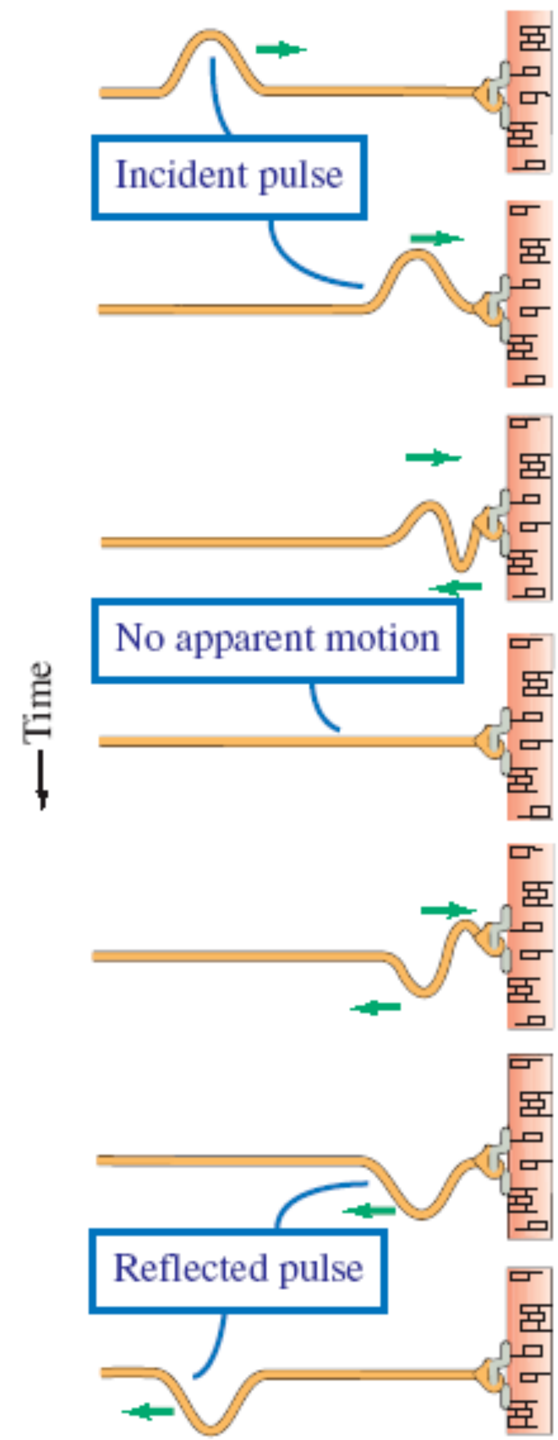


(b)

Pinch the end of the demonstrator to show neg amplitude

3) Electromagnetic waves such as light, TV, radio, etc. are also transverse waves

Pulses



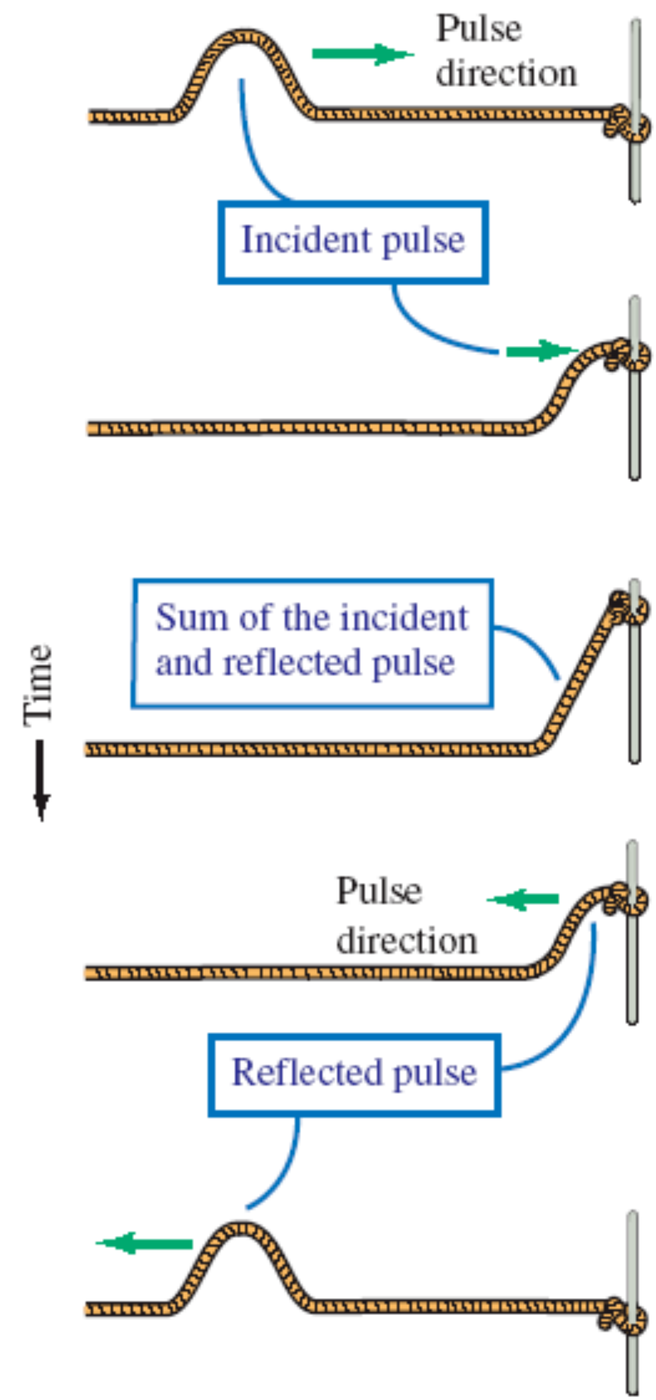
Reflection:

An incident pulse reflecting from a fixed point will be inverted.

Pulses

Reflection:

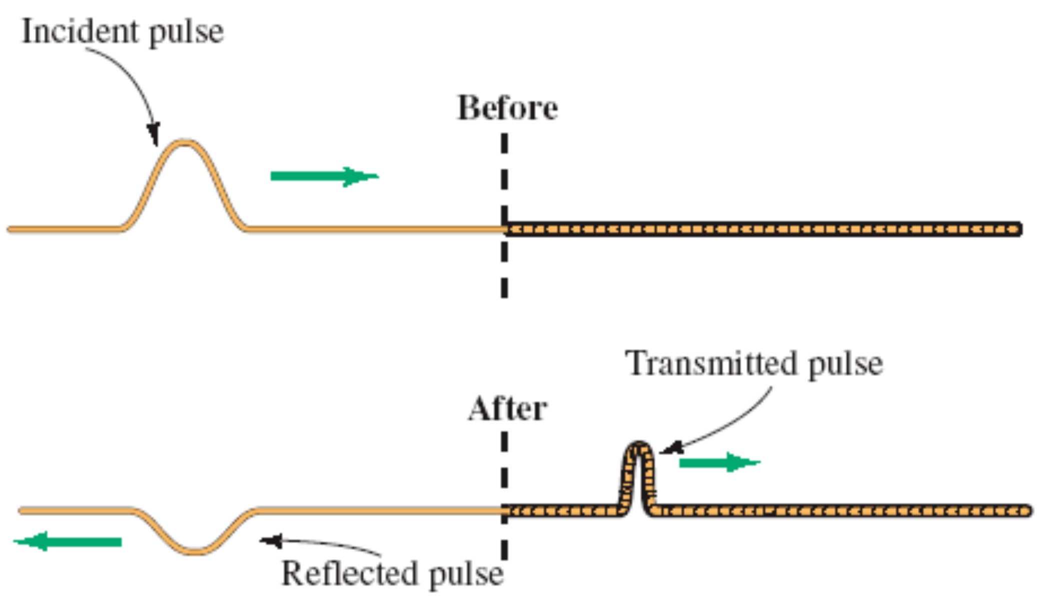
An incident pulse reflecting from a movable point will be upright.



Pulses

Transmission:

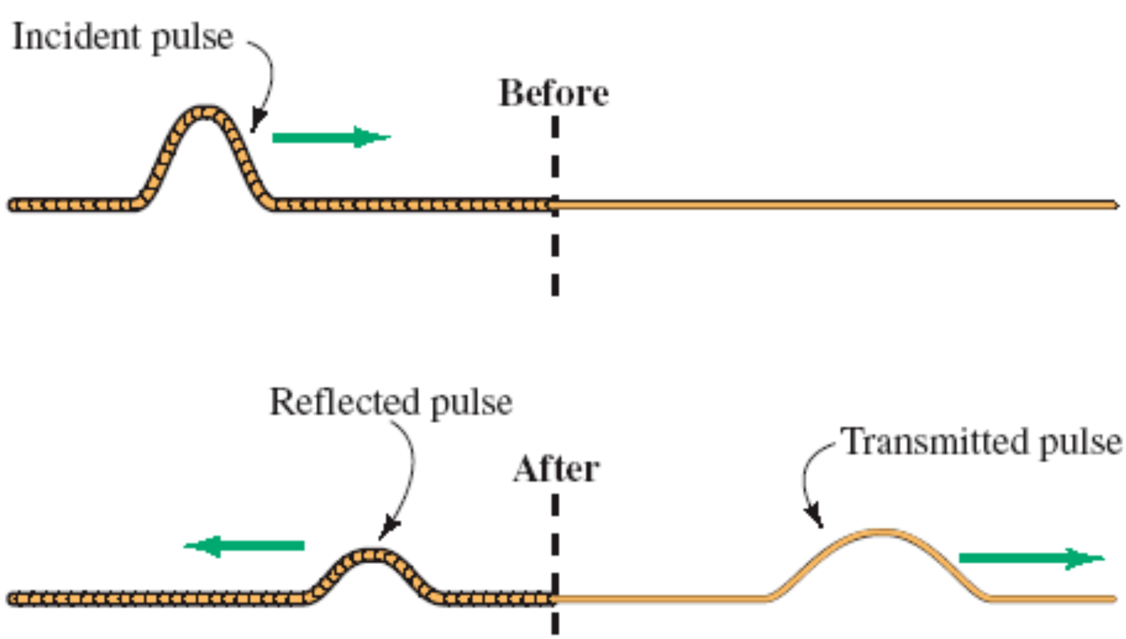
An incident pulse incident on a denser medium will have a reflection that is inverted.



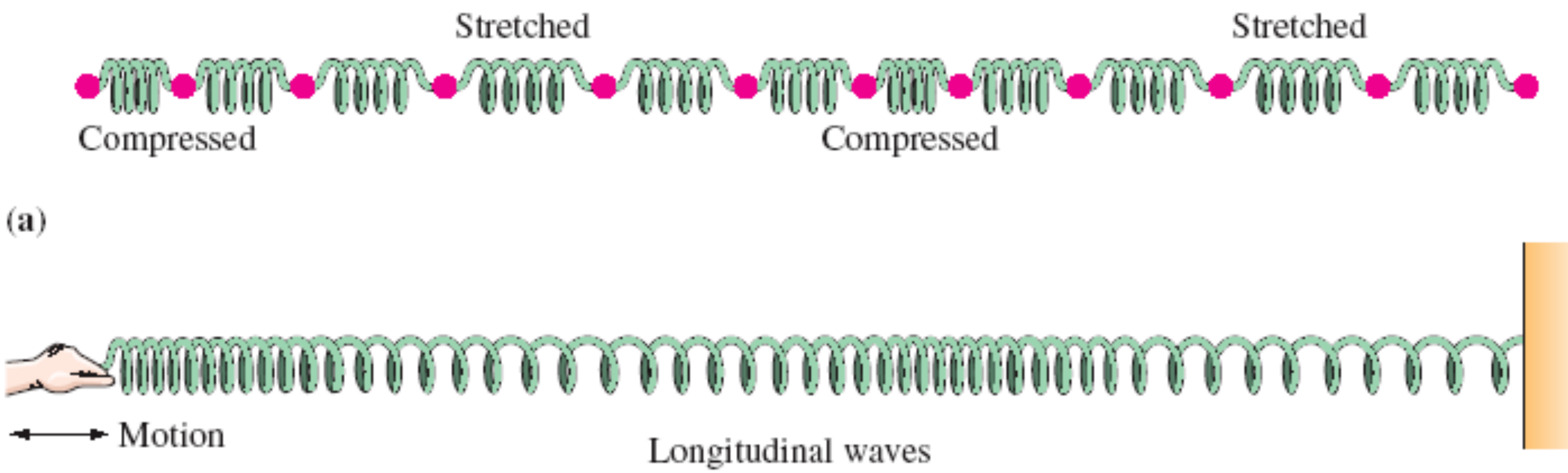
Pulses

Transmission:

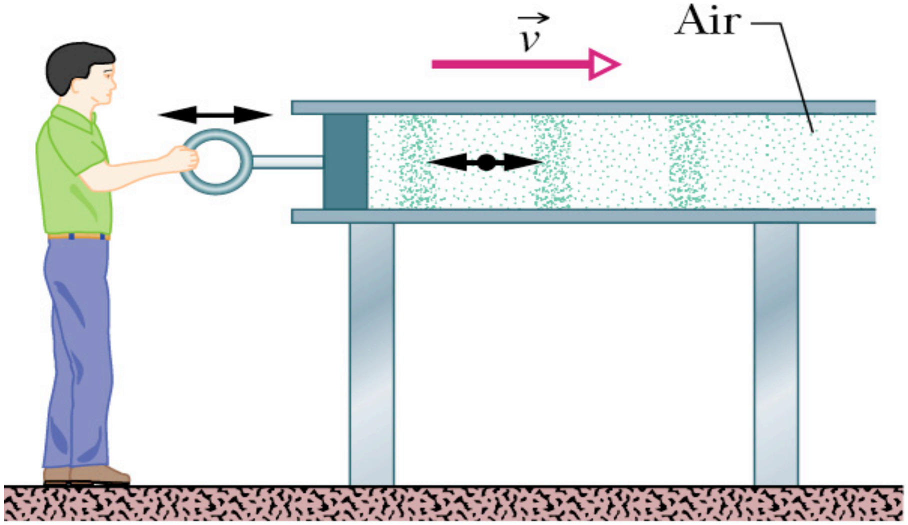
An incident pulse incident on a lighter medium will have a reflection that is upright.



Longitudinal Waves



Longitudinal Waves - Sound



Vibrations are back and forth or parallel to the velocity of the wave or direction of propagation. This is called longitudinal.

vibrations ⟷ → v

In a longitudinal sound wave propagating in a gas, the vibrations are the displacements of the air molecules or pressure or density.

Demo Longitudinal Waves: magnetic balls and slinky

Water waves are both transverse and longitudinal.
The particle in the water moves in a circular path as the wave goes by.
A floating object moves in a circular or elliptical path as the wave goes by.

Mathematical description of a Traveling wave

Traveling waves

Waves propagate from one place to another: From source to detector

Sound from an instrument to ear
Cell phone to cell tower and vice versa - E/M waves
Water waves - a disturbance in the water moves outward.

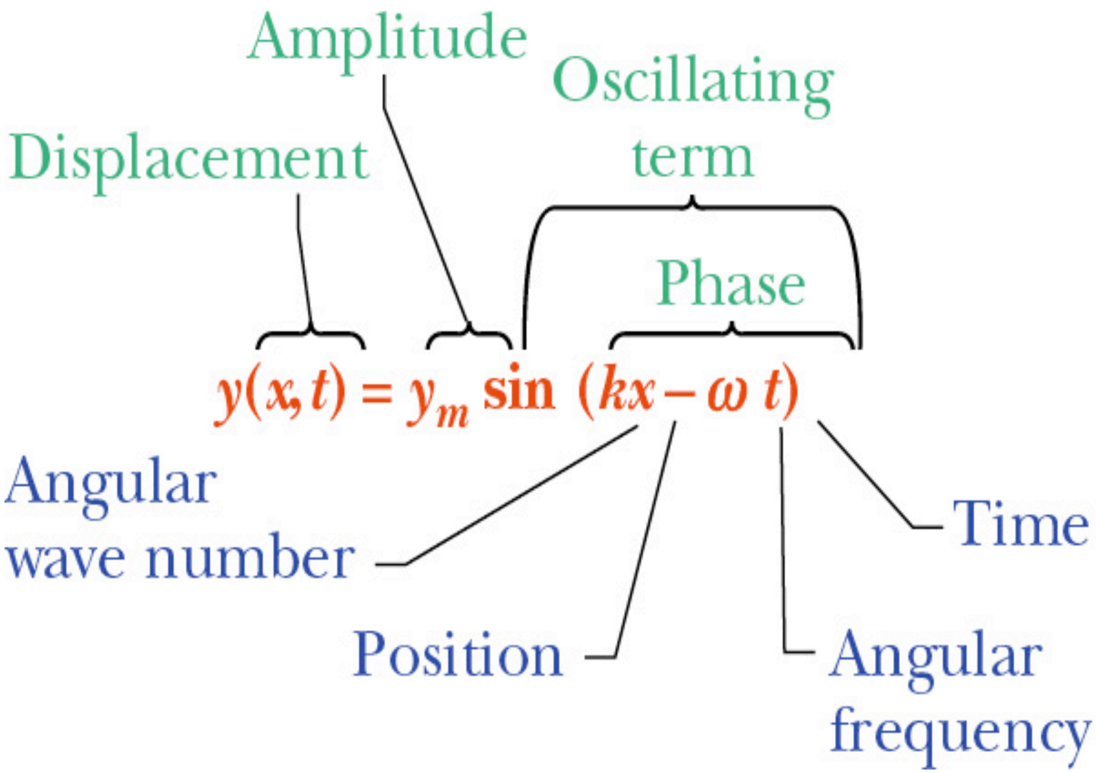
$$y(x,t) = y_m \sin(kx - \omega t)$$

A traveling wave can be represented as any function of kx-wt such that kx-wt is a constant. It can also be represented by kx+wt

$$\frac{\partial^2 y}{\partial^2 x} = \frac{1}{v^2}\frac{\partial^2 y}{\partial^2 t}$$

Satisfies the Wave Equation

Definitions: Similar to harmonic motion



y(x,t) can be any one of the following:

Density
Pressure
Particle position
Position of rope
Electric field
Magnetic field

v=speed of wave

$$v = \omega / k$$

Wave Equation

$$\frac{\partial^2 y}{\partial^2 x} = \frac{1}{v^2}\frac{\partial^2 y}{\partial^2 t}$$

A Solution

$$y(x,t) = y_m \sin(kx - \omega t)$$

$$v = \omega / k$$

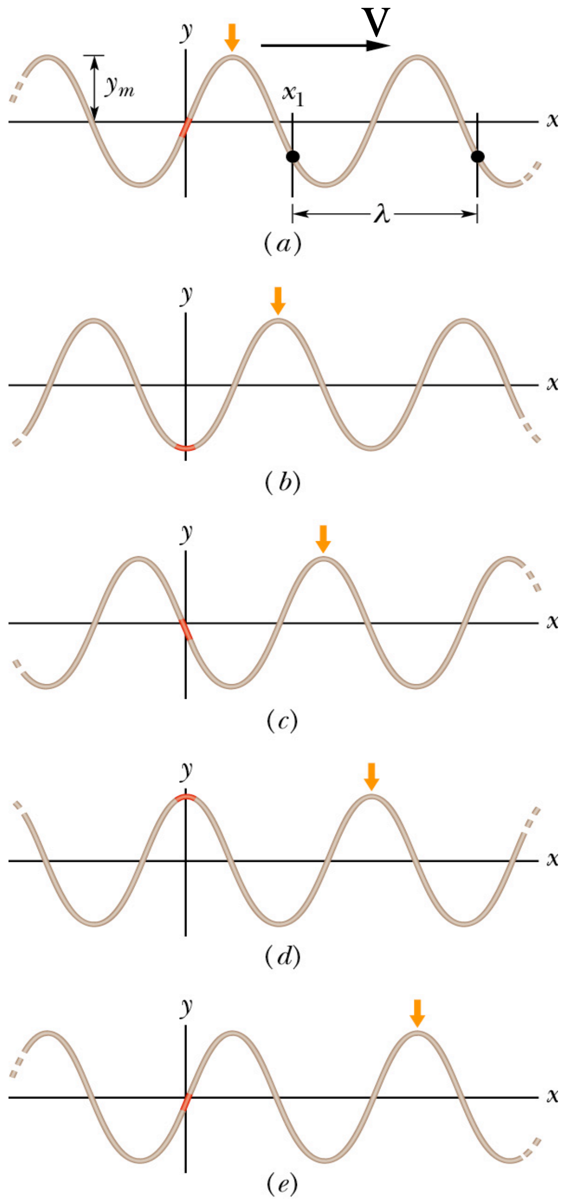
$$\frac{\partial y}{\partial x} = k y_m \cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial^2 x} = -k^2 y_m \sin(kx - \omega t)$$

$$\frac{1}{v^2}\frac{\partial y}{\partial t} = --\omega \frac{1}{v^2} y_m \cos(kx - \omega t)$$

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial^2 t} = -\omega^2 \frac{1}{v^2} y_m \sin(kx - \omega t) = -k^2 y_m \sin(kx - \omega t)$$

Traveling wave



$$y(x,t) = y_m \sin(kx - \omega t)$$

Snapshot at time t=0

$$y(x,0) = y_m \sin(kx)$$

$$= y_m \sin(kx + k\lambda)$$

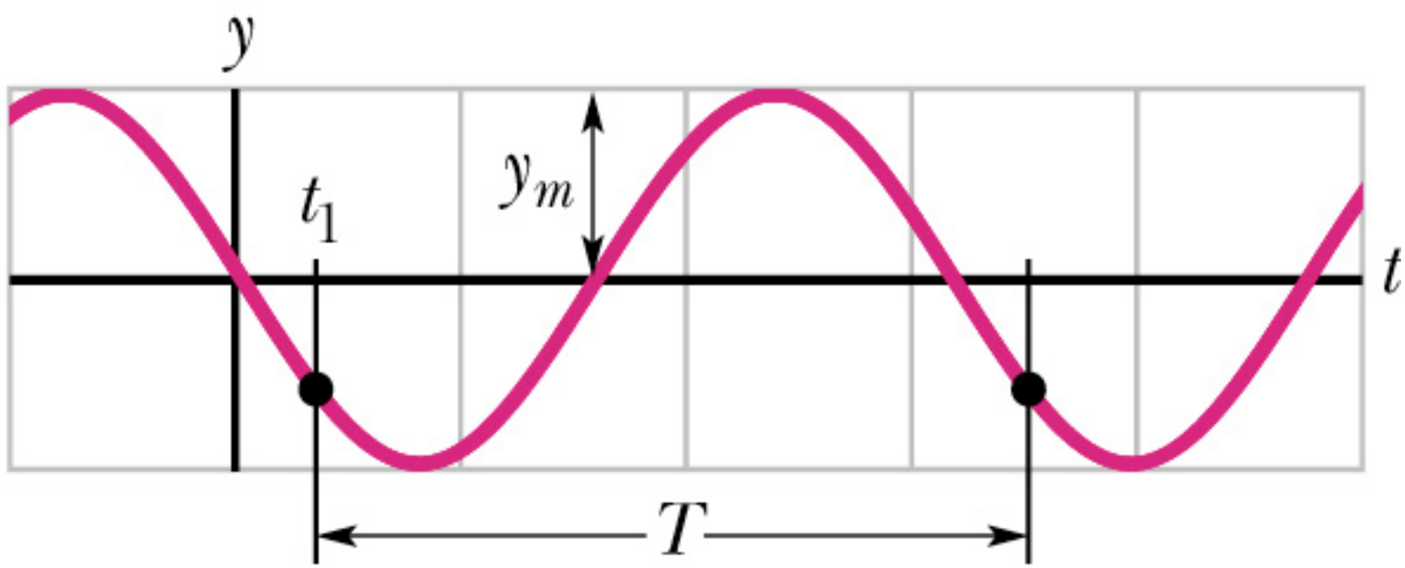
Repeats itself after 2π
or 1 wavelength

$$k\lambda = 2\pi$$

$$k = \frac{2\pi}{\lambda} = \text{angular wave number}$$

Illustrate traveling wave by mounting SHM on a cart and push it to the right with speed v.

Displacement as a function of time at x=0

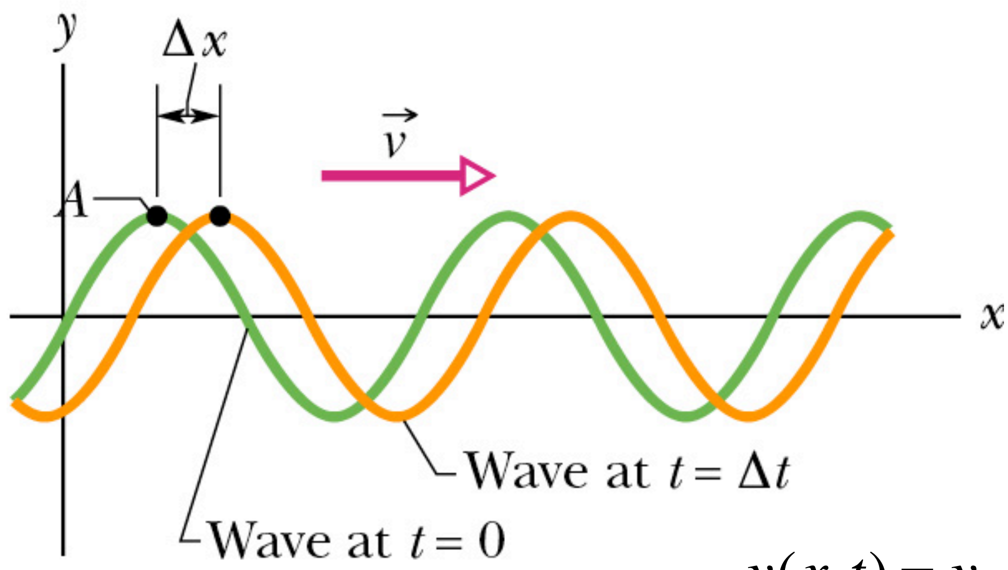


At x =0

$$y(0,t) = y_m \sin(-\omega t)$$
$$= -y_m \sin(\omega t)$$

$$= -y_m \sin(\omega t + \omega T)$$
$$\omega T = 2\pi$$
$$\omega = \frac{2\pi}{T} = \text{angular frequency}$$

Speed of Wave moving to the right



$$y(x,t) = y_m \sin(kx - \omega t)$$

Describes a wave moving towards positive x

$$y(x,t) = y_m \sin(kx + \omega t)$$

Describes a wave moving towards negative x

How can you tell which way the wave is moving?

$$v = \frac{\Delta x}{\Delta t}$$

$$kx - \omega t = \text{constant}$$

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = v$$

For traveling waves we showed that f λ = v

How the Wave Speed Depends on the Medium

$$\text{wave speed} = \sqrt{\frac{\text{restoring force factor}}{\text{mass factor}}}$$

The Speed of Sound in Air

$$v_{\text{Sound}} = \sqrt{\frac{B}{\rho}} \cong 330m / s = 740mi / h$$

Shear Waves in Solids

$$v_{\text{shear}} = \sqrt{\frac{G}{\rho}}$$

Waves on a string under Tension

$$v = \sqrt{\frac{T}{\mu}}$$

Energy transport of wave on a string

$$P_{avg} = (\frac{dK}{dt})_{avg} + (\frac{dU}{dt})_{avg}$$

$$(\frac{dK}{dt})_{avg} = (\frac{dU}{dt})_{avg}$$ for oscillating spring-like systems

$$P_{avg} = 2(\frac{dK}{dt})_{avg}$$

$$P_{avg} = \tfrac{1}{2}\mu v \omega^2 y_m^2$$

The point to remember is that the average power is proportional to the square of the amplitude and square of the frequency. This is a general result which is true for all waves.

Next slide for proof

Average Power of a traveling sine wave along a string

$$dK = \tfrac{1}{2} dm\ u^2$$

$$u = dy / dt$$

$$y = y_m \sin(kx - \omega t)$$

$$u = \omega y_m \cos(kx - \omega t)$$

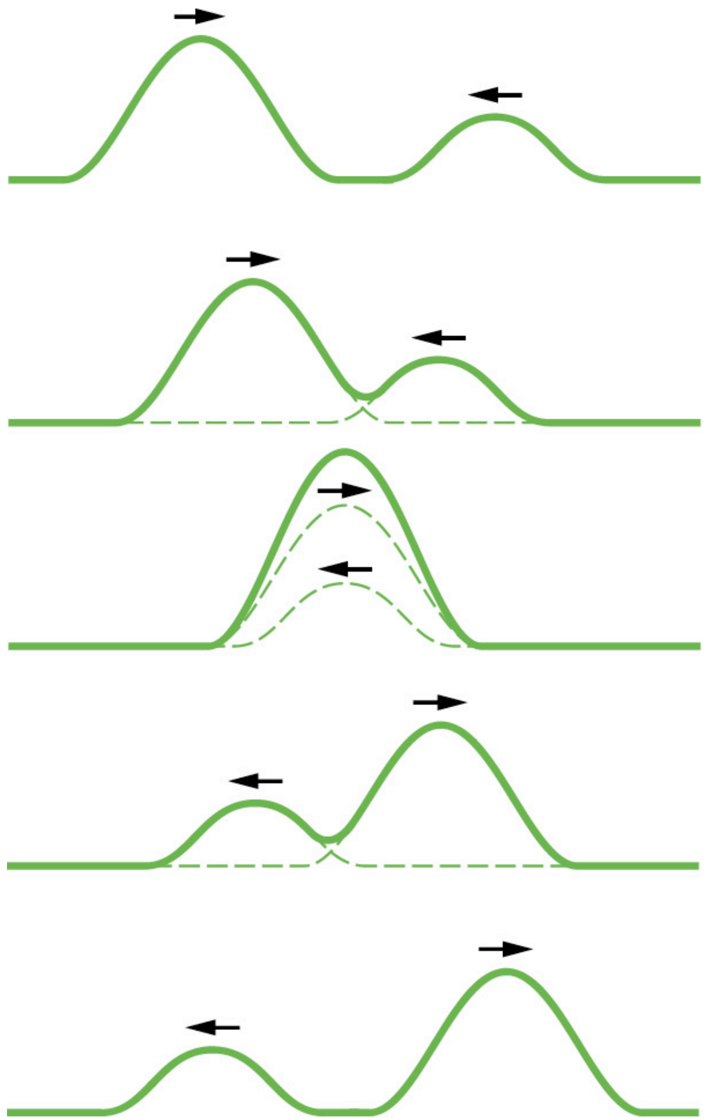
$$dK = \tfrac{1}{2} \mu dx\ (\text{-}\omega y_m)^2 \cos^2(kx - \omega t)$$

$$\left(\frac{dK}{dt}\right)_{avg} = \tfrac{1}{2} \mu \frac{dx}{dt} (\omega y_m)^2 \left[\cos^2(kx - \omega t)\right]_{avg}$$

$$P_{avg} = 2 \tfrac{1}{2} \mu v (\omega y_m)^2\ \tfrac{1}{2}$$

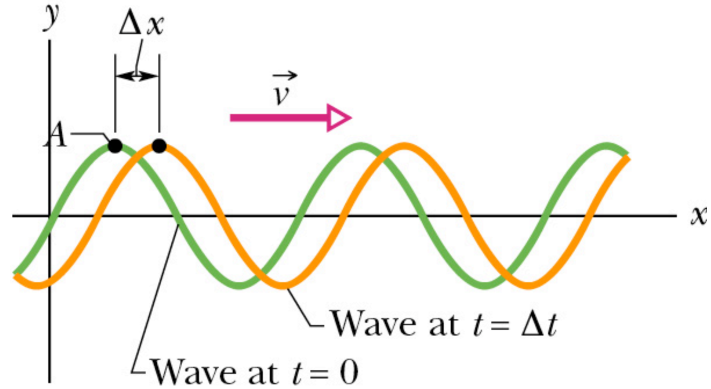
$$P_{avg} = \tfrac{1}{2} \mu v (\omega y_m)^2$$

Principle of Superposition



$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

Interference of Waves



$$y'(x,t) = y_1(x,t) + y_2(x,t)$$

$$y_m \sin(kx - \omega t) \qquad y_m \sin(kx - \omega t + \phi)$$

$$y'(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

Need to use a trigonometric identity

$$\sin\alpha + \sin\beta = 2\cos\tfrac{1}{2}(\alpha - \beta)\sin\tfrac{1}{2}(\alpha + \beta)$$

Interference of Waves

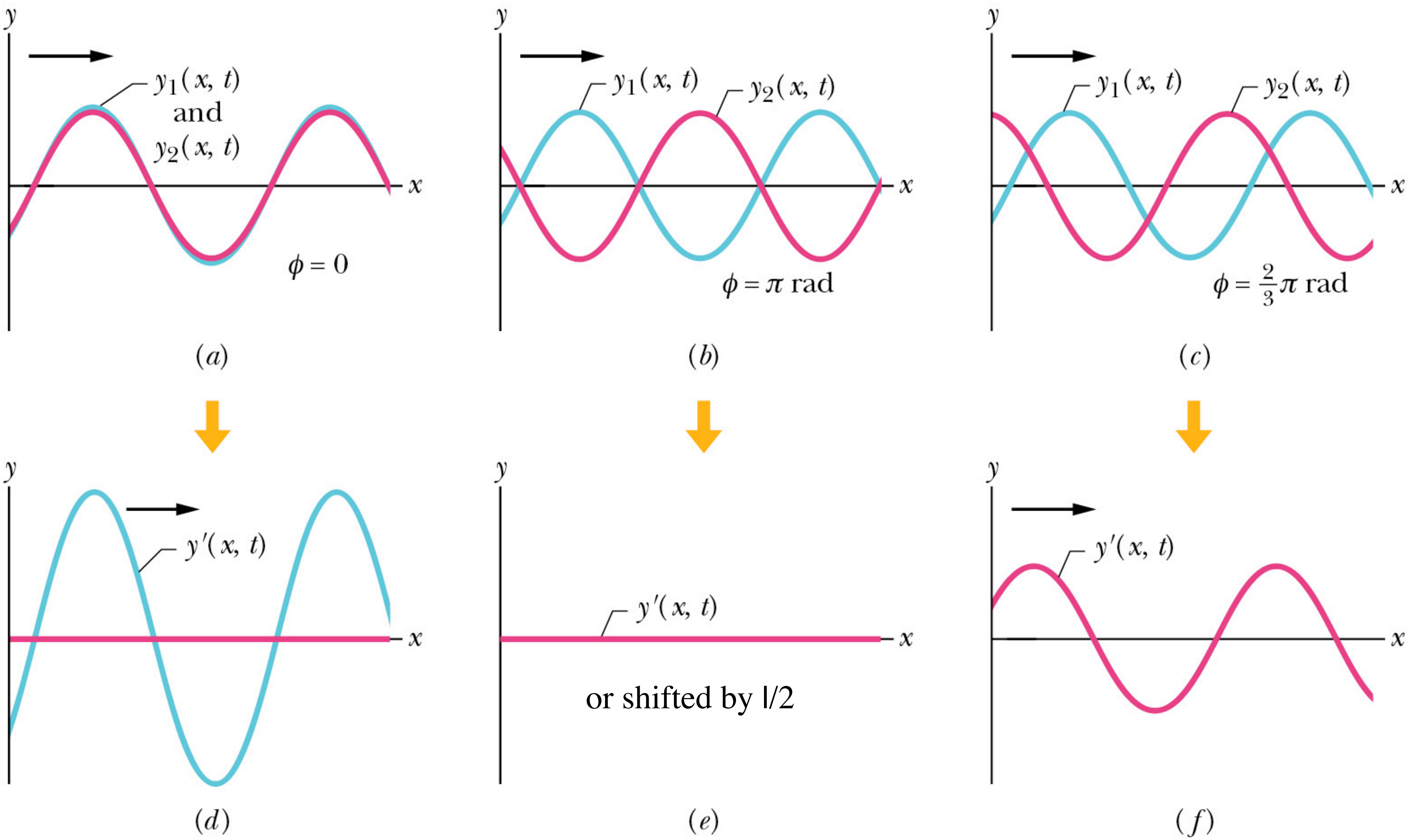
$$y'(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi)$$

$$\sin\alpha + \sin\beta = 2\cos\tfrac{1}{2}(\alpha - \beta)\sin\tfrac{1}{2}(\alpha + \beta)$$

$$\alpha = kx - \omega t \qquad\qquad \beta = kx - \omega t + \phi$$

$$y'(x,t) = 2y_m \cos\tfrac{1}{2}\phi \sin(kx - \omega t + \tfrac{1}{2}\phi)$$

$$Amplitude = 2y_m \cos\tfrac{1}{2}\phi$$



$y'(x,t) = 2y_m \sin(kx - \omega t)$

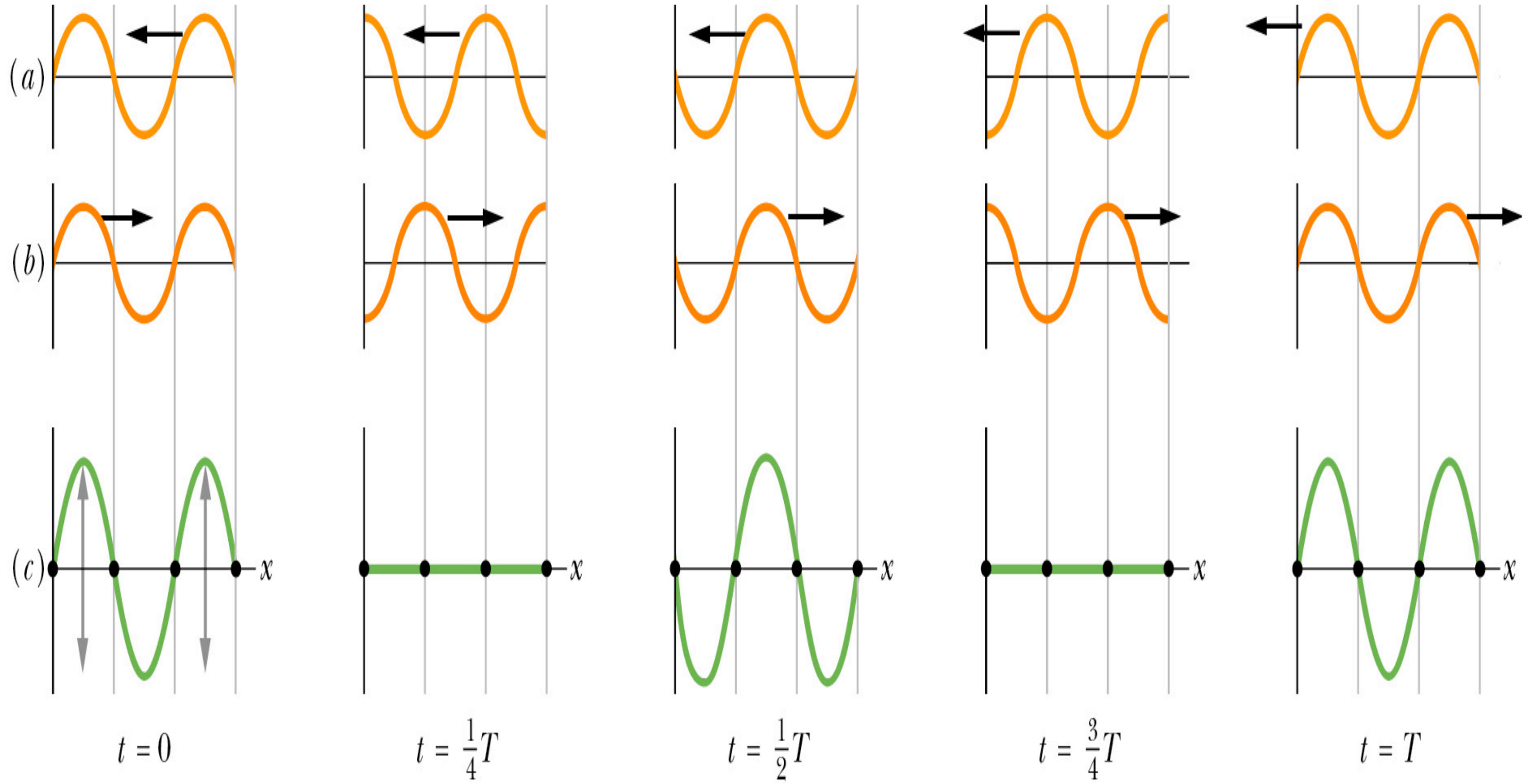
$y'(x,t) = 0$

$y'(x,t) = y_m \sin(kx - \omega t)$

$$y'(x,t) = 2y_m \cos\tfrac{1}{2}\phi \sin(kx - \omega t + \tfrac{1}{2}\phi)$$

$$Amplitude = 2y_m \cos\tfrac{1}{2}\phi$$

Standing waves

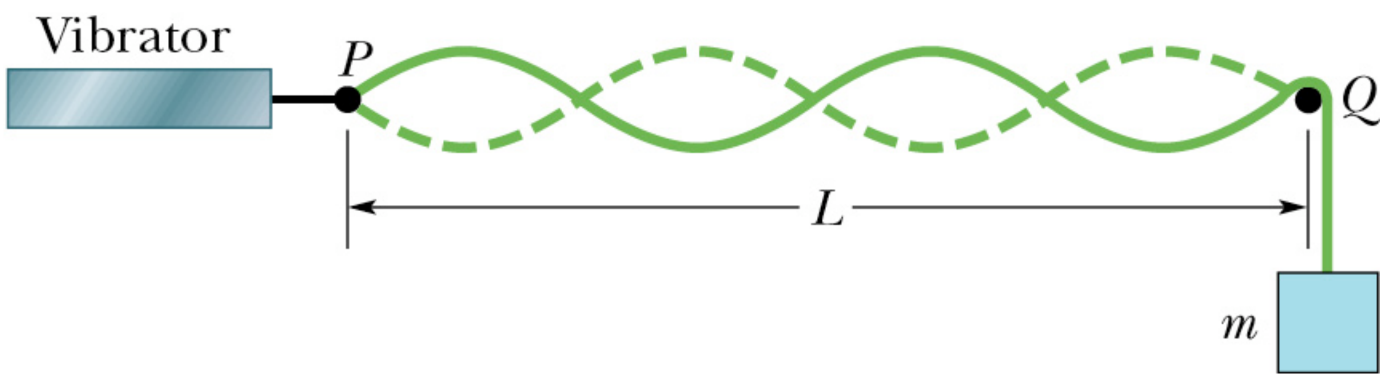


v ⟶ ⟵ v

$$y'(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

$$y'(x,t) = 2y_m \sin(kx)\cos\omega t \qquad \omega = \frac{2\pi}{T}$$

A string under tension connected to an oscillator

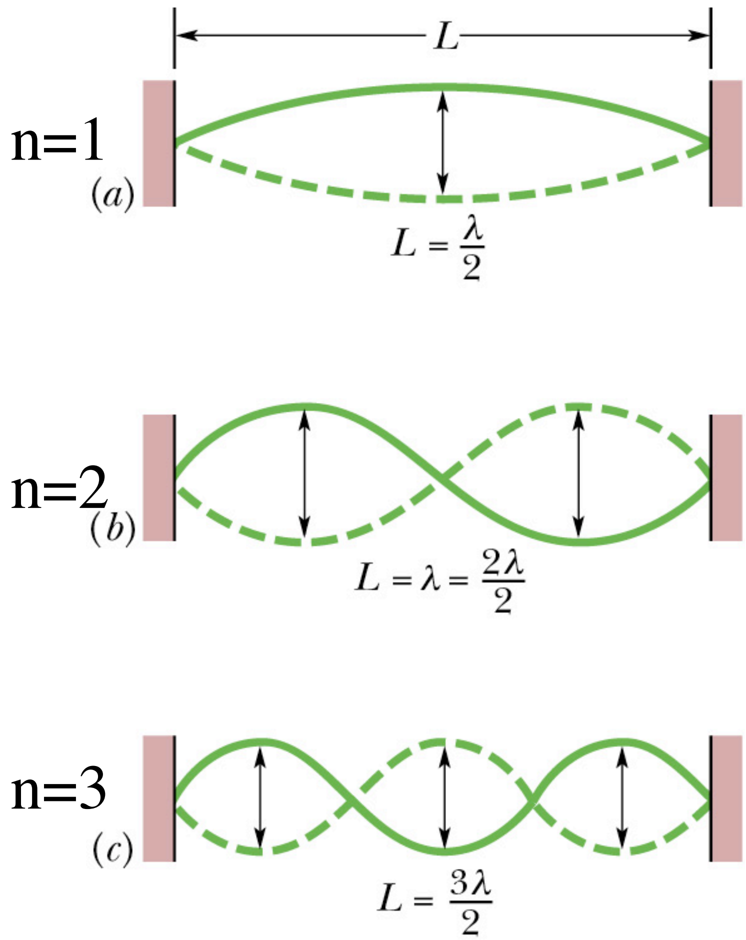


$$v = \sqrt{\frac{T}{\mu}}$$

Consider what happens if you continue to generate waves at a certain frequency.

Standing Waves and Resonance

Resonance condition: Note we must have a node at each end of the string where it is fastened to the wall. Node means y=0.



$$y'(x,t) = 2y_m \sin(kx)\cos\omega t$$

$$y(L,t) = 0$$

$$x = L$$

$$\sin(kx) = \sin(kL) = \sin(n\pi) = 0$$

$$kL = n\pi \qquad n = 1, 2, 3..$$

$$L = \frac{n\pi}{k}$$

$$k = \frac{2\pi}{\lambda}$$

$$L = \frac{n\lambda}{2} \text{ or } \lambda = \frac{2L}{n}$$

Allowed wavelengths

Resonant frequencies

$$\lambda = \frac{2L}{n}$$

$$f = v / \lambda = n \frac{v}{2L}$$

$$n = 1, 2, 3..$$

$$f_1 = \frac{v}{2L}$$

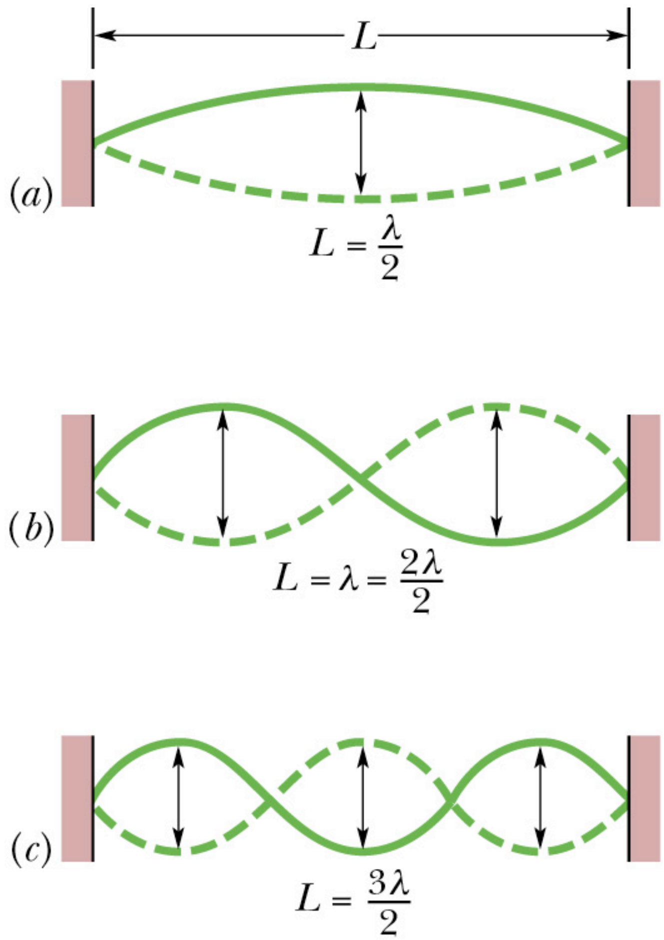
$$f_2 = \frac{2v}{2L}$$

$$f_3 = \frac{3v}{2L}$$

$$f_4 = ...$$

Standing Waves and Resonance

$$y'(x,t) = 2y_m \sin(kx)\cos\omega t$$



Resonant Frequencies

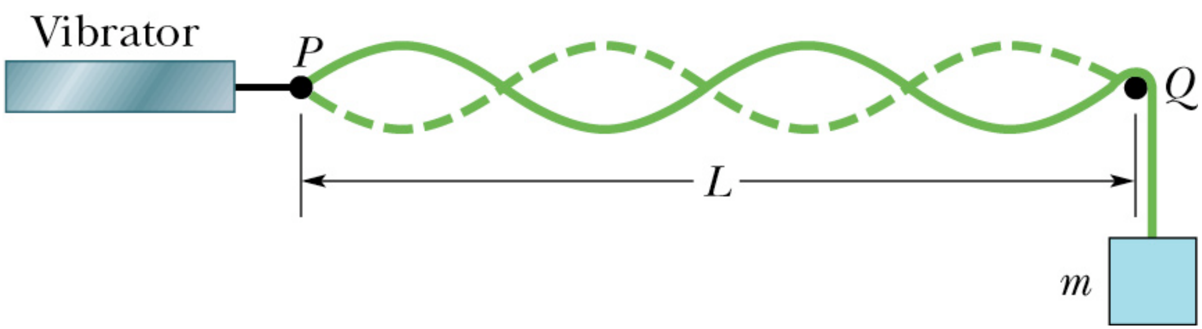
Fundamental or first harmonic $f_1 = \frac{v}{2L}$

Second harmonic $f_2 = \frac{2v}{2L}$

Third harmonic $f_3 = \frac{3v}{2L}$

$$f_n = \frac{nv}{2L}, n = 4, 5, 6...$$

A string under tension connected to an oscillator continued : Demo



$$v = \sqrt{\frac{T}{\mu}} \qquad v = f\lambda$$

$$L = 3m$$

$$m = 1kg$$

$$T = mg = (1kg)(10\ m/s^2) = 10N$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\mu = \frac{0.07kg}{3m} = 0.023\ \text{kg/m}$$

$$v = \sqrt{\frac{10N}{0.023\ \text{kg/m}}} = 20.85\ \text{m/s}$$

Prob 6 Chpt 16

The equation of a transverse wave traveling along a very long string is

$$y = 6.0\sin(0.020\pi x + 4.0\pi t)$$

where x and y are expressed in centimeters and t in seconds.
Determine:
(a) the amplitude,
(b) the wavelength,
©the frequency,
(d) The speed,
(e) the direction of propagation of the wave,
(f) the maximum transverse speed of a particle in the string,
(g) What is the transverse displacement at x=3.5 cm, when t=0.26 s.

Prob 48 Chpt 16

A rope under tension of 200 Newtons and fixed at both ends, oscillates in a second harmonic standing wave pattern. The displacement of the rope is given by

$$y = (0.10m)(\sin \pi x / 2)(\sin 12\pi t)$$

Where x=0 at one end of the rope, x is in meters, and t is in seconds. What are:

(a) the length of the rope,

(b) the speed of the waves on the rope,

(c) the mass of the rope?

(d) if the rope oscillates in a third Harmonic standing wave pattern, what will be the period of oscillation?

ConcepTest 14.2 The Wave

At a football game, the "wave" might circulate through the stands and move around the stadium. In this wave motion, people stand up and sit down as the wave passes. What type of wave would this be characterized as?

1) **polarized wave**
2) **longitudinal wave**
3) **lateral wave**
4) **transverse wave**
5) **soliton wave**

ConcepTest 14.3a Wave Motion I

Consider a wave on a string moving to the right, as shown below.

What is the direction of the velocity of a particle at the point labeled A ?

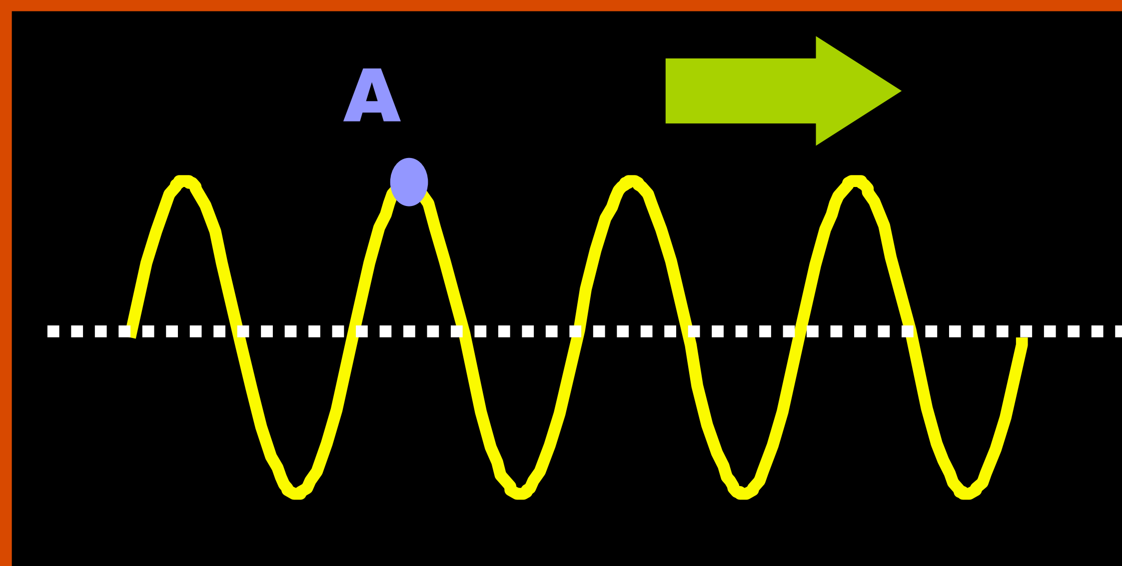
1) →

2) ↘

3) ↓

4) ↑

5) zero



ConcepTest 14.6a Wave Speed I

A wave pulse can be sent down a rope by jerking sharply on the free end. If the tension of the rope is increased, how will that affect the speed of the wave?

1) **speed increases**
2) **speed does not change**
3) **speed decreases**

ConcepTest 14.7a Sound Bite I

When a sound wave passes from air into water, what properties of the wave will change?

1) the frequency f
2) the wavelength l
3) the speed of the wave
4) both f and l
5) both v_{wave} and l

ConcepTest 14.8b Speed of Sound II

Do you expect an echo to return to you more quickly or less quickly on a hot day, as compared to a cold day?

1) **more quickly on a hot day**
2) **equal times on both days**
3) **more quickly on a cold day**

ConcepTest 14.12a Pied Piper I

You have a **long pipe** and a **short pipe**. Which one has the **higher frequency**?

(1) the long pipe

(2) the short pipe

(3) both have the same frequency

(4) depends on the speed of sound in the pipe

ConcepTest 14.2 The Wave

At a football game, the "wave" might circulate through the stands and move around the stadium. In this wave motion, people stand up and sit down as the wave passes. What type of wave would this be characterized as?

1) polarized wave
2) longitudinal wave
3) lateral wave
4) transverse wave
5) soliton wave

The people are moving up and down, and the wave is traveling around the stadium. Thus, the motion of the wave is perpendicular to the oscillation direction of the people, and so this is a transverse wave.

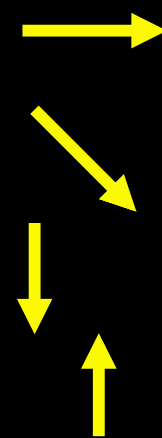
Follow-up: What type of wave occurs when you toss a pebble in a pond?

ConcepTest 14.3a Wave Motion I

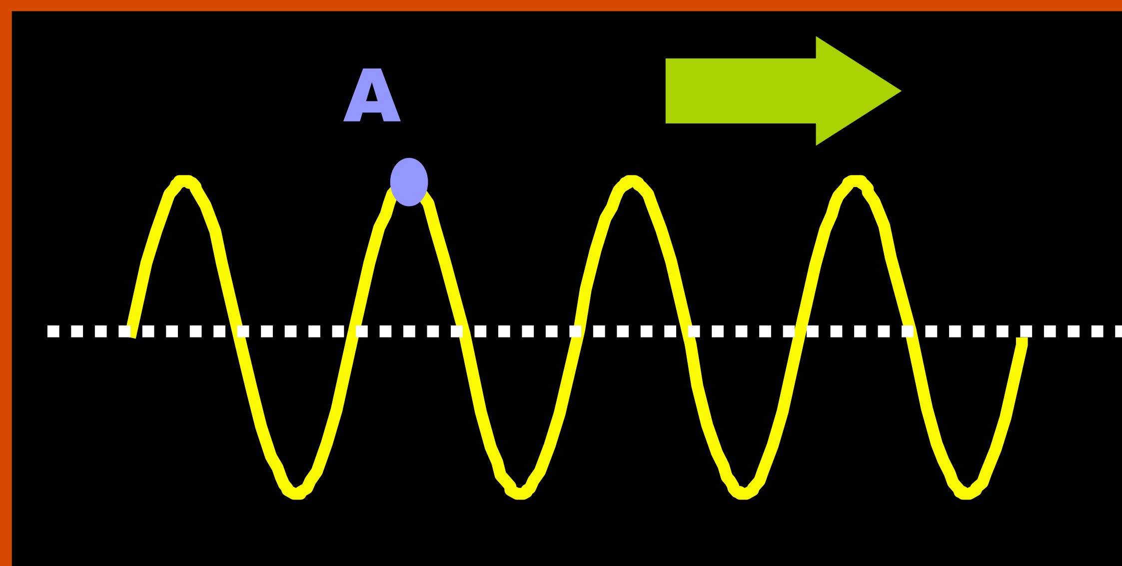
Consider a wave on a string moving to the right, as shown below.

What is the direction of the velocity of a particle at the point labeled A ?

1)
2)
3)
4)
5) zero



The velocity of an oscillating particle is (momentarily) *zero* at its maximum displacement.



Follow-up: What is the acceleration of the particle at point A?

ConcepTest 14.6a Wave Speed I

A wave pulse can be sent down a rope by jerking sharply on the free end. If the tension of the rope is increased, how will that affect the speed of the wave?

1) speed increases
2) speed does not change
3) speed decreases

The wave speed depends on the square root of the tension, so if the tension increases, then the wave speed will also increase.

ConcepTest 14.7a Sound Bite I

When a sound wave passes from air into water, what properties of the wave will change?

1) **the frequency *f***
2) **the wavelength l**
3) **the speed of the wave**
4) **both *f* and l**
5) **both v_{wave} and l**

Wave speed must change (different medium).
Frequency does not change (determined by the source).
Now, $v = f$l and since v has changed and f is constant then l must also change.

Follow-up: Does the wave speed increase or decrease in water?

ConcepTest 14.8b Speed of Sound II

Do you expect an echo to return to you more quickly or less quickly on a hot day, as compared to a cold day?

1) **more quickly on a hot day**
2) **equal times on both days**
3) **more quickly on a cold day**

The speed of sound in a gas increases with temperature. This is because the molecules are bumping into each other faster and more often, so it is easier to propagate the compression wave (sound wave).

ConcepTest 14.12a Pied Piper I

You have a long pipe and a short pipe. Which one has the higher frequency?

(1) the long pipe

(2) the short pipe

(3) both have the same frequency

(4) depends on the speed of sound in the pipe

A shorter pipe means that the standing wave in the pipe would have a shorter wavelength. Since the wave speed remains the same, the frequency has to be higher in the short pipe.