Power Systems Analysis
ET-321

References

- **Elements of power system analysis**
  (William Stevenson)

- **Power system analysis**
  (Hadi Sadaat)

POWER SYSTEM STABILITY

Steady State and Transient Stability

- Power system stability has been recognized as an important problem for secure system operation

- Historically, transient instability has been the dominant stability problem on most systems, and has been the focus of much of the industry's attention concerning system stability

- As power systems have evolved through continuing growth in interconnections, use of new technologies and controls, and the increased operation in highly stressed conditions, different forms of system instability have emerged

- For the satisfactory design and operation of power systems, a clear understanding of different types of instability and relationship between them is necessary

Steady State and Transient Stability

- Therefore, there is a need for the proper definition and classification of power system stability

- **Definition of Power System Stability:** Power system stability is the ability of an electric power system, for a given initial operating condition, to either regain a new state of operating equilibrium or return to the original operating condition (if no topological changes occurred in the system) after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact

Steady State and Transient Stability

- **Small Disturbances:** Small disturbances are incremental changes in the system load or generation. Small disturbances occur continually and the system adjusts itself to the changing conditions. The system must be able to operate satisfactorily under these conditions and successfully supply the maximum amount of load.

- **Large Disturbances:** Large disturbances are the disturbances of a severe nature, such as a short circuit on a transmission line or loss of a large generator. A large disturbance may lead to structural changes due to the isolation of the faulted elements. It must also be capable of surviving numerous disturbances of a severe nature, such as a short-circuit on a transmission line, loss of a large generator or load or loss of a tie between two subsystems.

Classification of Power System Stability

- Power system stability can be classified into different categories and subcategories (Figure 2.1)

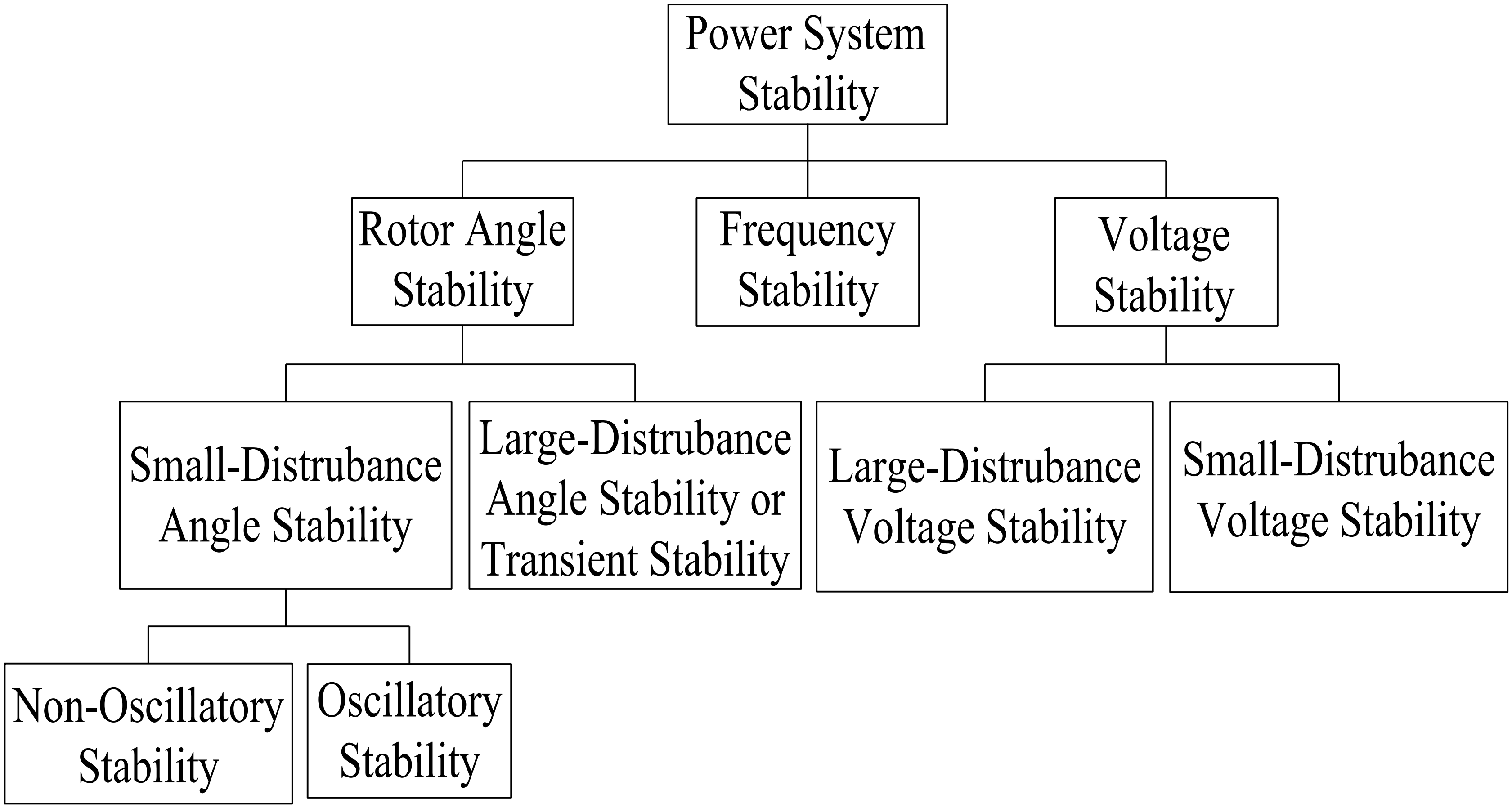


Figure 2.1 Classification of power system stability

Classification of Power System Stability

Rotor Angle Stability

- Rotor angle stability refers to the ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance.

- It depends on the ability to maintain/restore equilibrium between electromagnetic torque (generator output) and mechanical torque (generator input) of each synchronous machine in the system.

- Instability that may result occurs in the form of increasing angular swings of some generators leading to their loss of synchronism with other generators.

Rotor Angle Stability

- The change in electromagnetic torque (ΔT_e) of a synchronous machine following a perturbation can be resolved into two components: (i) *Synchronizing torque component*, in phase with rotor angle deviation ($\Delta\delta$), and (ii) *Damping torque component*, in phase with the speed deviation ($\Delta\omega$). Mathematically, this can be expressed as follows:

$$\Delta T_{\mathrm{e}} = T_{\mathrm{s}}\Delta\delta + T_{\mathrm{D}}\Delta\omega$$

where

$T_s\Delta\delta$ is the synchronizing torque component of torque change. T_s is the synchronizing torque coefficient.

$T_D\Delta\omega$ is the damping torque component of torque change. T_D is the damping torque coefficient.

Classification of Power System Stability

Rotor Angle Stability

- Stability depends on the existence of both components of torque for each of the synchronous machines.

- **Aperiodic or Nonoscillatory Instability :** Lack of sufficient synchronizing torque causes an increase in rotor angle through a nonoscillatory or aperiodic mode. This form of instability is known as *aperiodic* or *nonoscillator*y *instability*.

- **Oscillatory Instability :** Lack of damping torque causes rotor oscillations of increasing amplitude. This form of instability is known as *oscillatory instability*.

Classification of Power System Stability

Rotor Angle Stability

- Rotor angle stability can be classified into the following two subcategories:

  (1) Small-disturbance (or small-signal) or steady-state rotor angle stability
  (2) Large-disturbance rotor angle stability or transient stability

Small-Disturbance or Steady-State Rotor Angle Stability

- Small-disturbance (or small-signal) or steady-state rotor angle stability is concerned with the ability of the power system to maintain synchronism under small and slow disturbances, such as gradual power changes.

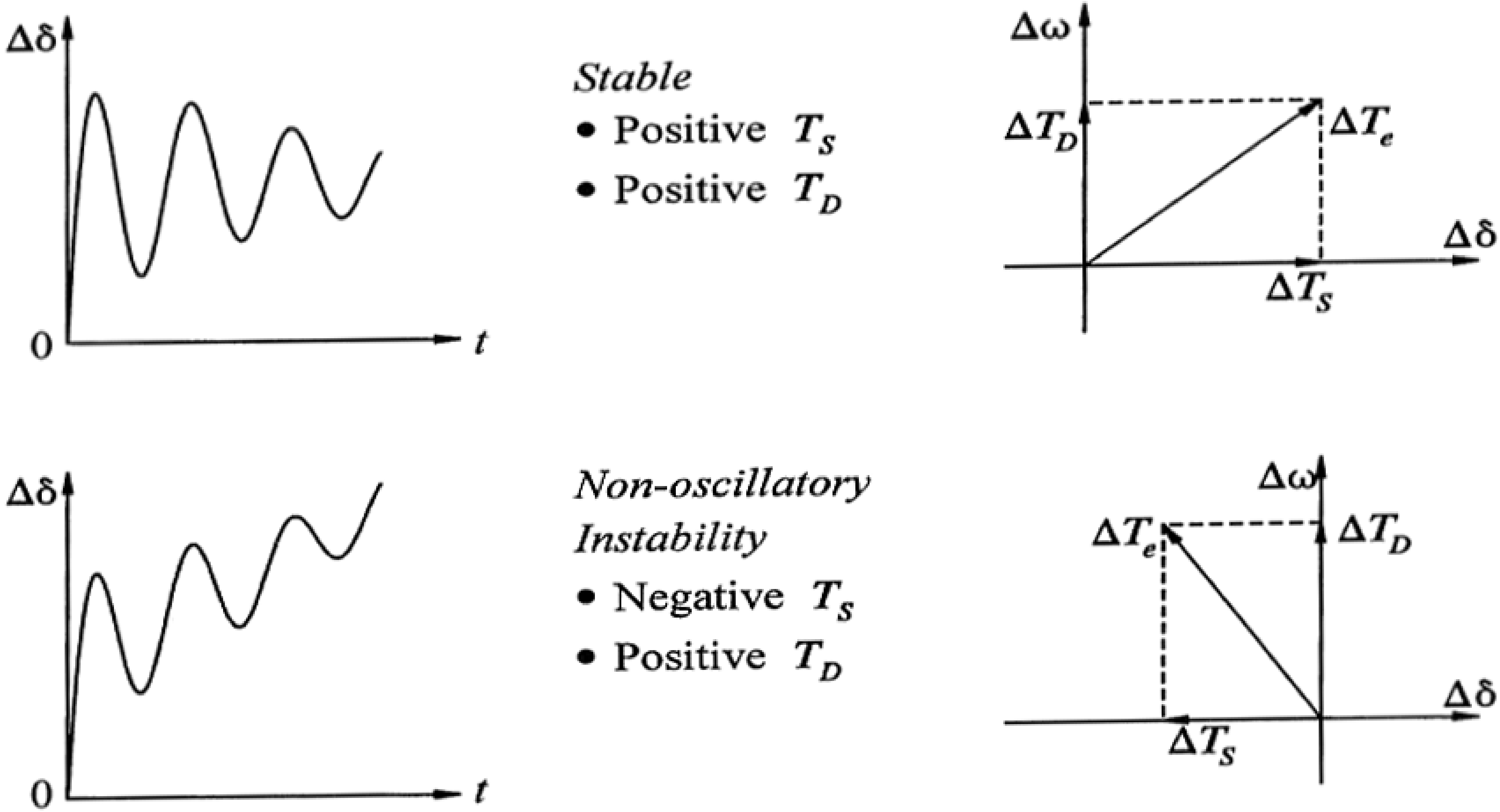
Classification of Power System Stability

Small-Disturbance or Steady-State Rotor Angle Stability

- The results of the system response to small disturbances are usually given in terms of eigenvalues and eigenvectors.

equations to be permissible for purposes of analysis. Instability that may result can be of two forms: (i) steady increase in rotor angle due to lack of sufficient synchronizing torque, or (ii) rotor oscillations of increasing amplitude due to lack of sufficient damping torque. The nature of system response to small disturbances depends on a number of factors including the initial operating, the transmission system strength, and the type of generator excitation controls used. For a generator connected radially to a large power system, in the absence of automatic voltage regulators (i.e., with constant field voltage) the instability is due to lack of sufficient synchronizing torque. This results in instability through a non-oscillatory mode, as shown in Figure 2.2(a). With continuously acting voltage regulators, the small-disturbance stability problem is one of ensuring sufficient damping of system oscillations. Instability is normally through oscillations of increasing amplitude. Figure 2.2(b) illustrates the nature of generator response with automatic voltage regulators.

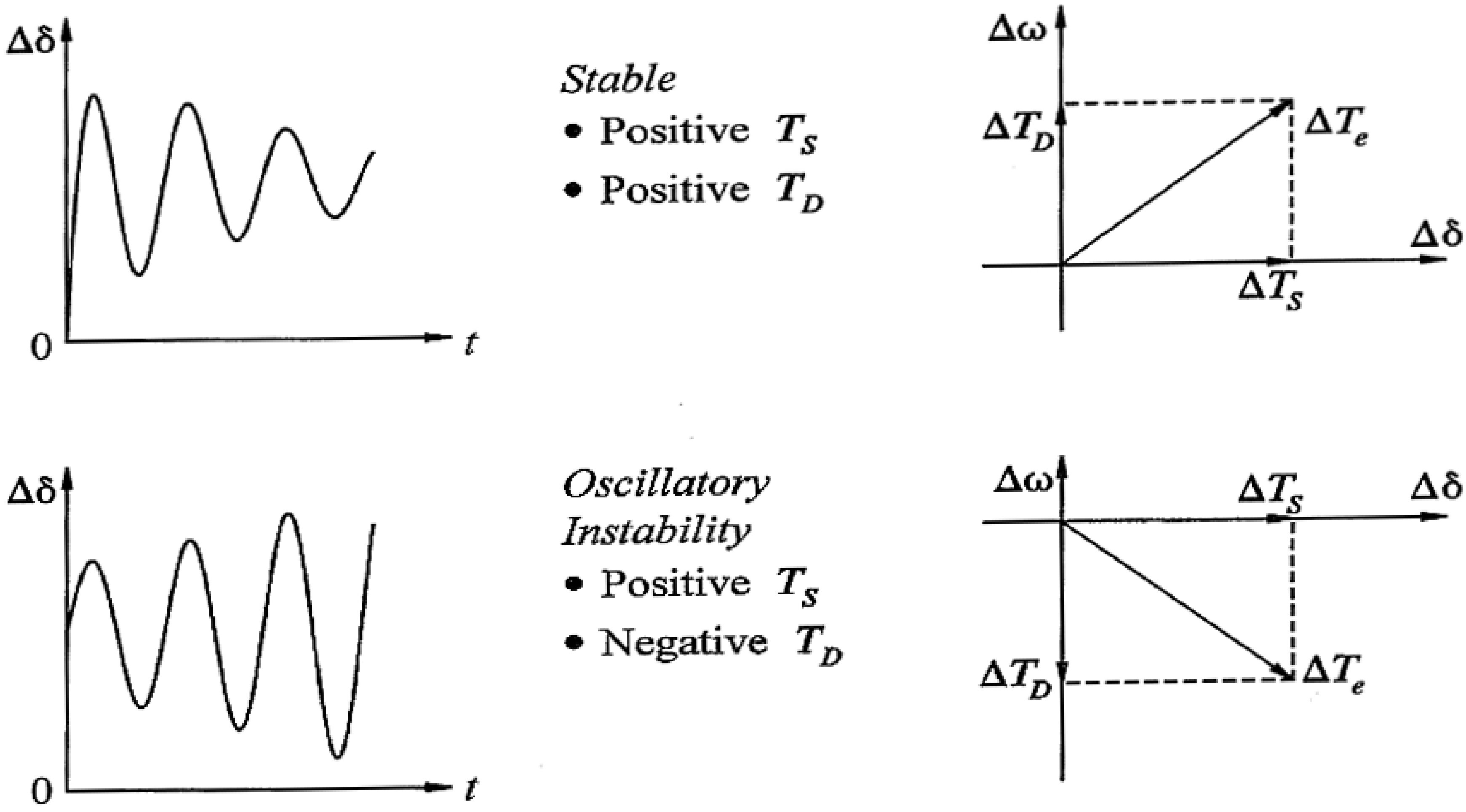
Classification of Power System Stability



(a) With constant field voltage

Figure 2.2 Nature of small-disturbance response

Classification of Power System Stability



(b) With excitation control

Figure 2.2 Nature of small-disturbance response

Classification of Power System Stability

Small-Disturbance Rotor Angle Stability

- In today's power systems, small-disturbance rotor angle stability problem is usually associated with insufficient damping of oscillations

- Small-disturbance rotor angle stability problems may be either local or global in nature. The descriptions of these problems are given below:

(i) Local problems: involve a small part of the power system, and are usually associated with rotor angle oscillations (swinging) of a single power plant (units at a generating station) against the rest of the power system. Such oscillations are called *local plant mode oscillations*.

Classification of Power System Stability

Small-Disturbance Rotor Angle Stability

(i) Local problems: The term local is used because the oscillations are localized at one station or a small part of the power system. Stability (damping) of these oscillations depends on the strength of the transmission system as seen by the power plant, generator excitation control systems and plant output [1]. When a generator is tied to a power system via a long radial line, it is susceptible to local mode oscillations

Classification of Power System Stability

Small-Disturbance Rotor Angle Stability

(ii) Global problems: Global problems are caused by interactions among large groups of generators. They are associated with rotor angle oscillations (swinging) of a group of generators in one area of an interconnected power system against a group of generators in another area. Such oscillations are called *inter-area mode oscillations*.

•The time frame of interest in small-disturbance stability studies is on the order of 10 to 20 seconds following a disturbance

Classification of Power System Stability

Large-Disturbance Rotor Angle Stability OR Transient Stability

- Large-disturbance rotor angle stability or transient stability is concerned with the ability of the power system to maintain synchronism when subjected to a severe disturbance, such as a short circuit on a transmission line, the sudden outage of a line or the sudden application or removal of loads. The resulting system response involves large excursions of generator rotor angles and is influenced by the nonlinear power-angle relationship.

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Classification of Power System Stability

Large-Disturbance Rotor Angle Stability OR Transient Stability

- Transient stability depends on both the initial operating state of the system and the severity of the disturbance. Usually, the system is altered so that the post-disturbance steady-state operation differs from that prior to the disturbance. Instability is usually in the form of aperiodic angular separation due to insufficient synchronizing torque, manifesting as *first swing instability*. However, in large power systems, transient instability may not always occur as first swing instability associated with a single mode; it could be a result of superposition of a slow inter-area swing mode and a local-plant swing mode causing a large excursion of rotor angle beyond the first swing

Classification of Power System Stability

Large-Disturbance Rotor Angle Stability OR Transient Stability

Figure 2.3 illustrates the behaviour of a synchronous machine for stable and unstable situations. It shows the rotor angle responses for a stable case and for two unstable cases. In the stable case (Case 1), the rotor angle increases to a maximum, then decreases and oscillates with decreasing amplitude until it reaches a steady state. In Case 2, the rotor angle continues to increase steadily until synchronism is lost. This form of instability is referred to as *first-swing* instability and is caused by insufficient synchronizing torque. In Case 3, the system is stable in the first swing but becomes unstable as a result of growing oscillations as the end state is approached. This form of instability generally occurs when the postfault steady-state condition itself is "small-signal" unstable, and not necessarily as a result of the transient disturbance.

Classification of Power System Stability

Large-Disturbance Rotor Angle Stability
OR Transient Stability

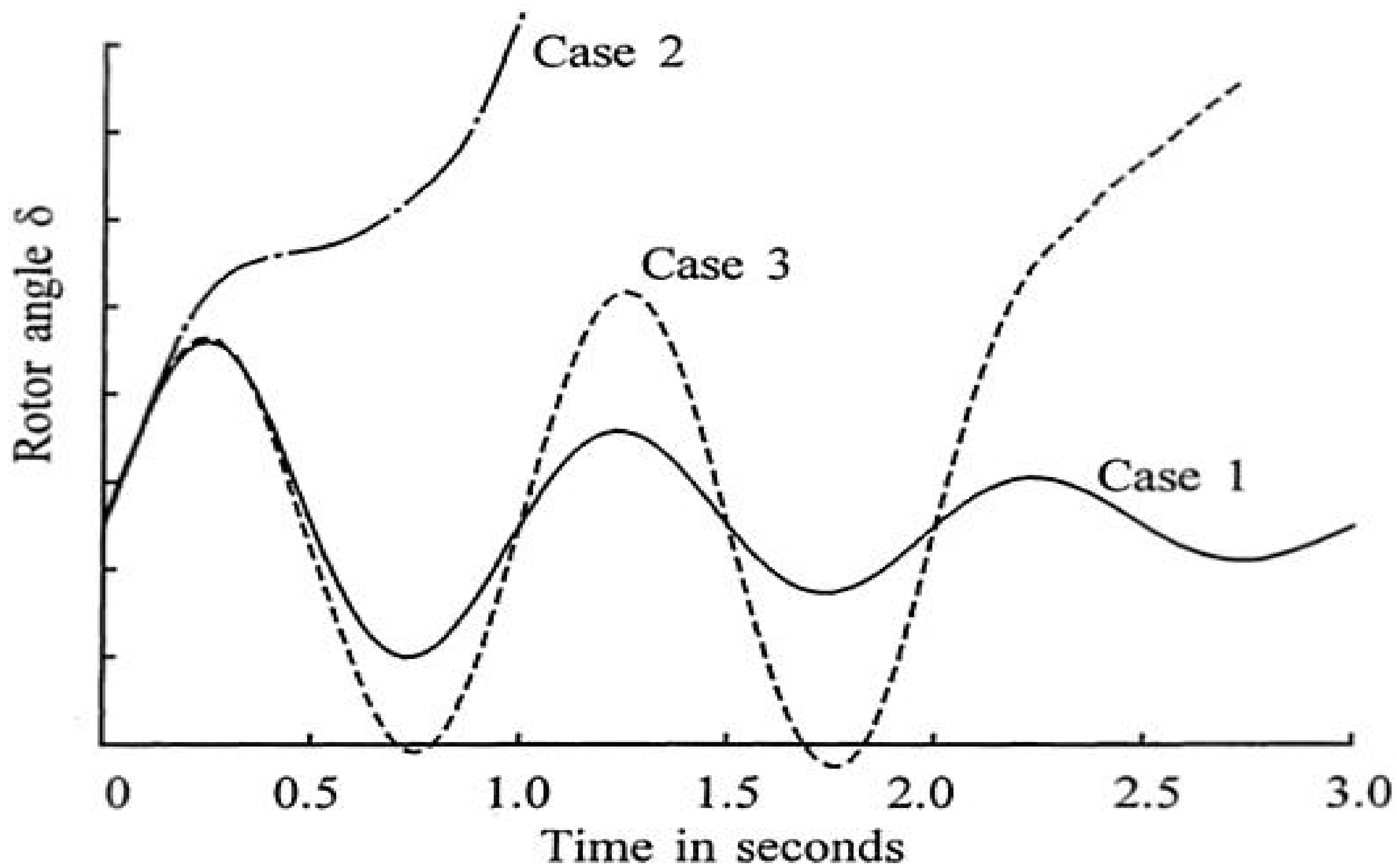

Figure 2.3 Rotor angle response to a transient disturbance

Classification of Power System Stability

Large-Disturbance Rotor Angle Stability
OR Transient Stability

- The time frame of interest in transient stability studies is usually 3 to 5 seconds following the disturbance. It may extend to 10–20 seconds for very large systems with dominant inter-area swings

- Small-disturbance rotor angle stability as well as transient stability is categorized as short term phenomena.

Swing Equation

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as the *power angle* or *torque angle*. During any disturbance, rotor will decelerate or accelerate with respect to the synchronously rotating air gap mmf, and a relative motion begins. The equation describing this relative motion is known as the *swing equation*. If, after this oscillatory period, the rotor locks back into synchronous speed, the generator will maintain its stability. If the disturbance does not involve any net change in power, the rotor returns to its original position. If the disturbance is created by a change in generation, load, or in network conditions, the rotor comes to a new operating power angle relative to the synchronously revolving field.

In order to understand the significance of the power angle we refer to the combined phasor/vector diagram of a two-pole cylindrical rotor generator illustrated in Figure 3.2. From this figure we see that the power angle δ_r is the angle between the rotor mmf F_r and the resultant air gap mmf F_{sr}, both rotating at synchronous speed (also see p-3(8)). It is also the angle between the no-load generated emf E and the resultant stator voltage E_{sr}. If the generator armature resistance and leakage flux are neglected, the angle between E and the terminal voltage V, denoted by δ, is considered as the power angle.

Swing Equation

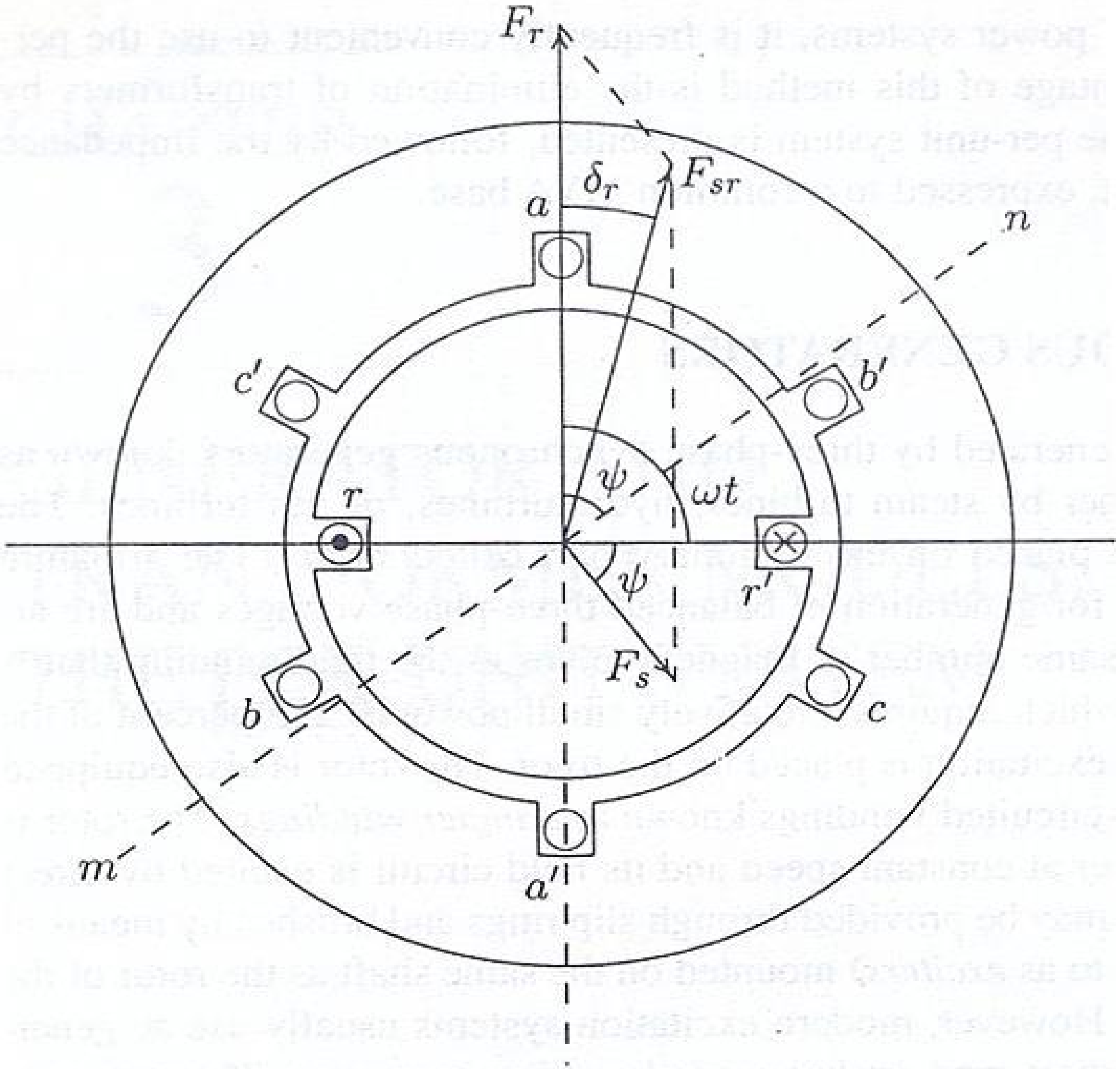

FIGURE 3.1
Elementary two-pole three-phase synchronous generator.

Swing Equation

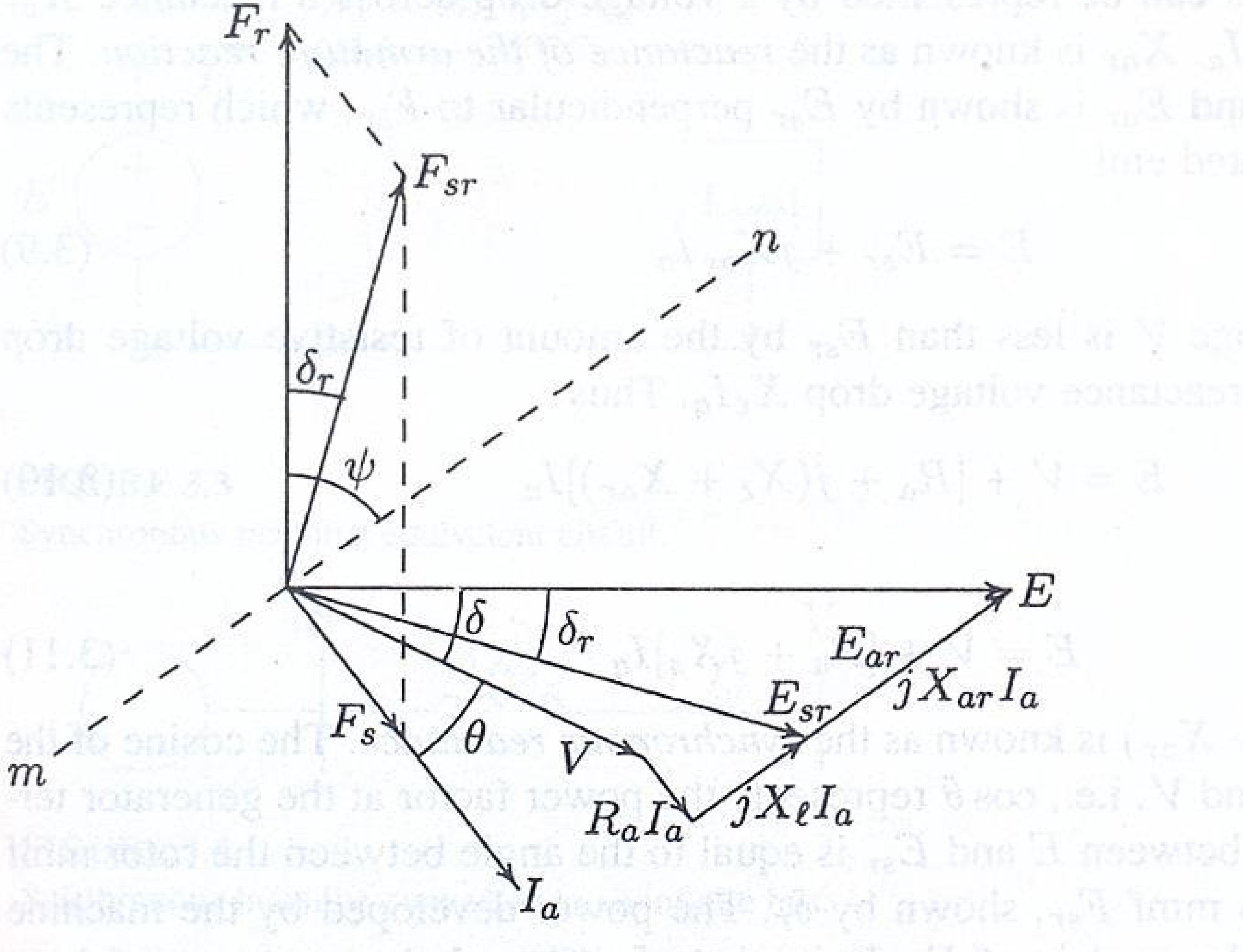

FIGURE 3.2
Combined phasor/vector diagram for one phase of a cylindrical rotor generator.

Swing Equation

The angle between E and E_{sr} is equal to the angle between the rotor mmf F_r and the air gap mmf F_{sr}, shown by δ_r. The power developed by the machine is proportional to the product of F_r, F_{sr} and $\sin\delta_r$. The relative positions of these mmfs dictates the action of the synchronous machine. When F_r is ahead of F_{sr} by an angle δ_r, the machine is operating as a generator and when F_r falls behind F_{sr}, the machine will act as a motor. Since E and E_{sr} are proportional to F_r and F_{sr}, respectively, the power developed by the machine is proportional to the products of E, E_{sr}, and $\sin\delta_r$. The angle δ_r is thus known as the *power angle*.

the space angle between the magnetic fields in the machine. Usually the developed power is expressed in terms of the excitation voltage E, the terminal voltage V, and $\sin\delta$. The angle δ is approximately equal to δ_r because the leakage impedance is very small compared to the magnetization reactance.

Swing Equation

Consider a synchronous generator developing an electromagnetic torque T_e and running at the synchronous speed ω_{sm}. If T_m is the driving mechanical torque, then under steady-state operation with losses neglected we have

$$T_m = T_e \tag{11.1}$$

A departure from steady state due to a disturbance results in an accelerating ($T_m > T_e$) or decelerating ($T_m < T_e$) torque T_a on the rotor.

$$T_a = T_m - T_e \tag{11.2}$$

In the above equation, T_m and T_e are positive for a generator and negative for a motor.

Swing Equation

shown in Fig. 14.1a. Under steady-state operation of the generator T_m and T_e are equal and the accelerating torque T_a is zero. In this case there is no acceleration or deceleration of the rotor masses and the resultant constant speed is the *synchronous speed*. The rotating masses which include the rotor of the generator and the prime mover are said to be in synchronism with the other machines operating at synchronous speed in the power system. The prime mover may be a hydro turbine or a steam turbine for which models of different levels of complexity exist to represent their effect on T_m. In this text T_m is considered constant at any given operating condition. This assumption is a fair one for generators even though input from the prime mover is controlled by governors. Governors do not act until after a change in speed is sensed and so are not considered effective during the time-period in which rotor dynamics are of interest in our stability studies here. The electrical torque T_e corresponds to the net air-gap power in the machine and thus accounts for the total output power of the generator plus $|I|^2R$ losses in the armature winding. In the synchronous motor the direction of

Swing Equation

Linear Motion			Rotation		
Quantity	Symbol/ Equation	MKS unit	Quantity	Symbol/ Equation	MKS unit
Length	s	meter (m)	Angular displacement	θ	radian (rad)
Mass	M	kilogram (kg)	Moment of inertia	$J=\int r^2 dm$	$kg \cdot m^2$
Velocity	$v=ds/dt$	meter/second (m/s)	Angular velocity	$\omega=d\theta/dt$	rad/s
Acceleration	$a=dv/dt$	m/s^2	Angular acceleration	$\alpha=d\omega/dt$	rad/s^2
Force	$F=Ma$	newton (N)	Torque	$T=J\alpha$	newton-meter (N·m) or J/rad
Work	$W=\int F ds$	joule (J)	Work	$W=\int T d\theta$	J, or W·s
Power	$p=dW/dt$ $=Fv$	watt (W)	Power	$p=dW/dt$ $=T\omega$	W

Swing Equation

The combined inertia of the generator and prime mover is accelerated by the unbalance in the applied torques. Hence, the equation of motion is

$$J\frac{d\omega_m}{dt} = T_a = T_m - T_e \tag{3.196}$$

where

J = combined moment of inertia of generator and turbine, kg·m^2

ω_m = angular velocity of the rotor, mech. rad/s

t = time, s

Swing Equation

The above equation can be normalized in terms of per unit *inertia constant* H, defined as the kinetic energy in watt-seconds at rated speed divided by the VA base. Using ω_{0m} to denote rated angular velocity in mechanical radians per second, the inertia constant is

$$H = \frac{W}{VA_{base}} = \frac{\int T\, d\theta}{VA_{base}} = \frac{\int J\alpha\, d\theta}{VA_{base}} = \frac{J\int \frac{d\omega}{dt} \cdot d\theta}{VA_{base}}$$

$$H = \frac{J\int d\omega \frac{d\theta}{dt}}{VA_{base}} = \frac{J\omega(\omega)}{VA_{base}} \Rightarrow H = \frac{1}{2}\frac{J\omega_{0m}^2}{\mathrm{VA}_{base}} \qquad (3.197)$$

The moment of inertia J in terms of H is

$$J = \frac{2H}{\omega_{0m}^2}\mathrm{VA}_{base}$$

Substituting the above in Equation 3.196 gives

Swing Equation

$$\frac{2H}{\omega_{0m}^2} \mathrm{VA}_{base} \frac{d\omega_m}{dt} = T_m - T_e$$

Rearranging yields

$$2H \frac{d}{dt}\left(\frac{\omega_m}{\omega_{0m}}\right) = \frac{T_m - T_e}{\mathrm{VA}_{base}/\omega_{0m}}$$

P_{base}/ω_{0m}

Noting that $T_{base} = \mathrm{VA}_{base}/\omega_{0m}$, the equation of motion in per unit form is

$$\boxed{2H \frac{d\bar{\omega}_r}{dt} = \bar{T}_m - \bar{T}_e}$$

(3

In the above equation,

$$\bar{\omega}_r = \frac{\omega_m}{\omega_{0m}} = \frac{\omega_r/p_f}{\omega_0/p_f} = \frac{\omega_r}{\omega_0}$$

$\because \omega_{sm} = \frac{2}{p_f}\omega_s$

$(p = 2)$

Swing Equation

where ω_r is angular velocity of the rotor in electrical rad/s, ω_0 is its rated value,
p_f is number of field poles.

If δ is the angular position of the rotor in electrical radians with respect
synchronously rotating reference and δ_0 is its value at $t=0$,

$$\delta = \omega_r t - \omega_0 t + \delta_0. \qquad (3.1$$

{P-55, 95, 99

Taking the time derivative, we have

$$\frac{d\delta}{dt} = \omega_r - \omega_0 = \Delta\omega_r \qquad (3.$$

and

$$\frac{d^2\delta}{dt^2} = \frac{d\omega_r}{dt} = \frac{d(\Delta\omega_r)}{dt} \qquad (3.2$$

$$= \omega_0 \frac{d\bar{\omega}_r}{dt} = \omega_0 \frac{d(\Delta\bar{\omega}_r)}{dt}$$

($\because \omega_r = \omega_0 \bar{\omega}_r$

Swing Equation

Substituting for $d\bar{\omega}_r/dt$ given by the above equation in Equation 3.198, we get

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \bar{T}_m - \bar{T}_e \qquad (3.202)$$

It is often desirable to include a component of damping torque, not accounted for in the calculation of T_e, separately. This is accomplished by adding a term proportional to speed deviation in the above equation as follows:

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = \bar{T}_m - \bar{T}_e - K_D\Delta\bar{\omega}_r \qquad (3.203)$$

From Equation 3.200,

$$\Delta\bar{\omega}_r = \frac{\Delta\omega_r}{\omega_0} = \frac{1}{\omega_0}\frac{d\delta}{dt}$$

Equation 3.203 represents the equation of motion of a synchronous machine. It is commonly referred to as the *swing equation* because it represents swings in rotor angle δ during disturbances. | p 135

Swing Equation

the *swing equation* of the machine, is the fundamental equation which governs the rotational dynamics of the synchronous machine in stability studies.

determine the stability of a machine within a power system. When the swing equation is solved we obtain the expression for δ as a function of time. A graph of the solution is called the *swing curve* of the machine and inspection of the swing curves of all the machines of the system will show whether the machines remain in synchronism after a disturbance. Stevenson (3rd edition) - 379

plotted. Such a graph is called the *swing curve*. If the swing curve indicates that the angle δ starts to decrease after reaching a maximum value, it is usually assumed that the system will not lose stability and that the oscillations of δ around the equilibrium point will become successively smaller and eventually be damped out. Kundur - 135

Swing Equation

3.9.5 Representation in System Studies

For analysis of power system dynamic performance, the component models are expressed in the state-space form (see Chapter 12, Section 12.1) or the block diagram form.

The state-space form requires the component models to be expressed as a set of first order differential equations. The swing equation 3.203, expressed as two first order differential equations, becomes

$$\left(\because \frac{d^2\delta}{dt^2} = \omega_0 \frac{d(\Delta\bar{\omega}_r)}{dt} \right)$$

$$\frac{d\Delta\bar{\omega}_r}{dt} = \frac{1}{2H}(\bar{T}_m - \bar{T}_e - K_D\Delta\bar{\omega}_r) \qquad (3.209)$$

$$\frac{d\delta}{dt} = \omega_0 \Delta\bar{\omega}_r \qquad (3.210)$$

In the above equations, time t is in seconds, rotor angle δ is in electrical radians, and ω_0 is equal to $2\pi f$. In later chapters, when we use the above equations we will not use superbars to identify per unit quantities. We will assume the variables $\Delta\omega_r$, T_m and T_e to be in per unit. However, t will be expressed in seconds and ω_0 in electrical radians per second.

Swing Equation

From (3.209),

$$\frac{d\Delta\bar{\omega}_r}{dt} + \frac{K_D \Delta\bar{\omega}_r}{2H} = \frac{1}{2H}(\bar{T}_m - \bar{T}_e)$$

$$s\Delta\bar{\omega}_r + \frac{K_D \Delta\bar{\omega}_r}{2H} = \frac{1}{2H}(\bar{T}_m - \bar{T}_e)$$

$$\Delta\bar{\omega}_r\left(s + \frac{K_D}{2H}\right) = \frac{1}{2H}(\bar{T}_m - \bar{T}_e)$$

$$\Delta\bar{\omega}_r\left(\frac{2Hs + K_D}{2H}\right) = \frac{1}{2H}(\bar{T}_m - \bar{T}_e)$$

$$\therefore \Delta\bar{\omega}_r = \left(\frac{1}{2Hs + K_D}\right)(\bar{T}_m - \bar{T}_e) \quad \text{(*)}$$

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From (3.210)

$$s\delta = \omega_0 \Delta\bar{\omega}_r$$

$$\therefore \delta = \frac{\omega_0}{s}\Delta\bar{\omega}_r \quad \text{- - - - - - } (\alpha)$$

Swing Equation

*) & (α)

The block diagram form representation of Equations 3.209 and 3.210 is shown in Figure 3.34.

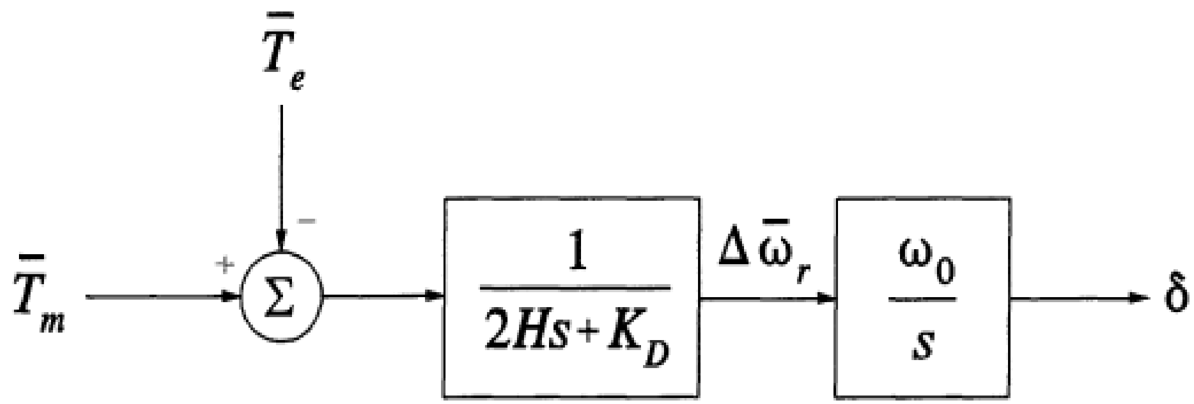


Figure 3.34 Block diagram representation of swing equations

In the block diagram, s is the Laplace operator; it replaces d/dt of Equations 3.209 and 3.210. As noted earlier, symbols T_M and M are often used in place of $2H$.

Application of swing curve & solution of problems

by switching at the instant when $\delta = \delta_c$. The angle δ_c is called the *critical clearing angle* because it is the largest possible value of δ for clearing to occur without exceeding the transient stability limit.

calculated. We can obtain a curve of δ versus t for this example and from it find the *critical clearing time*, which is the time for the machine to swing from its original position to its critical clearing angle.

The equal-area criterion is useful in understanding a two-machine system, but to find the critical clearing time we must find δ as a function of t. For large systems we depend on the digital computer which determines δ versus t for all the machines in which we are interested; and δ may be plotted versus t for a machine to obtain the swing curve of that machine. The angle δ is calculated as a function of time over a period long enough to determine whether δ will increase without limit or reach a maximum and start to decrease. Although the latter result usually indicates stability, on an actual system where a number of variables are taken into account it may be necessary to plot δ versus t over a long enough interval to be sure δ will not increase again without returning to a low value.

Application of swing curve & solution of problems

By determining swing curves for various clearing times the length of time permitted before clearing a fault can be determined. Standard interrupting times for circuit breakers and their associated relays are commonly 8, 5, 3, or 2 cycles after a fault occurs, and thus breaker speeds may be specified. Calculations should be made for a fault in the position which will allow the least transfer of power from the machine and for the most severe type of fault for which protection against loss of stability is justified.

A number of different methods are available for the numerical evaluation of second-order differential equations in step-by-step computations for small increments of the independent variable. The more elaborate methods are practical only when the computations are performed on a digital computer.

Infinite Bus

Application of swing curve & solution of problems

EXAMPLE 14.2 The single-line diagram of Fig. 14.10 shows a generator connected through parallel high-voltage transmission lines to a large metropolitan system considered as an infinite bus. Numbers on the diagram indicate the values of the reactances in per unit. The transient reactance of the generator is included in the values marked. Breakers adjacent to a fault on both sides are arranged to clear simultaneously. Specify in electrical degrees the critical clearing angle for

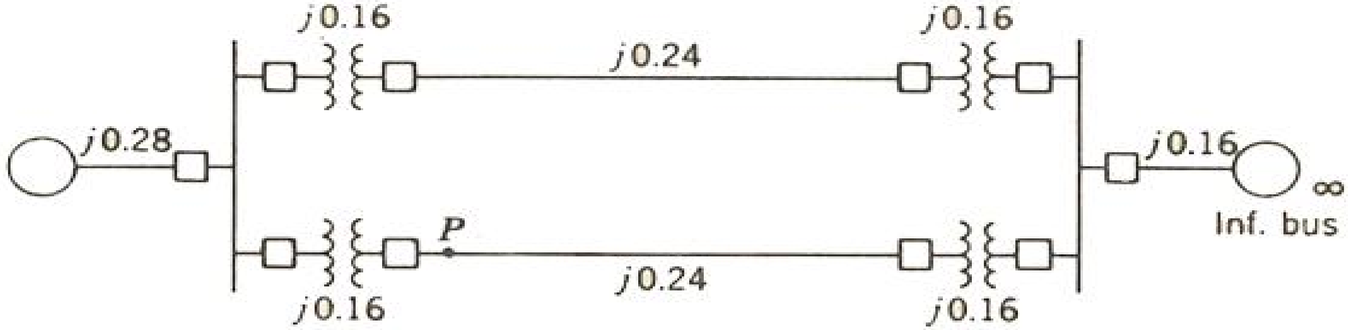


FIGURE 14.10
One-line diagram for Example 14.2.

the generator for a three-phase fault at the point P when the generator is delivering 1.0 per-unit power. Assume that the voltage behind transient reactance is 1.25 per unit for the generator and that the voltage at the infinite bus is 1.0 per unit.

Application of swing curve & solution of problems

EXAMPLE 14.3 Plot swing curves for the fault described on the system of Example 14.2 for clearing the fault by opening breakers simultaneously at the ends of the faulted line at 3 cycles and 4.5 cycles after the fault occurs. Also plot the swing curve over a period of 0.25 s if the fault is not cleared. For the generator assume $H = 3.0$ and carry out the calculations in per unit.

SOLUTION Since our calculations will express power in per unit, the term G in the equation for M must be in per unit instead of megavoltamperes and is the ratio of the megavoltamperes of the generator to the base megavoltamperes. Therefore for this example G is 1.0 per unit, and

$$M = \frac{GH}{180f} = \frac{1.0 \times 3.0}{180 \times 60} = 2.78 \times 10^{-4}\ \mathrm{s^2/electrical\ degree}$$

Application of swing curve & solution of problems

For the time interval $\Delta t = 0.05$ s,

$$\frac{(\Delta t)^2}{M} = \frac{25 \times 10^{-4}}{2.78 \times 10^{-4}} = 9.0 \text{ electrical degrees}$$

From Example 14.2, when the fault occurs,

$$\delta = 35.2°$$

and during the fault,

$$P_e = 0.42 \sin \delta$$

Therefore

$$P_a = P_s - P_e = 1.0 - 0.42 \sin \delta$$

At the beginning of the first interval there is a discontinuity in the accelerating power. Just before the fault occurs, $P_a = 0$, and just after the fault occurs

$$P_a = 1.0 - 0.42 \sin 35.2° = 1.0 - 0.242 = 0.758 \text{ per unit}$$

The average value of P_a is $\frac{1}{2} \times 0.758 = 0.379$ per unit.

$$\frac{(\Delta t)^2}{M} P_a = 9 \times 0.379 = 3.41$$

$$\Delta \delta_n = 0 + 3.41 = 3.41°$$

Application of swing curve & solution of problems

When $t = 0.05$ s,

$$\delta_n = 35.2° + 3.41° = 38.61°$$

$$P_a = 1.0 - 0.42 \sin 38.61° = 1.0 - 0.262 = 0.738 \text{ per unit}$$

$$\frac{(\Delta t)^2}{M} P_a = 9 \times 0.738 = 6.64$$

$$\Delta\delta_n = 3.41° + 6.64° = 10.05°, \text{ or } 10.1°$$

When $t = 0.10$ s,

$$\delta_n = 38.6° + 10.1° = 48.7°$$

Steps in the computations are shown in Table 14.4.

In the table P_e, P_a, and δ_n are values computed at the time t shown in the first column, but $\Delta\delta_n$ is the *change* in torque angle *during* the interval that begins at the time indicated. For example, in the row of figures for $t = 0.10$ s, the angle 48.7° is the first value calculated and is found by adding the change in angle during the preceding interval to the angle at the beginning of the preceding interval. Next, P_e is calculated for $\delta = 48.7°$. Then, P_a and the product of P_a and $(\Delta t)^2/M$ are calculated. The value of the product is 6.17°, which is added to

The swing curves are plotted in Fig. 14.14 for all three cases. Evidently the system is stable for clearing at 4.5 cycles or less.

Application of swing curve & solution of problems

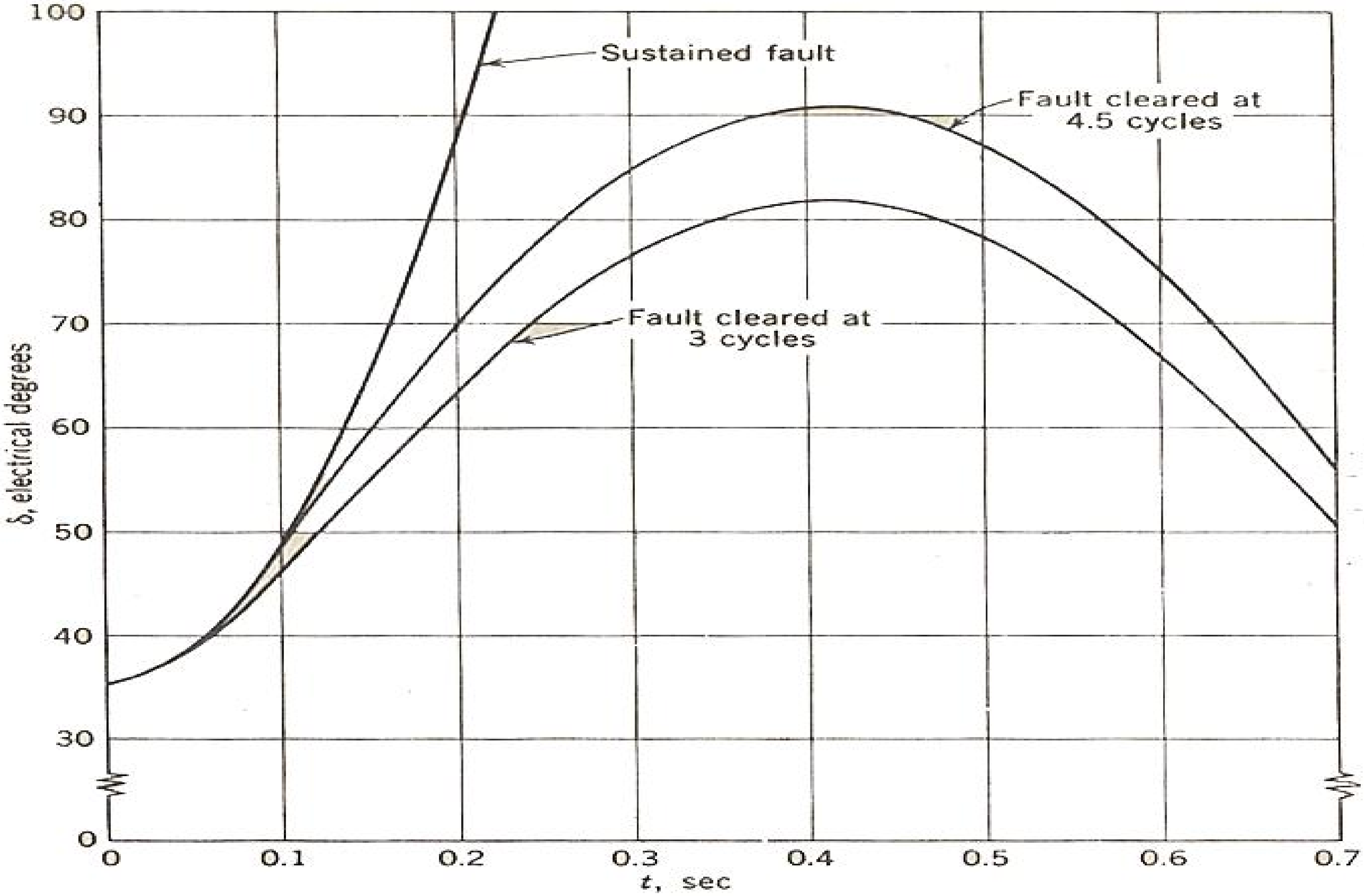


FIGURE 14.14
Swing curves for Example 14.3 for a sustained fault and for clearing in 3.0 and 4.5 cycles.

Example 11.6

In the system of Example 11.5 a three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.

(a) The fault is cleared in 0.3 second. Obtain the numerical solution of the swing equation for 1.0 second using the modified Euler method (function **swingmeu**) with a step size of $\Delta t = 0.01$ second. From the swing curve, determine the system stability.

(b) The **swingmeu** function automatically calls upon the **cctime** function and determines the critical clearing time. Repeat the simulation and obtain the swing plots for the critical clearing time, and when fault is cleared in 0.5 second.

(c) Obtain a *SIMULINK* block diagram model for the swing equation, and simulate for a fault clearing time of 0.3 and 0.5 second. Repeat the simulation until a critical clearing time is obtained.

Application of swing curve & solution of problems

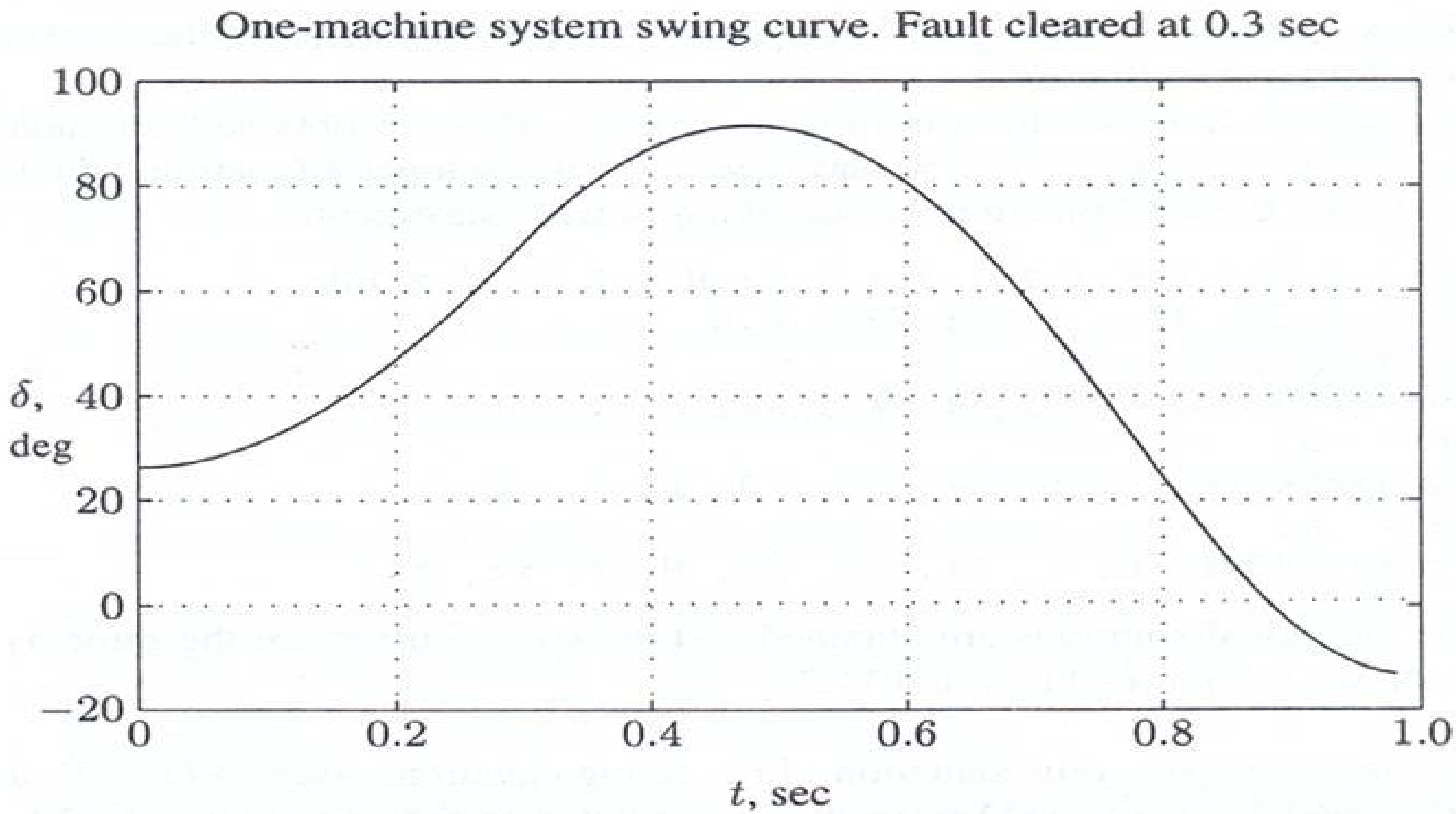


FIGURE 11.27
Swing curve for machine of Example 11.6. Fault cleared at 0.3 sec.

Application of swing curve & solution of problems

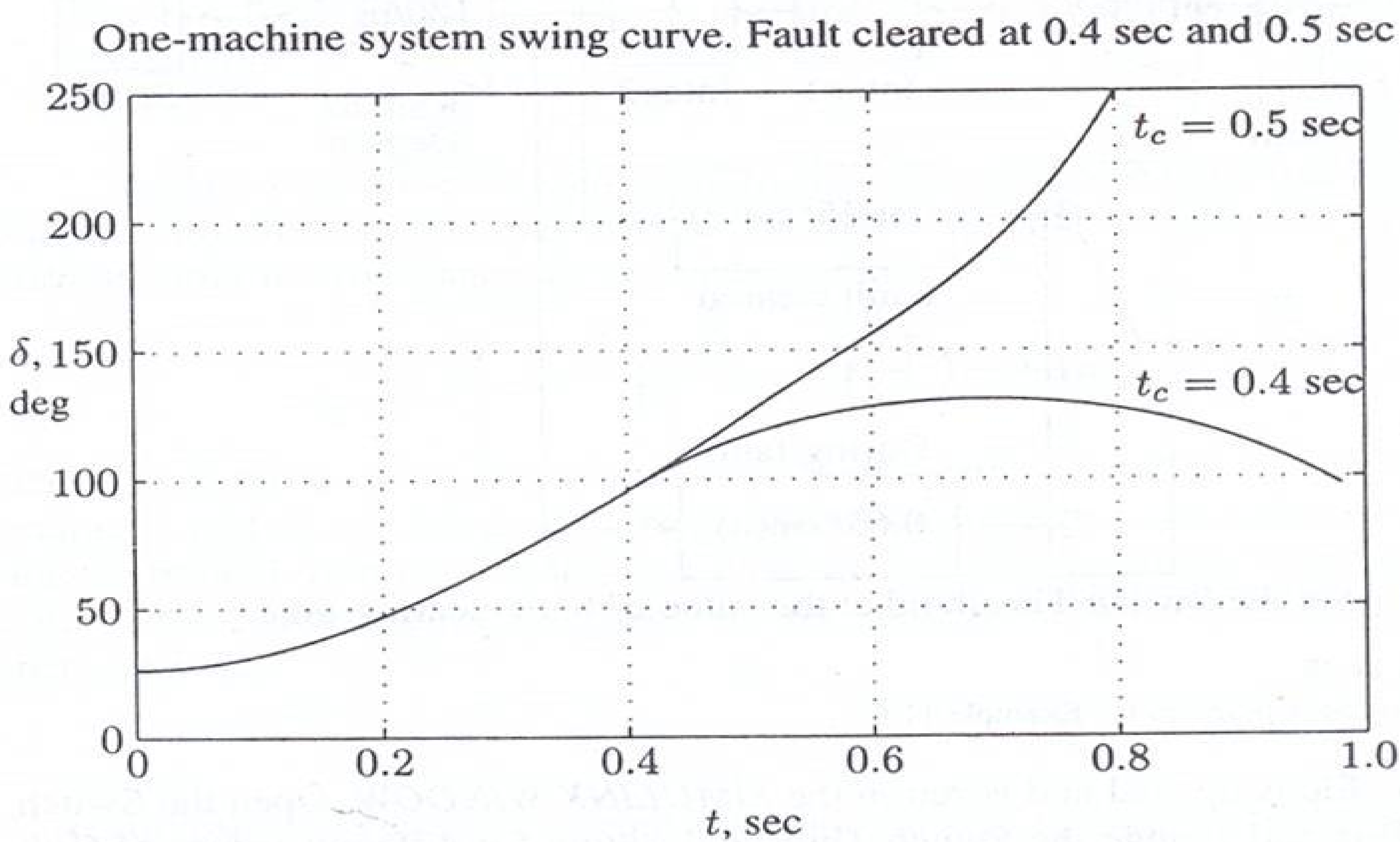


FIGURE 11.28
Swing curves for machine of Example 11.6, for fault clearance at 0.4 sec and 0.5 sec.

Equal area criterion in machine dynamics

11.4 STEADY-STATE STABILITY — SMALL DISTURBANCES

The steady-state stability refers to the ability of the power system to remain in synchronism when subjected to small disturbances. It is convenient to assume that the disturbances causing the changes disappear. The motion of the system is free, and stability is assured if the system returns to its original state. Such a behavior can be determined in a linear system by examining the characteristic equation of the system. It is assumed that the automatic controls, such as voltage regulator and governor, are not active.

2 To illustrate the steady-state stability problem, we consider the dynamic behavior of a one-machine system connected an infinite bus bar as shown in Figure 11.1. Substituting for the electrical power from (11.29) into the swing equation given in (11.21) results in

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta \tag{11.36}$$

The swing equation is a nonlinear function of the power angle. However, for small disturbances, the swing equation may be linearized with little loss of accuracy

Equal area criterion in machine dynamics

11.5 TRANSIENT STABILITY — EQUAL-AREA CRITERION

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This may be sudden application of load, loss of generation, loss of large load, or a fault on the system. In most disturbances, oscillations are of such magnitude that linearization is not permissible and the nonlinear swing equation must be solved.

n Sec. 14.4 we developed swing equations which are nonlinear in nature. ormal solution of such equations cannot be explicitly found. Even in the case of single machine swinging with respect to an infinite bus it is very difficult to btain literal-form solutions and therefore digital computer methods are ormally used. To examine the stability of a two-machine system without solv-ıg the swing equation, a direct approach is possible

Equal area criterion in machine dynamics

In a system where one machine is swinging with respect to an infinite bus, it is not necessary to plot and inspect the swing curves to determine whether the torque angle of the machine increases indefinitely or oscillates around an equilibrium position. Solution of the swing equation, with the usual assumptions of constant P_s, a purely reactive network, and constant voltage behind transient reactance, shows that δ oscillates around the equilibrium point with constant amplitude if the transient stability limit is not exceeded. The principle by which stability under transient conditions is determined without solving the swing equation is called the *equal-area criterion* of stability. Although not applicable to multimachine systems, the method helps in understanding how certain factors influence the transient stability of any system.

Equal area criterion in machine dynamics

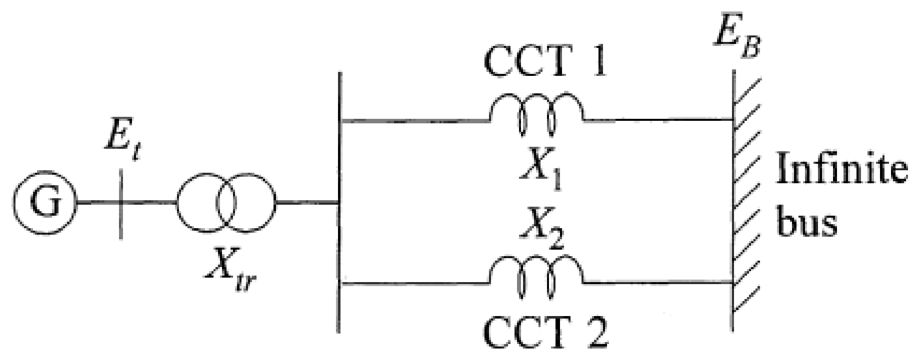
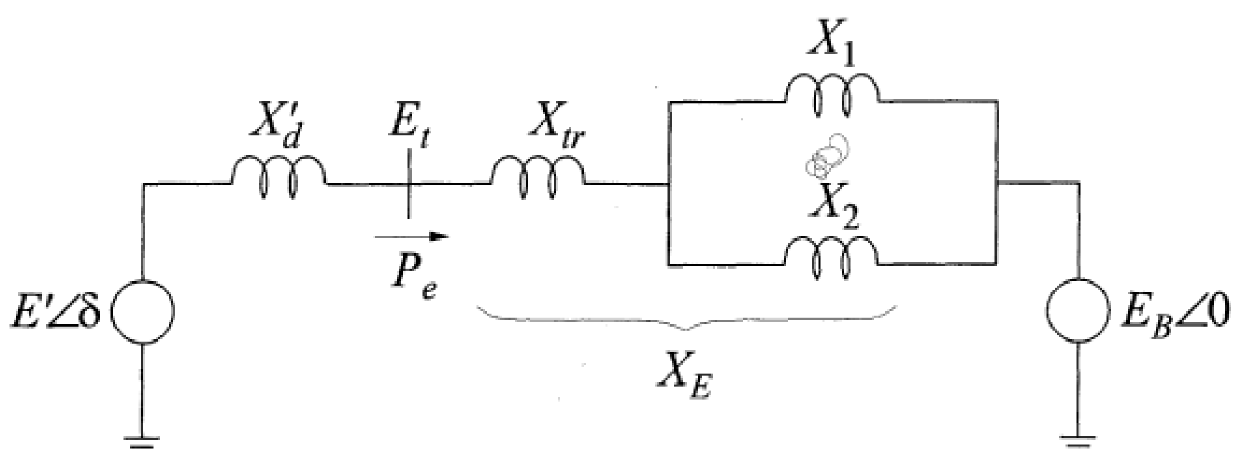
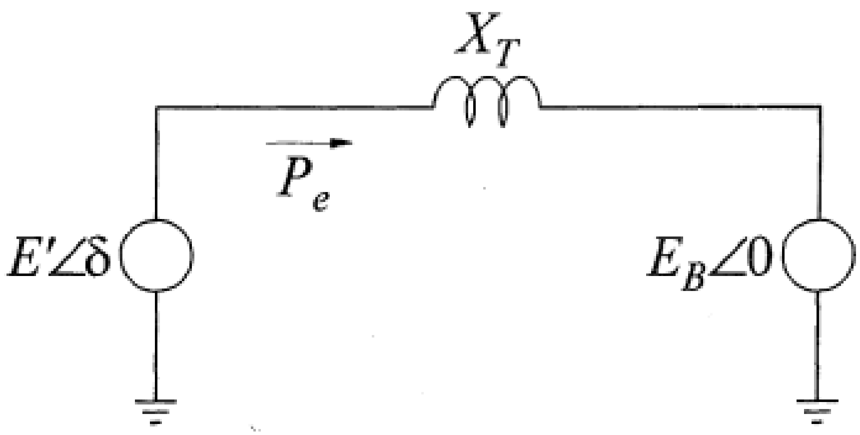


Figure 13.1 Single-machine infinite bus system



(a) Equivalent circuit

Equal area criterion in machine dynamics



$$\tilde{E}' = \tilde{E}_{t0} + jX'_d\tilde{I}_{t0}$$
$$X_T = X'_d + X_E$$

(b) Reduced equivalent circuit

13.2 System representation with generator represented by classical model

$$P_e = \frac{E'E_B}{X_T}\sin\delta = P_{max}\sin\delta \qquad (13.1)$$

where

$$P_{max} = \frac{E'E_B}{X_T} \qquad (13.2)$$

Equal area criterion in machine dynamics

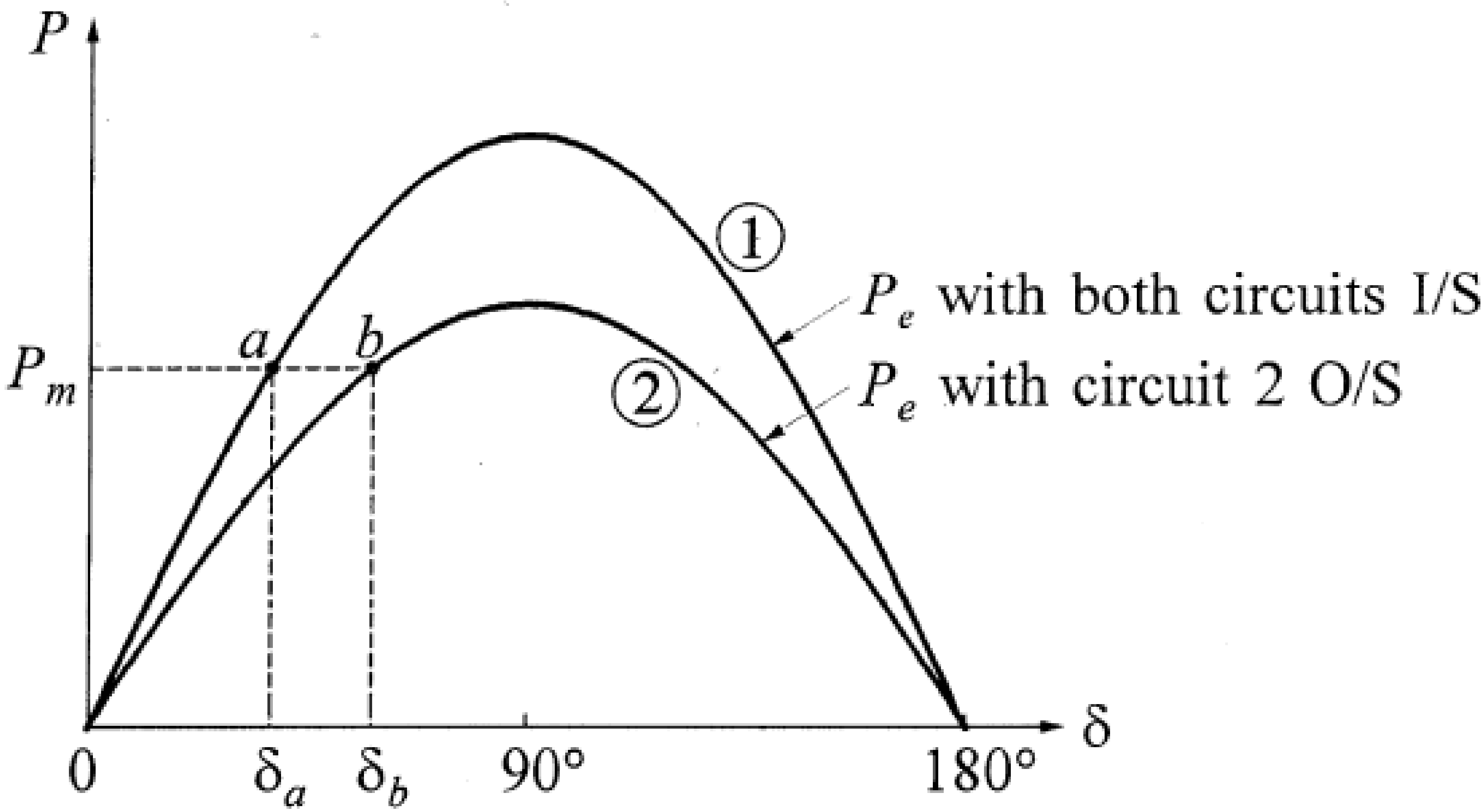
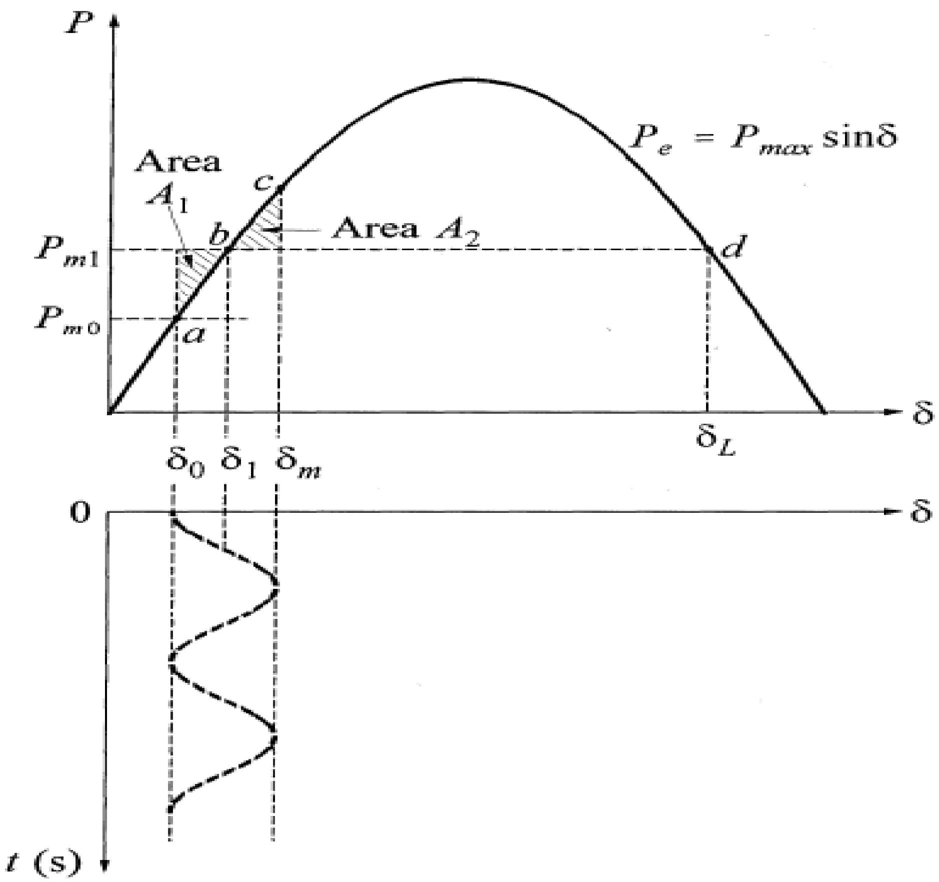


Figure 13.3 Power-angle relationship

Equal area criterion in machine dynamics

Response to a step change in P_m

Let us now examine the transient behaviour of the system, with both circuits in service, by considering a sudden increase in the mechanical power input from an initial value of P_{m0} to P_{m1} as shown in Figure 13.4(a). Because of the inertia of the



(a) Power-angle variations

(b) Rotor angle time response

Figure 13.4 Response to a step change in mechanical power input

Equal area criterion in machine dynamics

rotor, the rotor angle cannot change instantly from the initial value of δ_0 to δ_1 corresponding to the new equilibrium point b at which $P_e = P_{m1}$. The mechanical power is now in excess of the electrical power. The resulting accelerating torque causes the rotor to accelerate from the initial operating point a toward the new equilibrium point b, tracing the P_e-δ curve at a rate determined by the swing equation. The difference between P_{m1} and P_e at any instant represents the accelerating power.

When point b is reached, the accelerating power is zero, but the rotor speed is higher than the synchronous speed ω_0 (which corresponds to the frequency of the infinite bus voltage). Hence, the rotor angle continues to increase. For values of δ higher than δ_1, P_e is greater than P_{m1} and the rotor decelerates. At some peak value δ_m, the rotor speed recovers to the synchronous value ω_0, but P_e is higher than P_{m1}. The rotor continues to decelerate with the speed dropping below ω_0; the operating point retraces the $P_e - \delta$ curve from c to b and then to a. The rotor angle oscillates indefinitely about the new equilibrium angle δ_1 with a constant amplitude as shown by the time plot of δ in Figure 13.4(b).

In our representation of the power system in the above analysis, we have neglected all resistances and the classical model is used to represent the generator. In effect, this neglects all sources of damping. Therefore, the rotor oscillations continue unabated following the perturbation. In practice, as discussed in Chapter 12, there are many sources of positive damping including field flux variations and rotor amortisseur circuits. In a system which is small-signal stable, the oscillations damp out.

Equal area criterion in machine dynamics

For the system model considered above, it is not necessary to formally solve the swing equation to determine whether the rotor angle increases indefinitely or oscillates about an equilibrium position. Information regarding the maximum angle excursion (δ_m) and the stability limit may be obtained graphically by using the power-angle diagram shown in Figure 13.4. Although this method is not applicable to multimachine systems with detailed representation of synchronous machines, it helps in understanding basic factors that influence the transient stability of any system.

The *equation of motion* or the *swing equation* (see Chapter 3, Section 3.9) may be written as

$$\frac{2H}{\omega_0}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta \tag{13.3}$$

where

P_m = mechanical power input, in pu
P_{max} = maximum electrical power output, in pu
H = inertia constant, in MW·s/MVA
δ = rotor angle, in elec. rad
t = time, in s

Equal area criterion in machine dynamics

From Equation 13.3, we have the following relationship between the rotor angle and the accelerating power:

$$\frac{d^2\delta}{dt^2} = \frac{\omega_0}{2H}(P_m - P_e) \qquad (13.4)$$

Now P_e is a nonlinear function of δ, and therefore the above equation cannot be solved directly. If both sides are multiplied by $2d\delta/dt$, then

$$\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = \frac{\omega_0(P_m - P_e)}{H}\frac{d\delta}{dt} \qquad (13.5)$$

Integrating gives

$$\left[\frac{d\delta}{dt}\right]^2 = \int \frac{\omega_0(P_m - P_e)}{H}\, d\delta \qquad (13.6)$$

Equal area criterion in machine dynamics

The speed deviation $d\delta/dt$ is initially zero. It will change as a result of the disturbance. For stable operation, the deviation of angle δ must be bounded, reaching a maximum value (as at point *c* in Figure 13.4) and then changing direction. This requires the speed deviation $d\delta/dt$ to become zero at some time after the disturbance. Therefore, from Equation 13.6, as a criterion for stability we may write

$$\int_{\delta_0}^{\delta_m} \frac{\omega_0}{H}(P_m - P_e)\,d\delta = 0 \tag{13.7}$$

where δ_0 is the initial rotor angle and δ_m is the maximum rotor angle, as illustrated in Figure 13.4. Thus, the area under the function $P_m - P_e$ plotted against δ must be zero if the system is to be stable. In Figure 13.4, this is satisfied when area A_1 is equal to area A_2. Kinetic energy is gained by the rotor during acceleration when δ changes from δ_0 to δ_1. The energy gained is

$$E_1 = \int_{\delta_0}^{\delta_1}(P_m - P_e)\,d\delta = \text{area } A_1 \tag{13.8}$$

Equal area criterion in machine dynamics

The energy lost during deceleration when δ changes from δ_1 to δ_m is

$$E_2 = \int_{\delta_1}^{\delta_m} (P_e - P_m)\, d\delta = \text{area } A_2 \tag{13.9}$$

As we have not considered any losses, the energy gained is equal to the energy lost; therefore, area A_1 is equal to area A_2. This forms the basis for the equal-area criterion. It enables us to determine the maximum swing of δ and hence the stability of the system without computing the time response through formal solution of the swing equation.

The criterion can be readily used to determine the maximum permissible increase in P_m for the system of Figure 13.1. The stability is maintained only if an area A_2 at least equal to A_1 can be located above P_{m1}. If A_1 is greater than A_2, then $\delta_m > \delta_L$, and stability will be lost. This is because, for $\delta > \delta_L$, P_{m1} is larger than P_e and the net torque is accelerating rather than decelerating.