Linear motion

Linear motion

The motion of an object along a straight path is called a linear motion.

A train running on a straight railway track

Vehicles moving on a straight highway

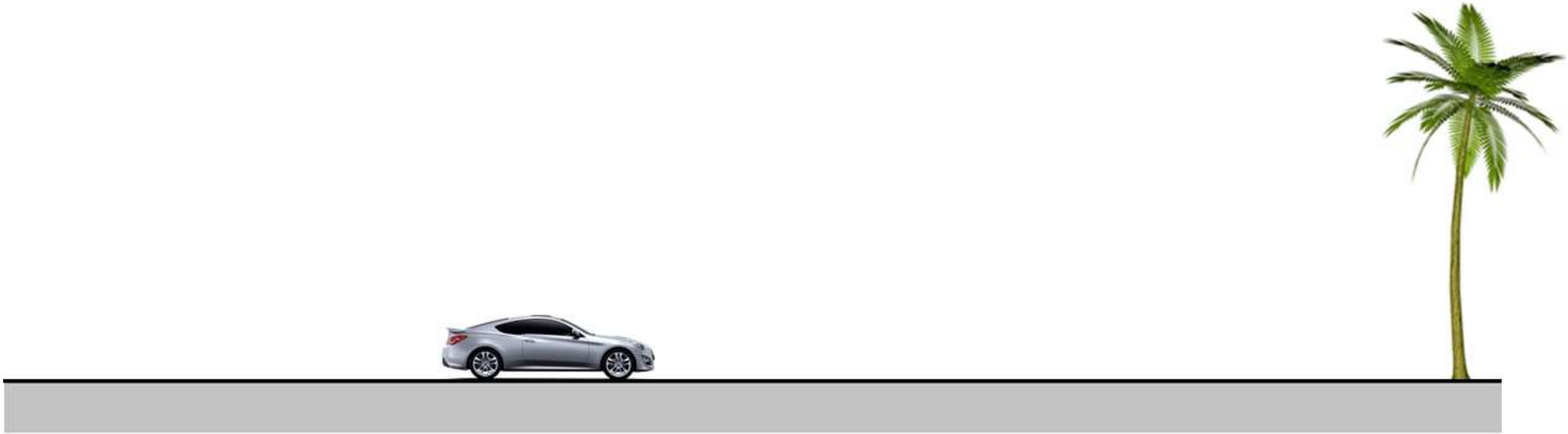
Sprinters running on a straight athletic track

Motion of a bowling ball

Motion diagrams

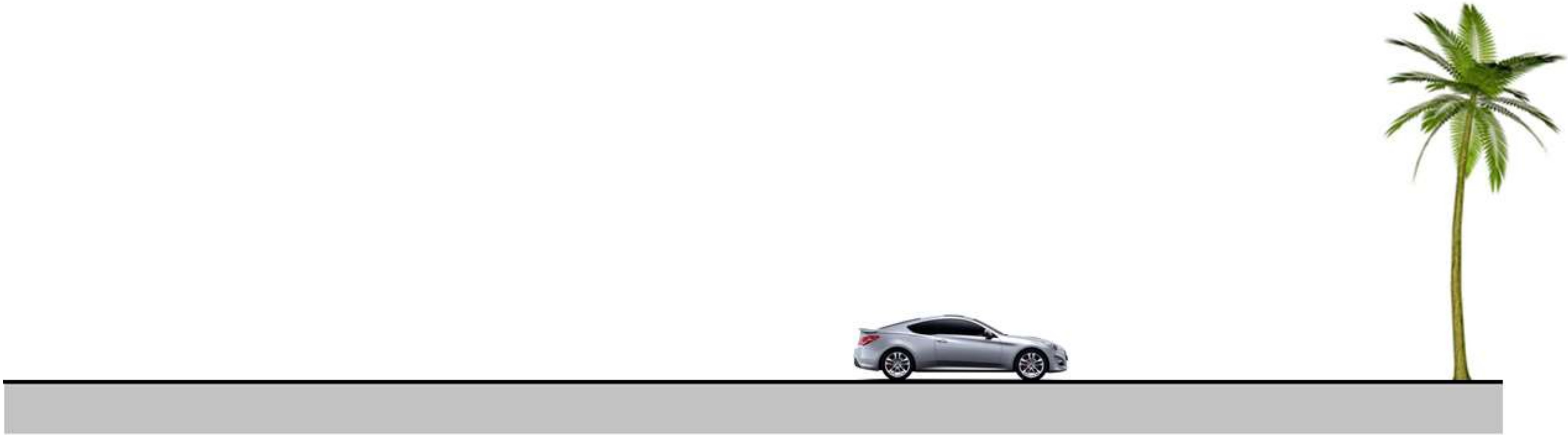
Frame-1

Motion diagrams



Frame-2

Motion diagrams

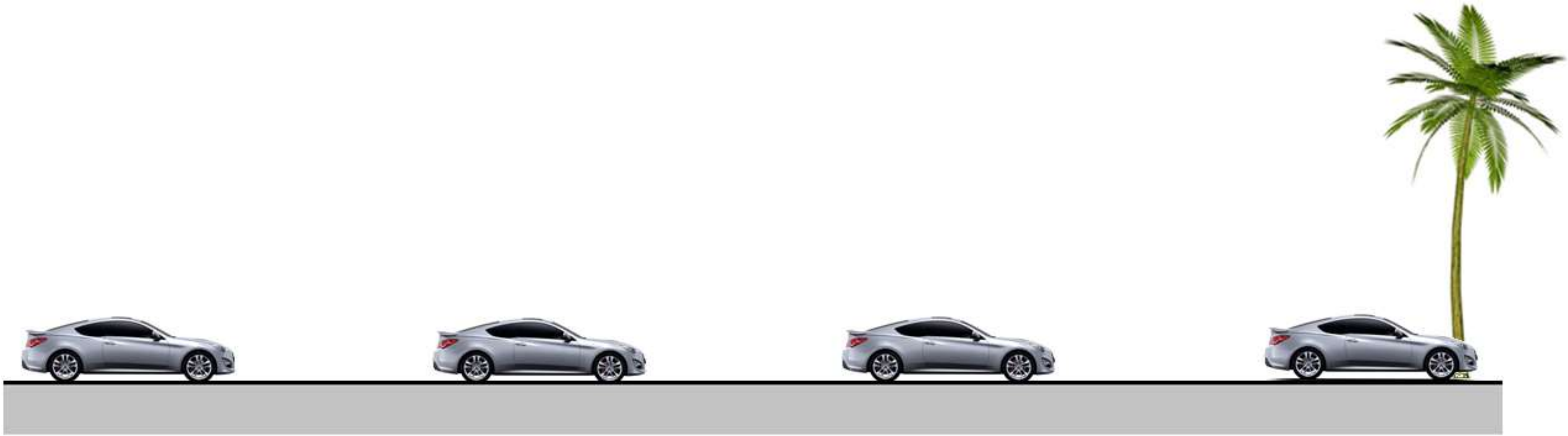


Frame-3

Motion diagrams

Frame-4

Motion diagrams



Motion diagram of a car moving towards a tree

Motion diagrams

A racing car running on a track

Images that are equally spaced indicate an object moving with constant speed.

Motion diagrams

A cyclist starting a race

An increasing distance between the images shows that the object is speeding up.

Motion diagrams

A car stopping for a red light

A decreasing distance between the images shows that the object is slowing down.

Particle model

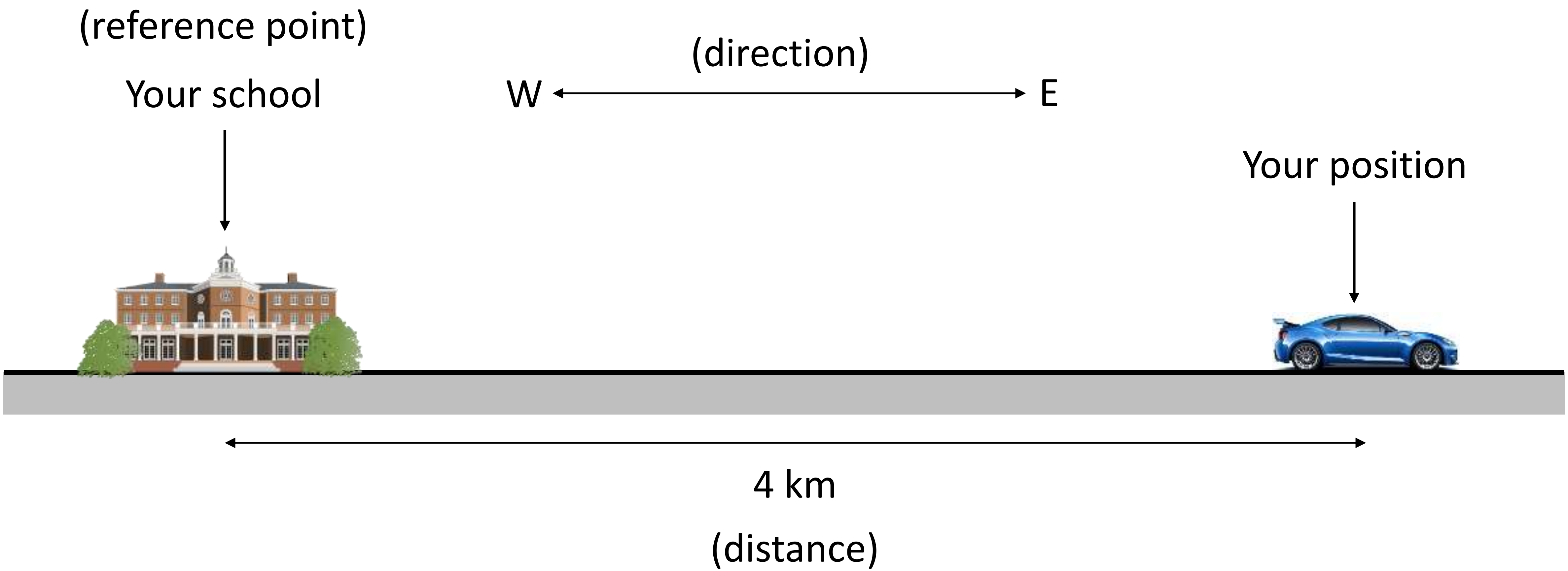
Motion diagram of a racing car moving with constant speed

Same motion diagram using the particle model

Position

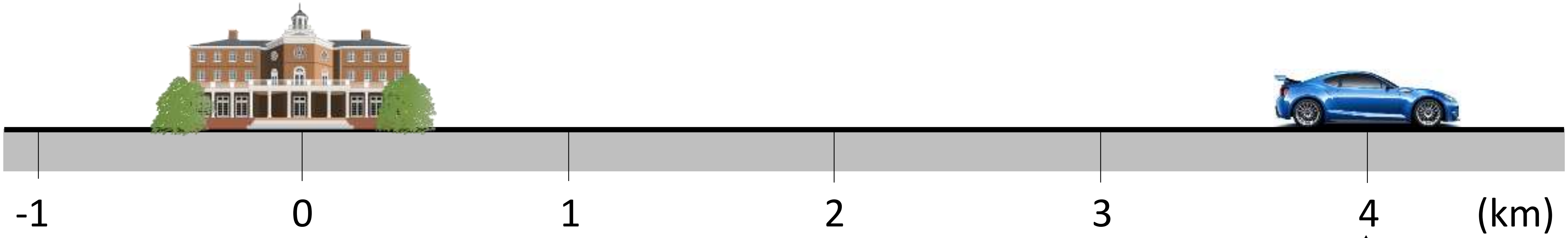
The location of an object at a particular instant is called its position.

Position


(reference point)
Your school
(direction)
W
E
Your position
4 km
(distance)

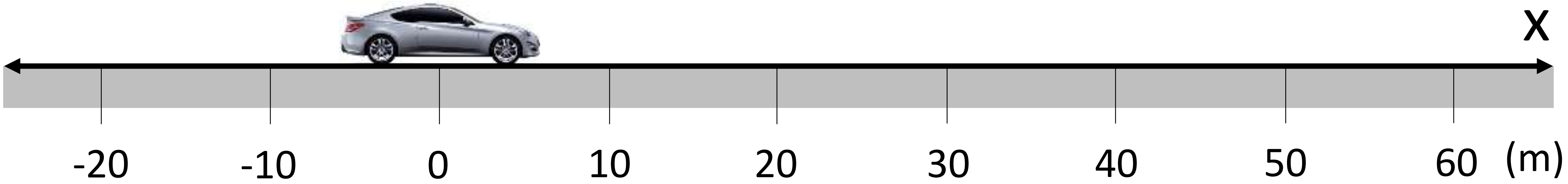
Coordinate system

The school defines the origin of
the coordinate system



Your position is at
+4 km

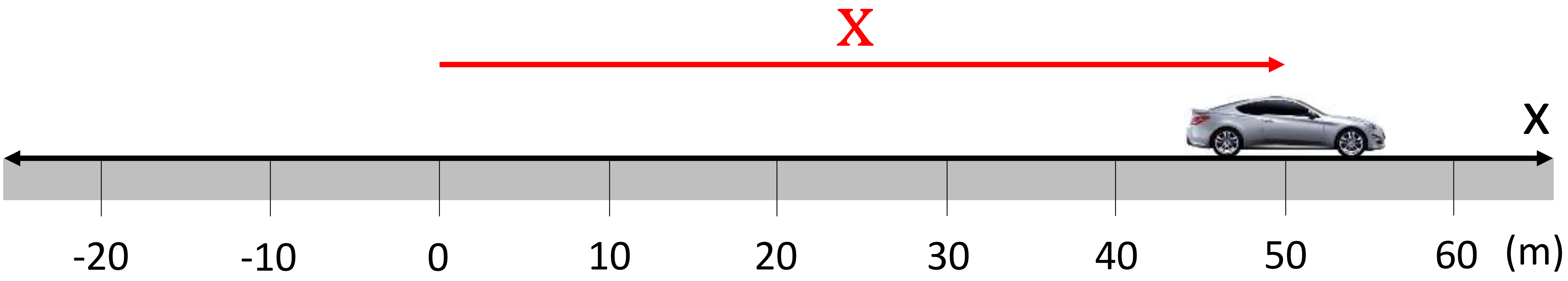
Case-I: Position is zero



The car is at origin

$$x = 0 \text{ m}$$

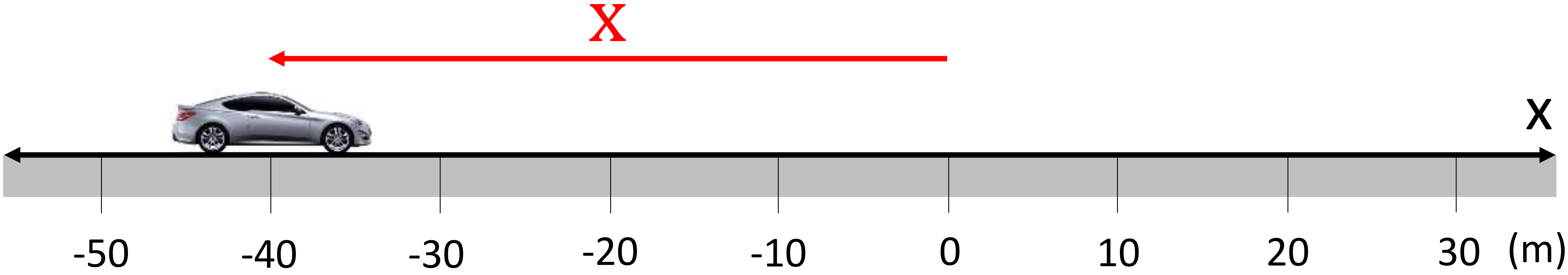
Case-II: Position is positive



The car is at positive side of origin

$$x = 50 \text{ m}$$

Case-III: Position is negative



The car is at negative side of origin

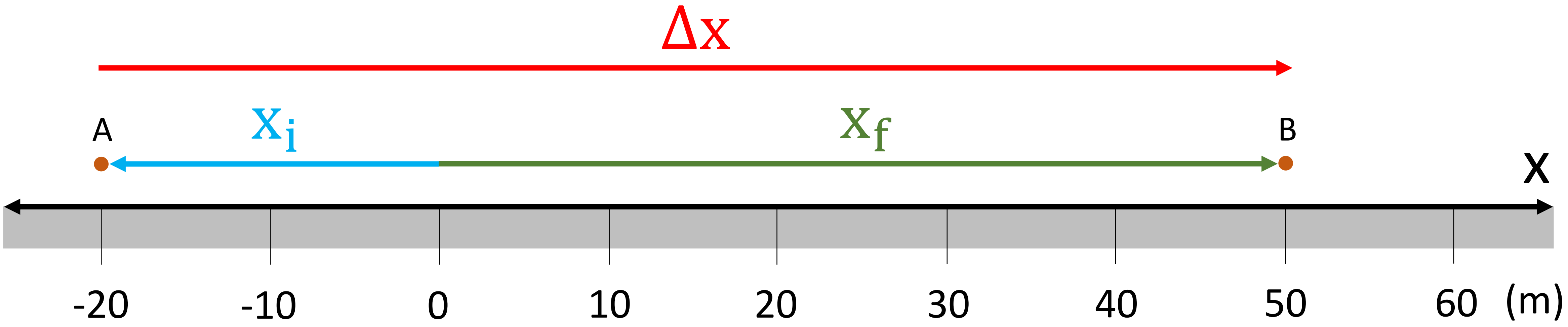
$$x = -40 \text{ m}$$

Displacement

The displacement of an object is the change in the position of an object in a particular direction.

$$\Delta x = x_f - x_i$$

Case-I: Displacement is positive



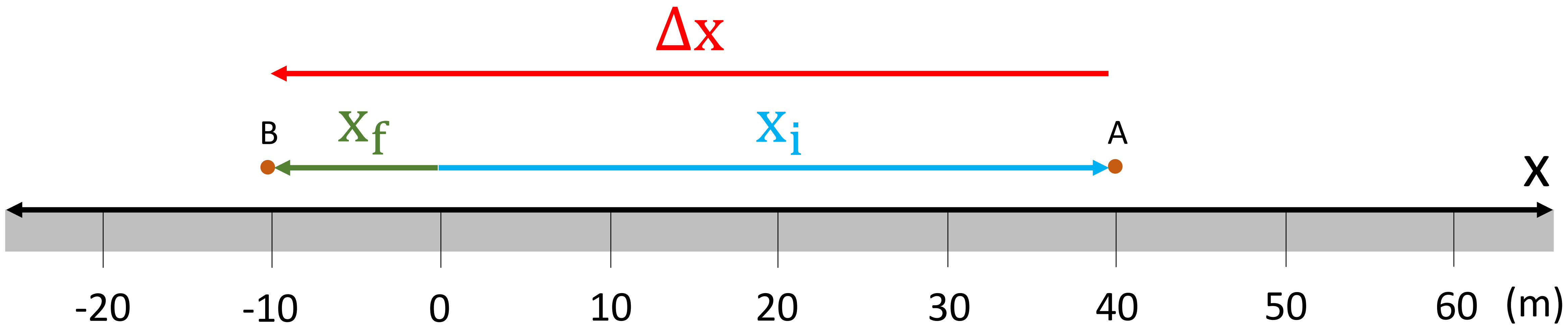
$$\Delta x = 50 - (-20)$$

$$\Delta x = 70 \text{ m}$$

If object is moving towards right, the displacement is positive because

$$x_f > x_i$$

Case-II: Displacement is negative



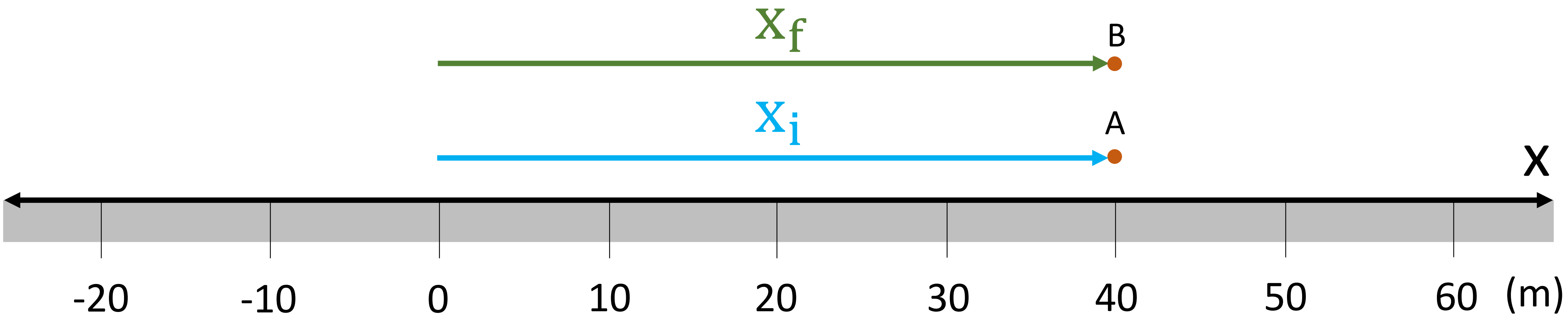
$$\Delta x = -10 - (40)$$

$$\Delta x = -50 \text{ m}$$

If object is moving towards left, the displacement is negative because

$$\boxed{x_f < x_i}$$

Case-III: Displacement is zero



$$\Delta x = 40 - (40)$$

$$\Delta x = 0 \text{ m}$$

If object come to its initial position, the displacement is zero because

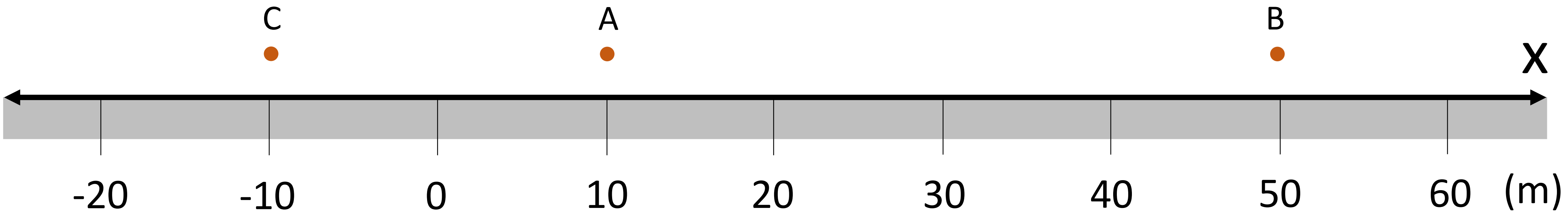
$$\boxed{x_f = x_i}$$

Distance

The actual path covered by an object is called a distance.

$$\color{red}{d} = \text{actual path covered}$$

Distance



$$|\Delta x_{AB}| = |x_B - x_A|$$
$$= |50 - 10|$$
$$= |40| = 40\ \text{m}$$

$$|\Delta x_{BC}| = |x_C - x_B|$$
$$= |-10 - 50|$$
$$= |-60| = 60\ \text{m}$$

$$d = |\Delta x_{AB}| + |\Delta x_{BC}|$$
$$= 40 + 60$$
$$= 100\ \text{m}$$

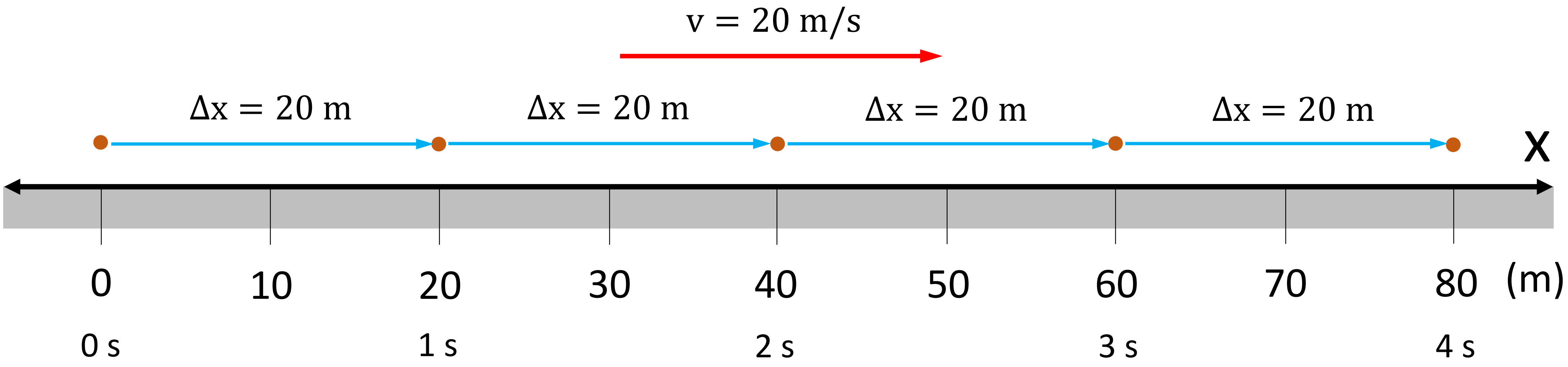
Velocity

The rate of change of position of an object in a given direction is called its velocity.

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

- Vector quantity (magnitude & direction).
- SI unit is metre/second (m/s).
- Dimensional formula is $[L^1T^{-1}]$.

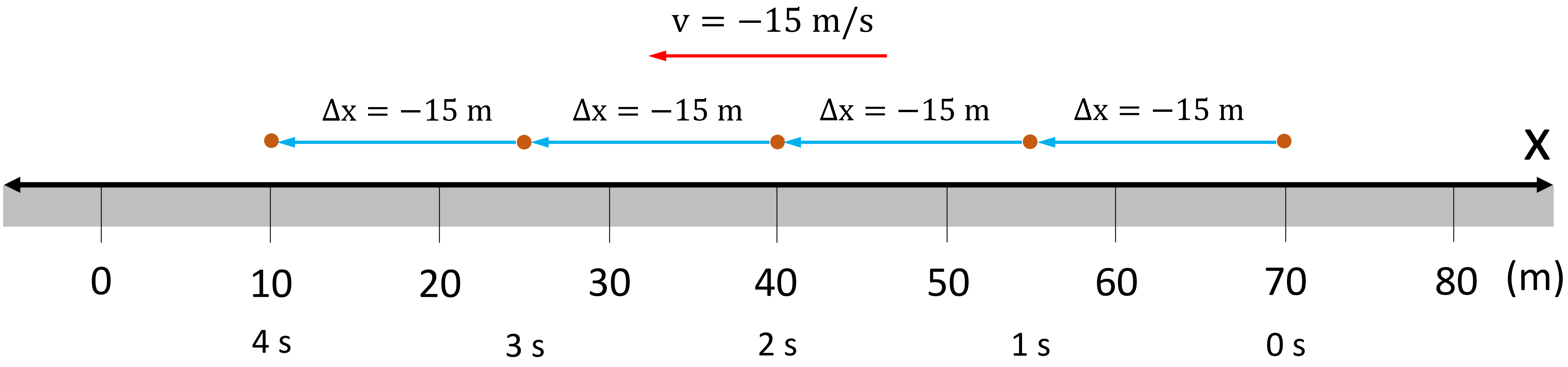
Case-I: Velocity is positive



$$\text{velocity} = \frac{20}{1} = 20\ \text{m/s}$$

Velocity is positive because change in position is positive.

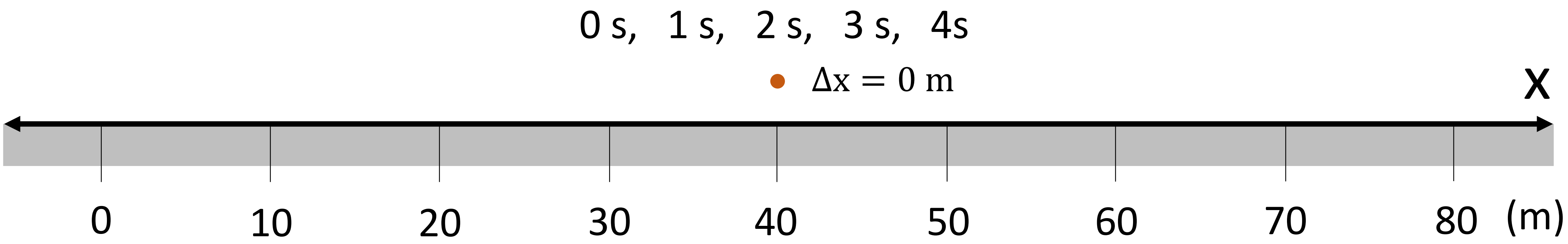
Case-II: Velocity is negative



$$\text{velocity} = \frac{-15}{1} = -15\ \text{m/s}$$

Velocity is negative because change in position is negative.

Case-III: Velocity is zero



$$\text{velocity} = \frac{0}{1} = 0 \text{ m/s}$$

Velocity is zero because change in position is zero.

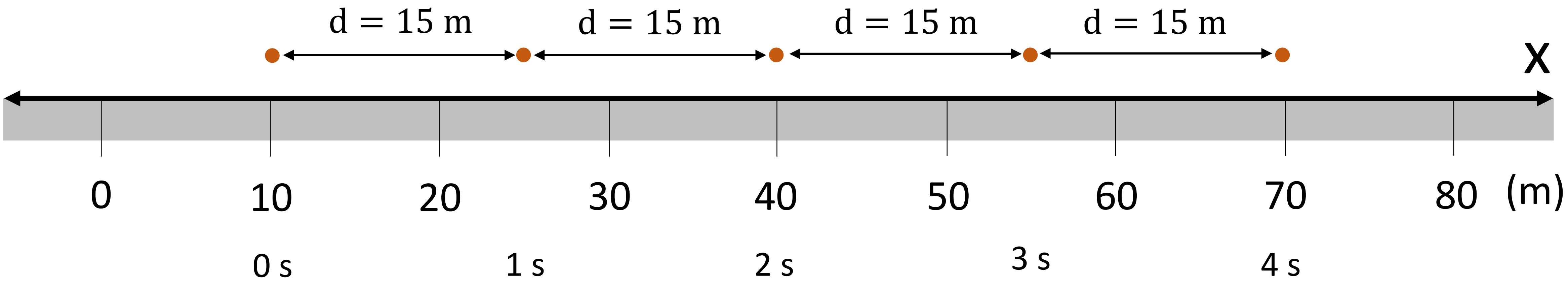
Speed

The rate at which an object covers a distance is called a speed.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

- Scalar quantity (only magnitude).
- SI unit is metre/second (m/s).
- Dimensional formula is $[L^1T^{-1}]$.

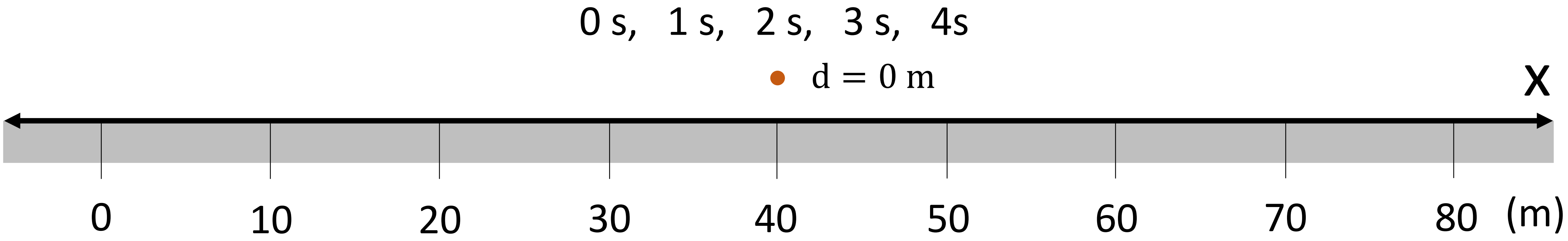
Case-I: Speed is positive



$$\text{speed} = \frac{15}{1} = 15\ \text{m/s}$$

If an object moves it cover some distance and hence the speed is positive.

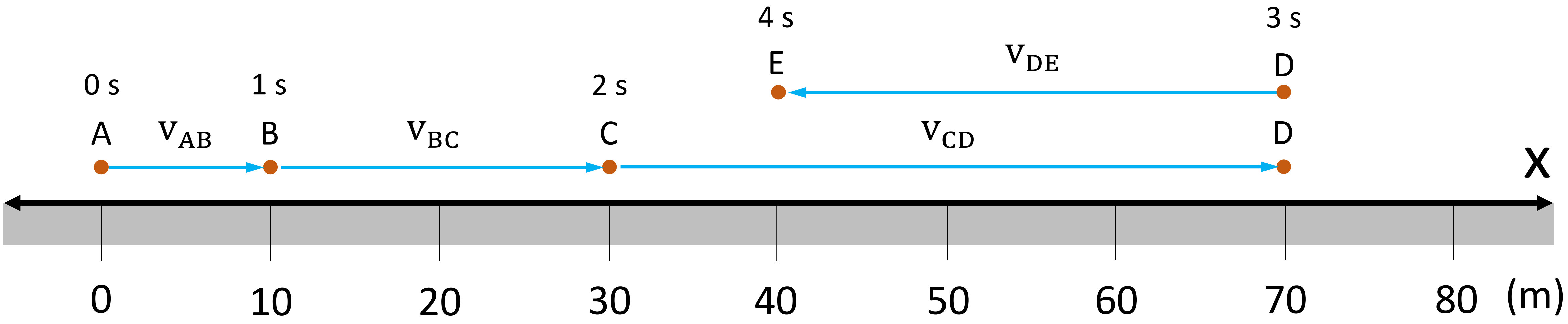
Case-II: Speed is zero



$$\text{speed} = \frac{0}{1} = 0 \text{ m/s}$$

If an object is at rest the distance covered is zero and hence the speed is zero.

Average velocity



$$v_{AB} = \frac{10 - 0}{1 - 0} = \frac{10}{1} = 10 \text{ m/s}$$

$$v_{BC} = \frac{30 - 10}{2 - 1} = \frac{20}{1} = 20 \text{ m/s}$$

$$v_{CD} = \frac{70 - 30}{3 - 2} = \frac{40}{1} = 40 \text{ m/s}$$

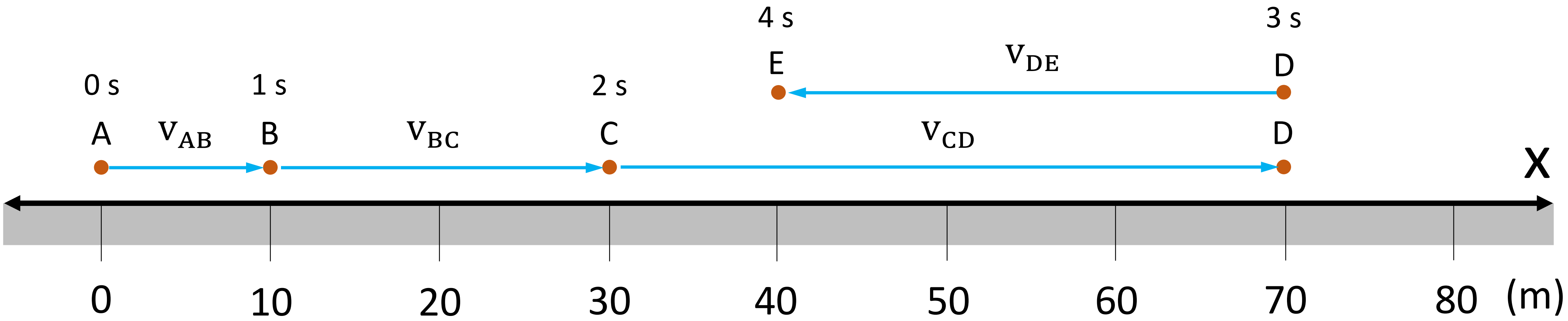
$$v_{DE} = \frac{40 - 70}{4 - 3} = \frac{-30}{1} = -30 \text{ m/s}$$

Average velocity

For an object moving with variable velocity, average velocity is defined as the ratio of its total displacement to the total time interval in which that displacement occurs.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}}$$

Average velocity



$$v_{avg} = \frac{10 + 20 + 40 - 30}{1 + 1 + 1 + 1}$$

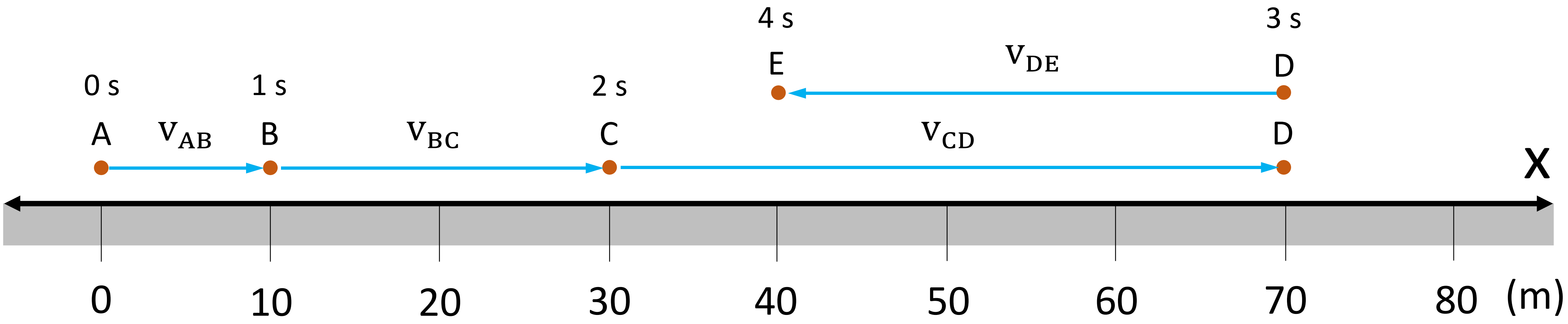
$$v_{avg} = \frac{40}{4} = 10\ \mathrm{m/s}$$

Average velocity

If x_1 and x_2 are the positions of an object at times t_1 and t_2, then the average velocity from time t_1 to t_2 is given by

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

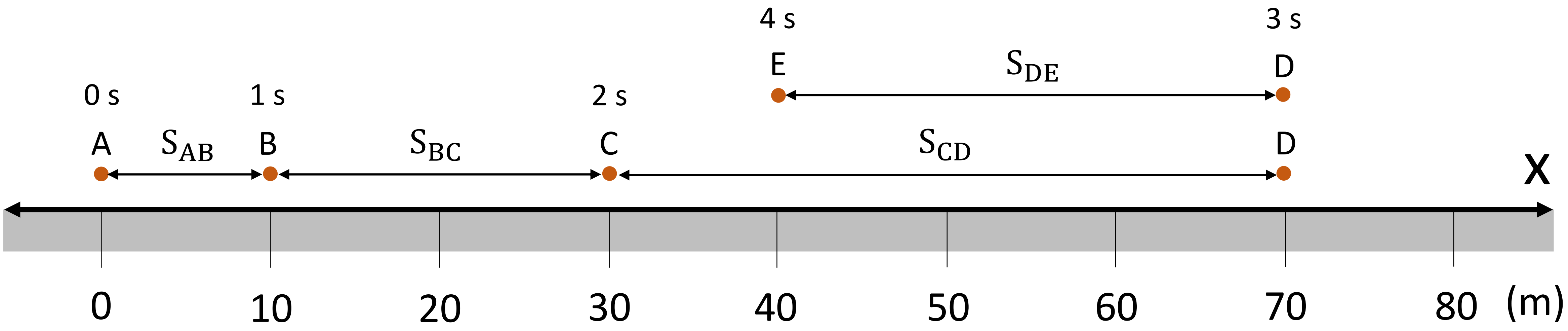
Average velocity



$$v_{avg} = \frac{40 - 0}{4 - 0} = \frac{40}{4}$$

$$v_{avg} = 10\ \mathrm{m/s}$$

Average speed



$$S_{AB} = \frac{|10-0|}{1-0} = \frac{|10|}{1} = 10\ \mathrm{m/s}$$

$$S_{BC} = \frac{|30-10|}{2-1} = \frac{|20|}{1} = 20\ \mathrm{m/s}$$

$$S_{CD} = \frac{|70-30|}{3-2} = \frac{|40|}{1} = 40\ \mathrm{m/s}$$

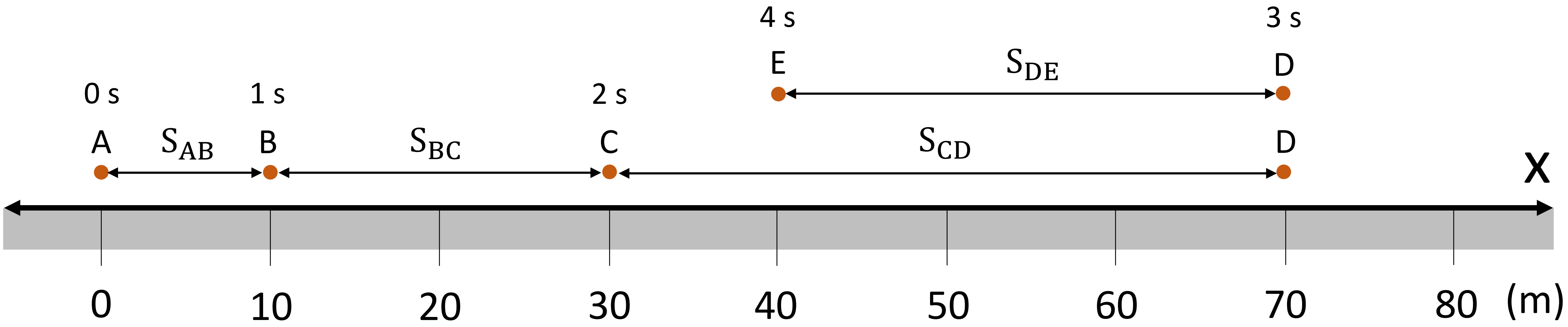
$$S_{DE} = \frac{|40-70|}{4-3} = \frac{|-30|}{1} = 30\ \mathrm{m/s}$$

Average speed

For an object moving with variable speed, the average speed is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Average speed

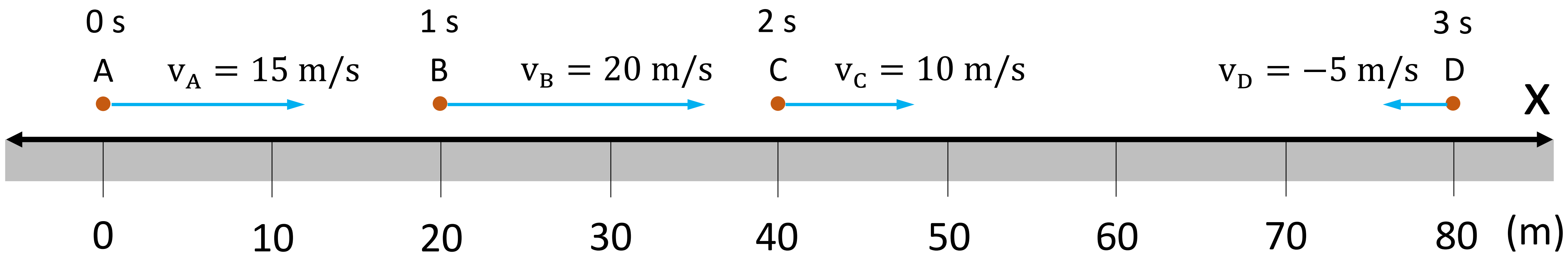


$$S_{avg} = \frac{10 + 20 + 40 + 30}{1 + 1 + 1 + 1}$$

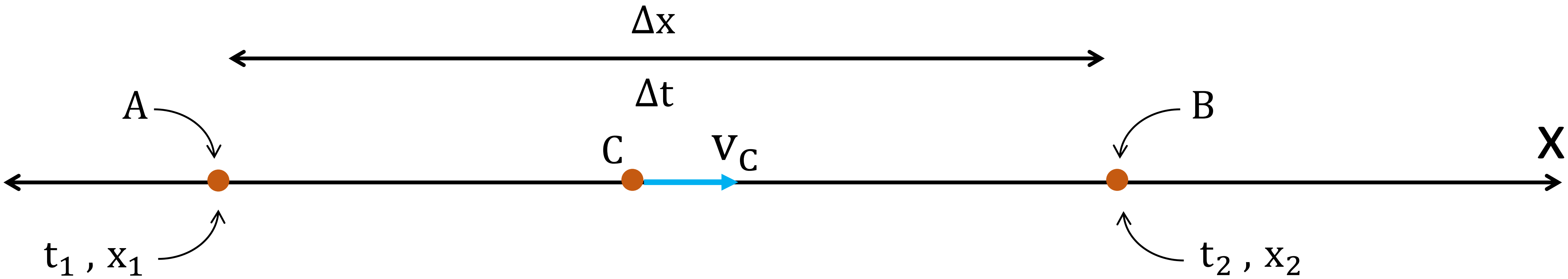
$$S_{avg} = \frac{100}{4} = 25 \text{ m/s}$$

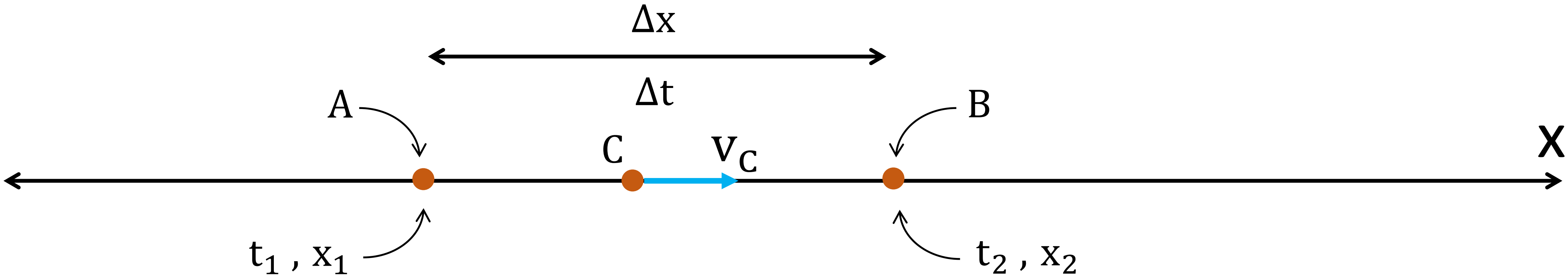
Instantaneous velocity

The velocity of an object at a particular instant of time or at a particular point of its path is called its instantaneous velocity.

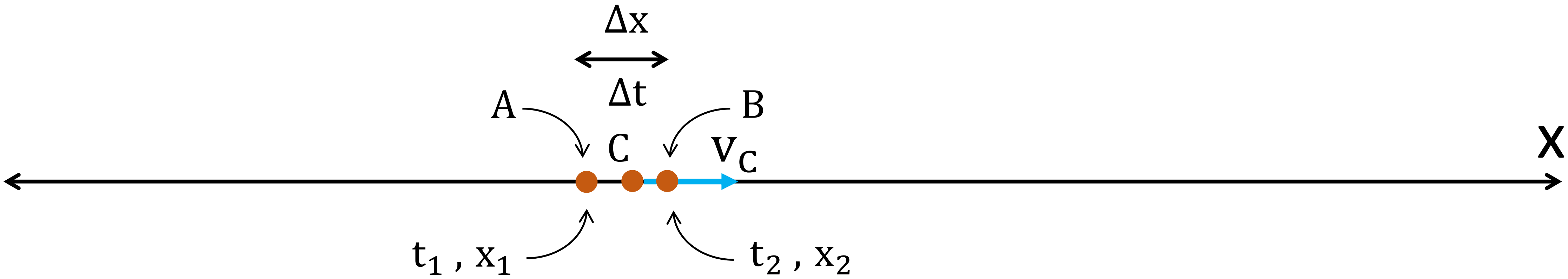


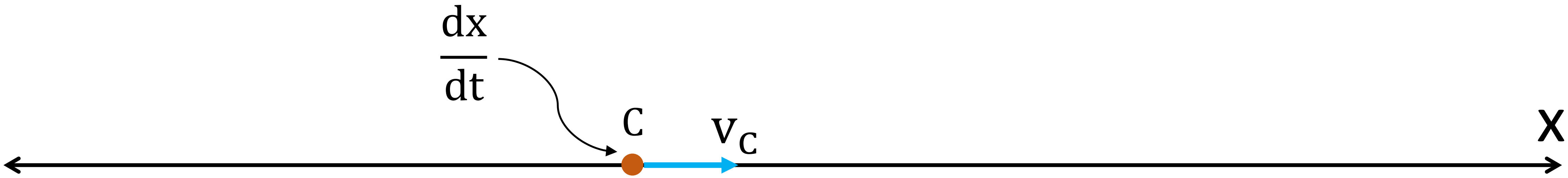
Instantaneous velocity





Instantaneous velocity





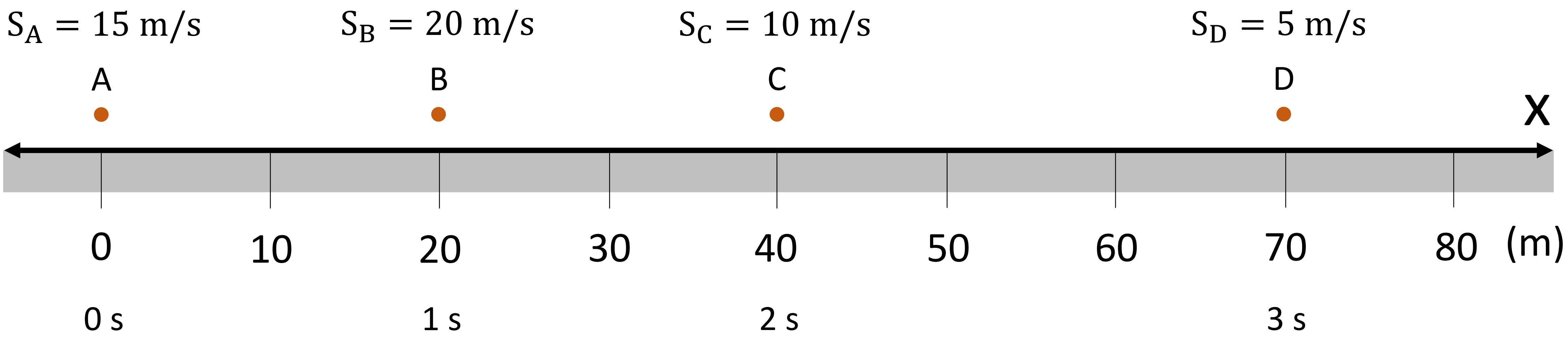
Instantaneous velocity

Instantaneous velocity is equal to the limiting value of the average velocity of the object in a small time interval taken around that instant, when time interval approaches zero.

$$v = \lim_{\Delta t \to 0} \frac{x_2 - x_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed

The magnitude of instantaneous velocity is called instantaneous speed.



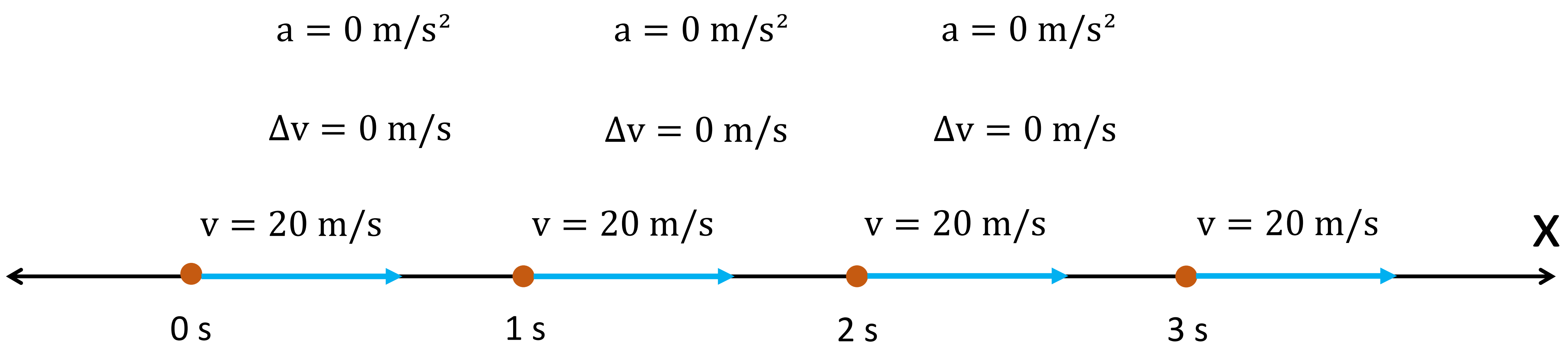
Acceleration

The rate of change of velocity of an object with time is called its acceleration.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

- Vector quantity (magnitude & direction).
- SI unit is metre/second/second (m/s^2).
- Dimensional formula is $[L^1T^{-2}]$.

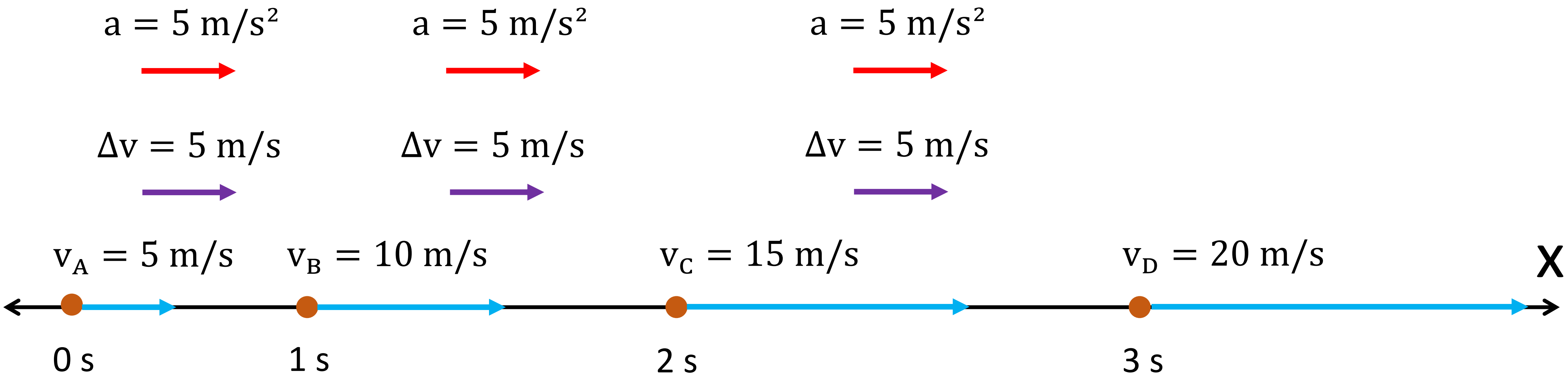
Case-I: Velocity is constant



$$\text{acceleration} = 0\ \text{m/s}^2$$

Acceleration is zero because change in velocity is zero.

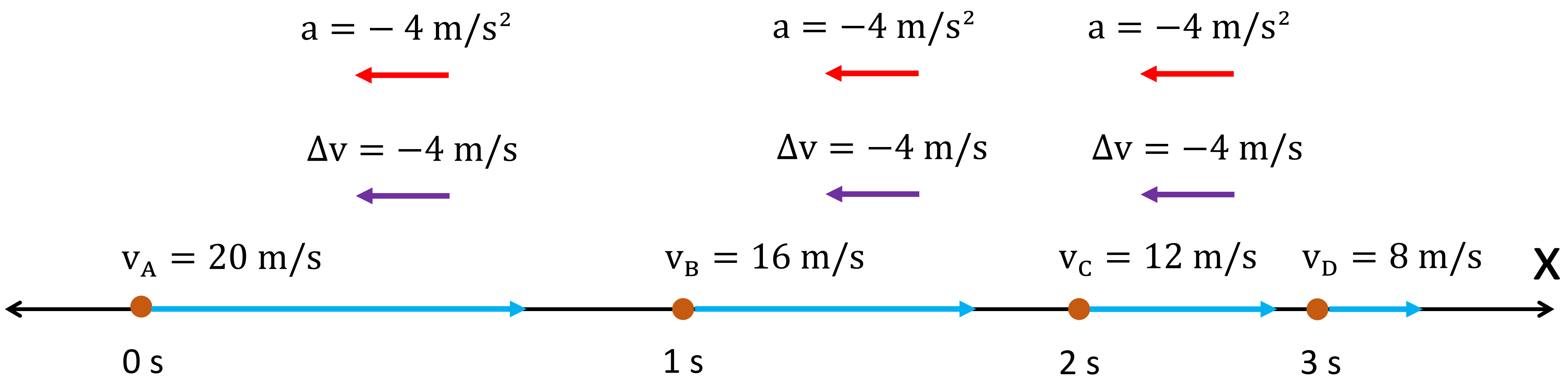
Case-II: Velocity is increasing towards right



$$\text{acceleration} = 5\ \text{m/s}^2$$

Acceleration is positive because change in velocity is positive.

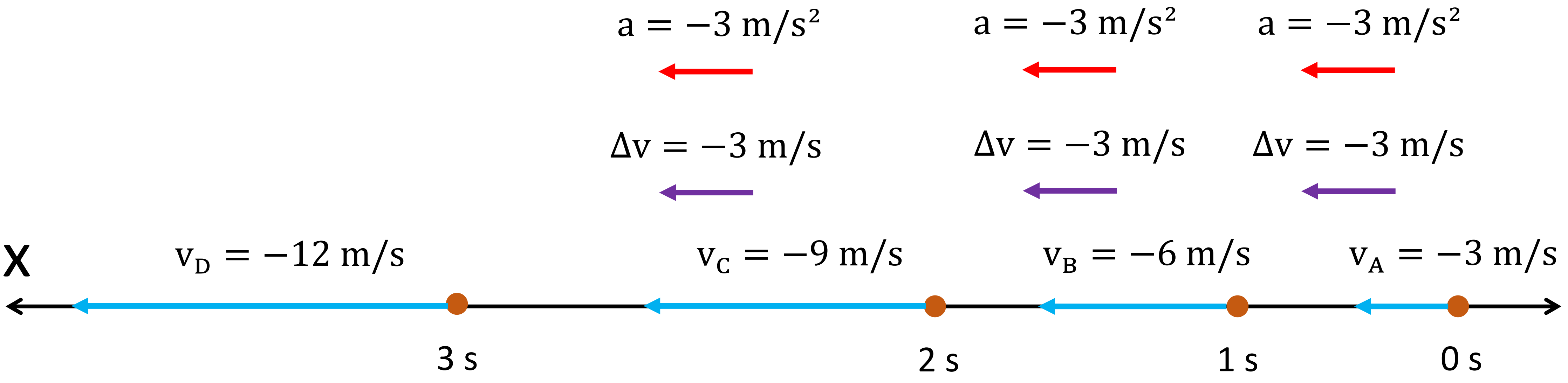
Case-III: Velocity is decreasing towards right



$$\text{acceleration} = -4\ m/s^2$$

Acceleration is negative because change in velocity is negative.

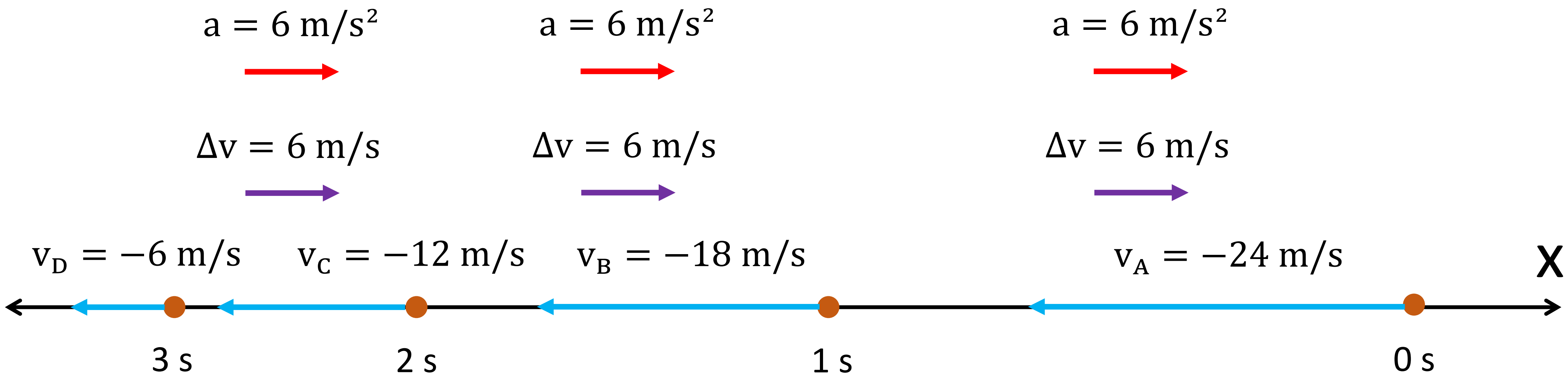
Case-IV: Velocity is increasing towards left



$$\text{acceleration} = -3\ m/s^2$$

Acceleration is negative because change in velocity is negative.

Case-V: Velocity is increasing towards left



$$\text{acceleration} = 6\ m/s^2$$

Acceleration is positive because change in velocity is positive.

Important note

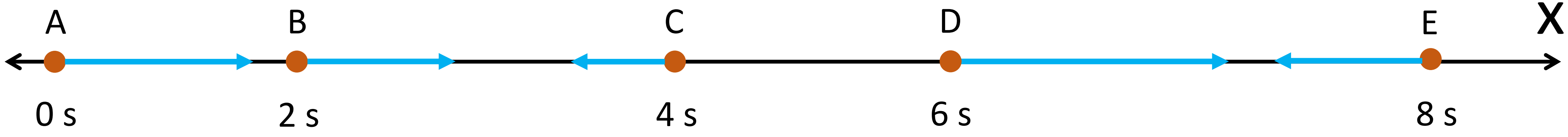
- If the signs of the velocity and acceleration of an object are the same, the speed of the object increases.

- If the signs are opposite, the speed decreases.

Average acceleration

$\Delta v_{AB} = -5\ \text{m/s}$ $\quad \Delta v_{BC} = -20\ \text{m/s}$ $\quad \Delta v_{CD} = 35\ \text{m/s}$ $\quad \Delta v_{DE} = -40\ \text{m/s}$

$v_A = 20\ \text{m/s}$ $\quad v_B = 15\ \text{m/s}$ $\quad v_C = -5\ \text{m/s}$ $\quad v_D = 30\ \text{m/s}$ $\quad v_E = -10\ \text{m/s}$



$$a_{AB} = \frac{15 - 20}{2 - 0} = \frac{-5}{2} = -2.5\ \text{m/s}^2$$

$$a_{BC} = \frac{-5 - 15}{4 - 2} = \frac{-20}{2} = -10\ \text{m/s}^2$$

$$a_{CD} = \frac{30 - (-5)}{6 - 4} = \frac{35}{2} = 17.5\ \text{m/s}^2$$

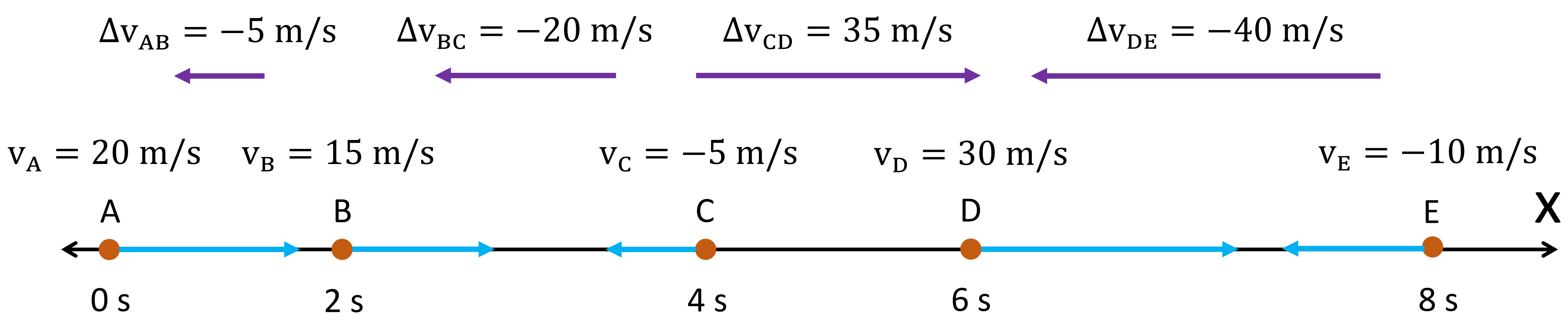
$$a_{DE} = \frac{-10 - 30}{8 - 6} = \frac{-40}{2} = -20\ \text{m/s}^2$$

Average acceleration

For an object moving with variable velocity, the average acceleration is defined as the ratio of the total change in velocity of the object to the total time interval taken.

$$\text{Average acceleration} = \frac{\text{total change in velocity}}{\text{total time}}$$

Average acceleration



$$a_{avg} = \frac{-5 - 20 + 35 - 40}{2 + 2 + 2 + 2}$$

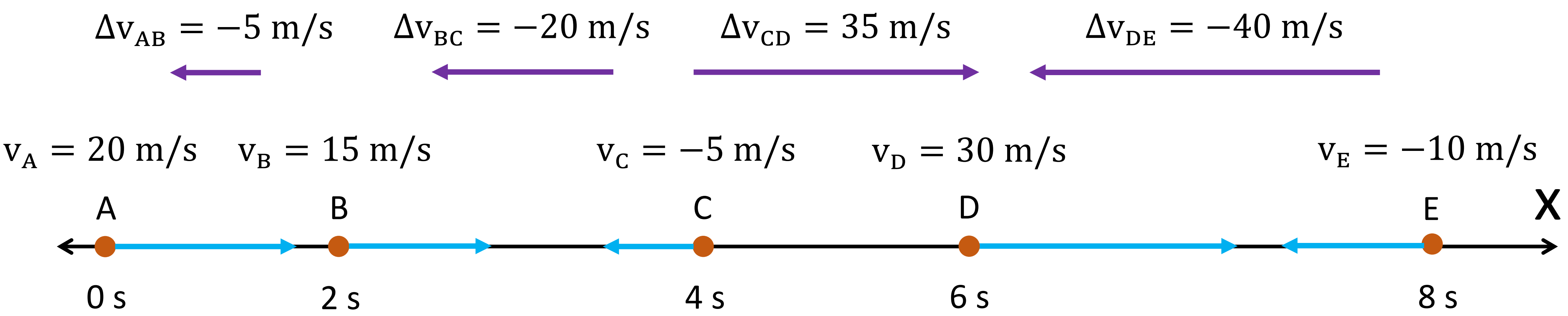
$$a_{avg} = \frac{-30}{8} = -3.75\ \text{m/s}^2$$

Average acceleration

If v_1 and v_2 are the velocities of an object at times t_1 and t_2, then the average acceleration from time t_1 to t_2 is given by

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Average acceleration

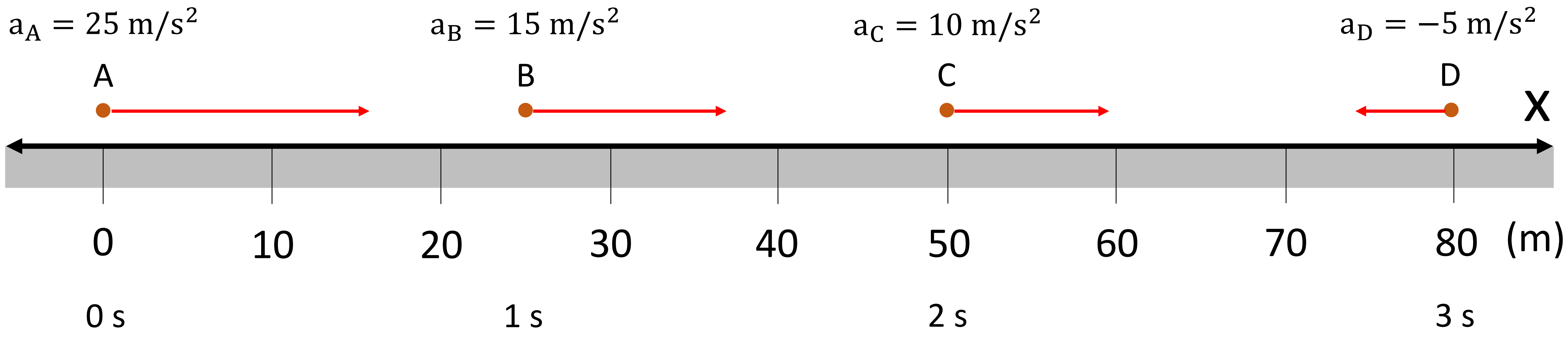


$$a_{avg} = \frac{-10 - 20}{8 - 0} = \frac{-30}{8}$$

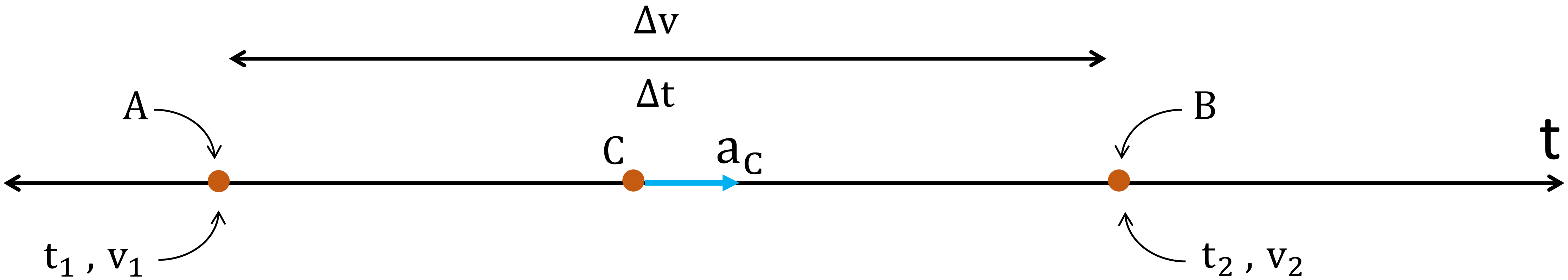
$$a_{avg} = -3.75\ \mathrm{m/s^2}$$

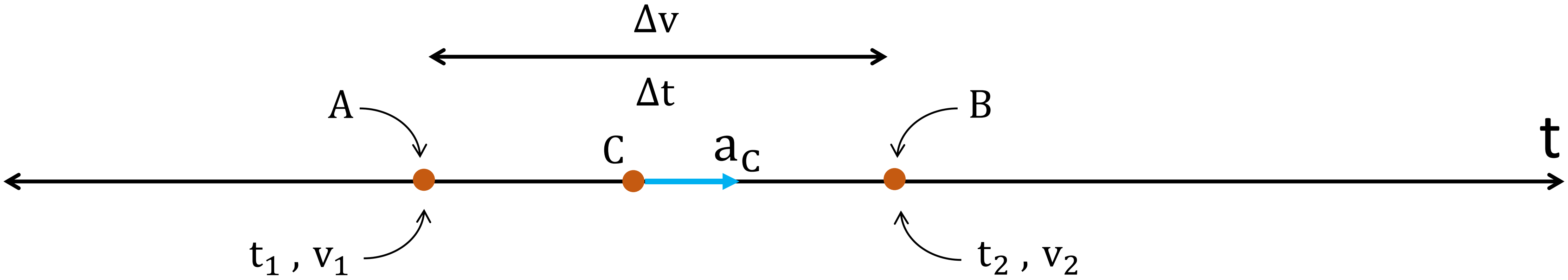
Instantaneous acceleration

The acceleration of an object at a particular instant of time or at a particular point of its path is called its instantaneous acceleration.

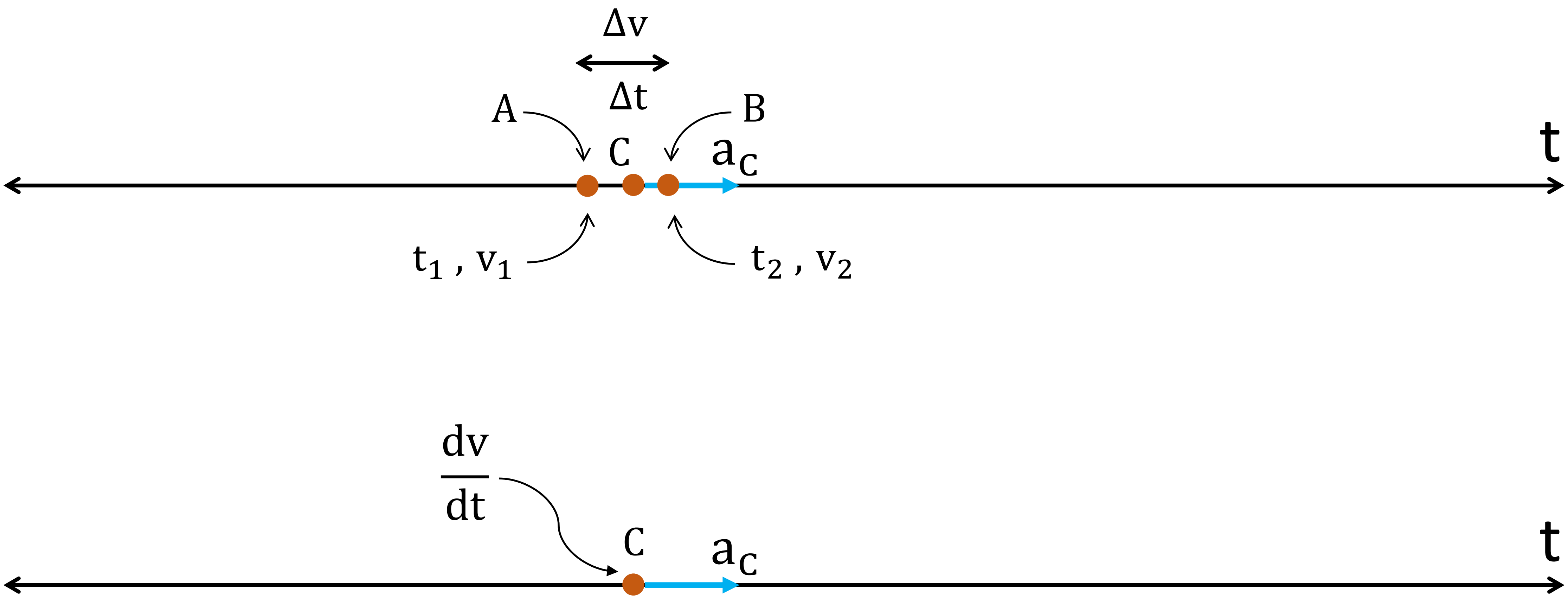


Instantaneous acceleration





Instantaneous acceleration



Instantaneous acceleration

Instantaneous acceleration is equal to the limiting value of the average acceleration of the object in a small time interval taken around that instant, when time interval approaches zero.

$$a = \lim_{\Delta t \to 0} \frac{v_2 - v_1}{t_2 - t_1} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Uniformly accelerated motion

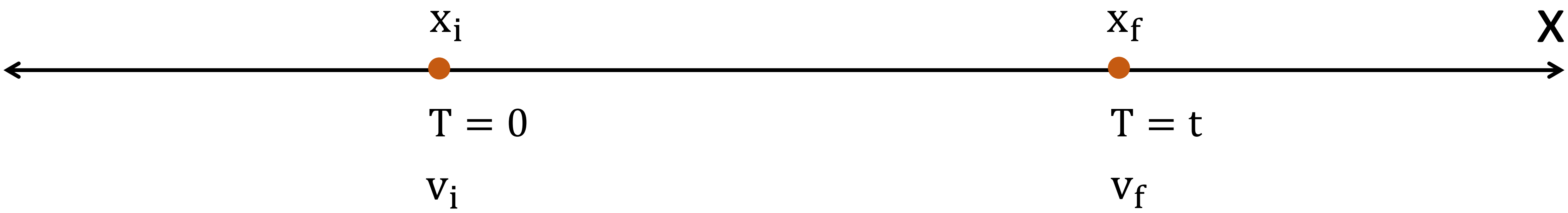
The motion in which the velocity of an object changes with uniform rate or constant rate is called uniformly accelerated motion.

Uniformly accelerated motion

A car slowing down after a red signal

A car speeding up after a green signal

Kinematical equations of motion



1. Velocity after a certain time:

By definition,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$a = \frac{v_f - v_i}{t - 0}$$

$$v_f - v_i = a\,t$$

$$\boxed{v_f = v_i + a\,t}$$

This is the first kinematical equation.

Kinematical equations of motion

2. Displacement in a certain time:

By definition,

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}}$$

$$v_{avg} = \frac{x_f - x_i}{t - 0}$$

But,

$$v_{avg} = \frac{v_i + v_f}{2}$$

$$\therefore \quad \frac{x_f - x_i}{t} = \frac{v_i + v_f}{2}$$

$$\frac{x_f - x_i}{t} = \frac{v_i + v_i + at}{2} = \frac{2v_i + a\,t}{2}$$

$$\frac{x_f - x_i}{t} = \frac{2v_i}{2} + \frac{a\,t}{2}$$

$$\frac{x_f - x_i}{t} = v_i + \frac{1}{2}a\,t$$

$$\boxed{x_f - x_i = v_i t + \frac{1}{2}a\,t^2}$$

This is the second kinematical equation.

Kinematical equations of motion

3. Velocity after certain displacement:

We know that,

$$v_f = v_i + a\,t$$

$$v_f - v_i = a\,t \qquad \dots\dots..(1)$$

Also,

$$\frac{v_f + v_i}{2} = \frac{x_f - x_i}{t}$$

$$v_f + v_i = x_f - x_i\left(\frac{2}{t}\right) \qquad \dots\dots..(2)$$

multiplying equations (1) & (2), we get

$$(v_f - v_i)(v_f + v_i) = a\,t\,(x_f - x_i)\left(\frac{2}{t}\right)$$

$$\mathrm{L.H.S} = v_f^2 - v_i^2$$

$$\mathrm{R.H.S} = 2\,a\,(x_f - x_i)$$

$$v_f^2 - v_i^2 = 2\,a\,(x_f - x_i)$$

$$\boxed{v_f^2 = v_i^2 + 2\,a\,(x_f - x_i)}$$

This is the third kinematical equation.

Equations of motion by calculus method

1. First equation of motion:

By definition,

$$a = \frac{dv}{dt}$$

$$dv = a\,dt \qquad \dots\dots..(1)$$

When time = 0, velocity = v_i

When time = t, velocity = v_f

Integrating equation (1) within the above limits of time & velocity, we get

$$\int_{v_i}^{v_f} dv = \int_0^t a\,dt = a\int_0^t dt$$

$$[v]_{v_i}^{v_f} = a\,[t]_0^t$$

$$v_f - v_i = a\,(t - 0)$$

$$v_f - v_i = a\,t$$

$$\boxed{v_f = v_i + a\,t}$$

Equations of motion by calculus method

2. Second equation of motion:

By definition,

$$v = \frac{dx}{dt}$$

$$dx = v\,dt \qquad \ldots\ldots..(1)$$

When time = 0, position = x_i

When time = t, position = x_f

Integrating equation (1) within the above limits of time & position, we get

$$\int_{x_i}^{x_f} dx = \int_0^t v\,dt = \int_0^t (v_i + a\,t)\,dt$$

$$\int_{x_i}^{x_f} dx = \int_0^t v_i\,dt + \int_0^t a\,t\,dt$$

$$[x]_{x_i}^{x_f} = v_i\,[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$x_f - x_i = v_i\,(t - 0) + \frac{1}{2}a\,(t^2 - 0^2)$$

$$\boxed{x_f - x_i = v_i t + \frac{1}{2}a\,t^2}$$

Equations of motion by calculus method

3. Second equation of motion:

By definition,

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$$

$$v\,dv = a\,dx \qquad \ldots\ldots..(1)$$

When time = 0, velocity = v_i, position = x_i

When time = t, velocity = v_f, position = x_f

Integrating equation (1) within the above limits of velocity & position, we get

$$\int_{v_i}^{v_f} v\,dv = \int_{x_i}^{x_f} a\,dx$$

$$\left[\frac{v^2}{2}\right]_{v_i}^{v_f} = a\,[x]_{x_i}^{x_f}$$

$$[v^2]_{v_i}^{v_f} = 2a\,[x]_{x_i}^{x_f}$$

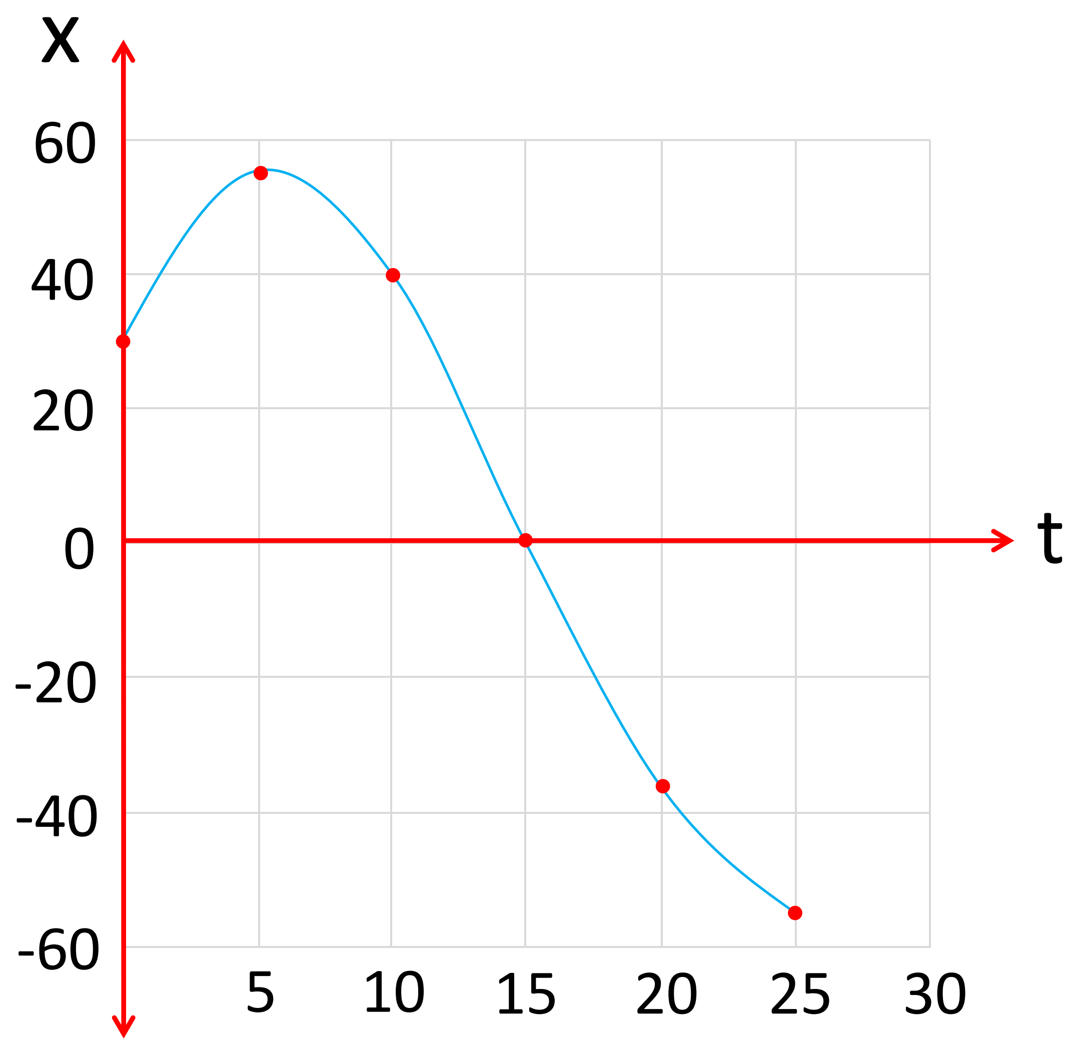
$$v_f^2 - v_i^2 = 2\,a\,(x_f - x_i)$$

$$\boxed{v_f^2 = v_i^2 + 2\,a\,(x_f - x_i)}$$

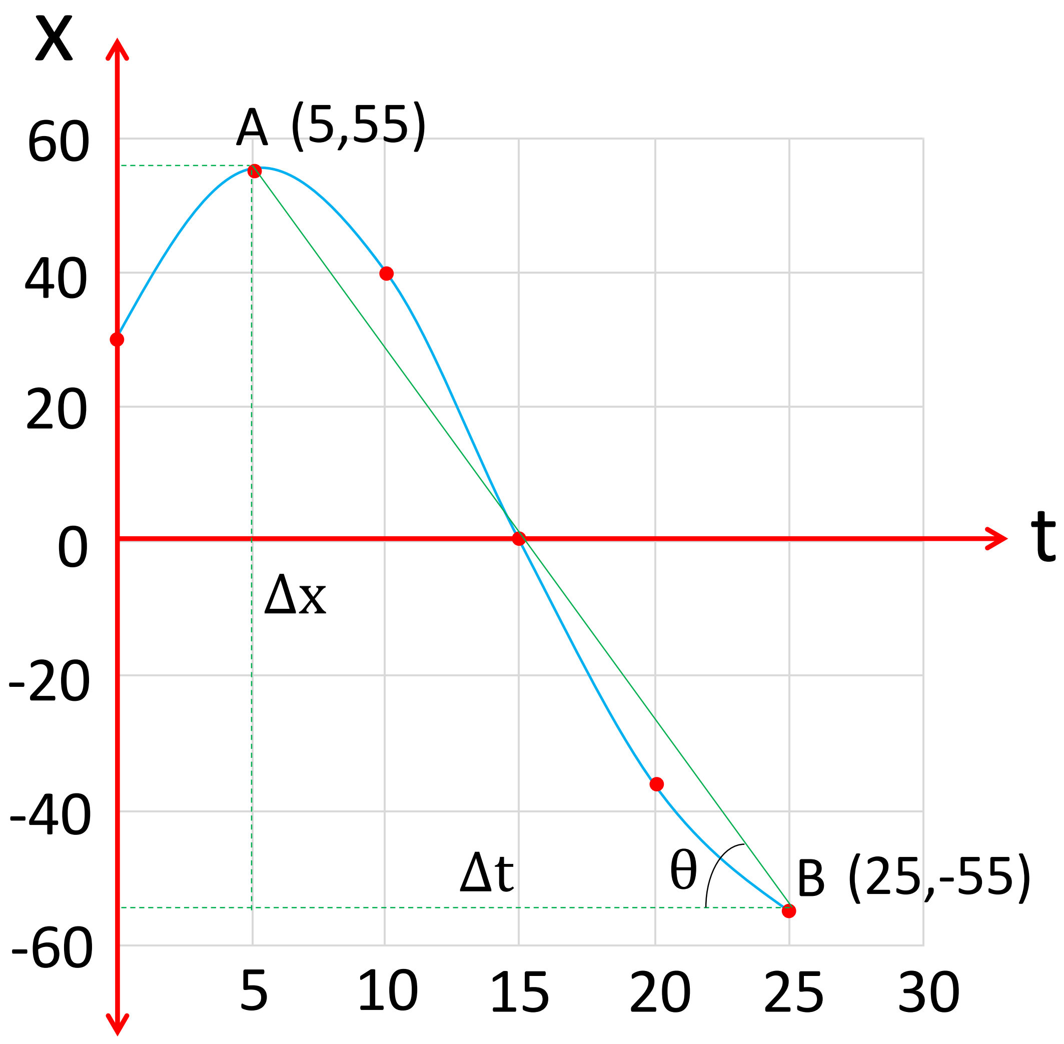
Graphical representation of motion

Position-time graph

Time (s)	Position (m)
0	30
5	55
10	40
15	0
20	-35
25	-55



Average velocity from position-time graph



Slope of line AB gives average velocity between points A & B.

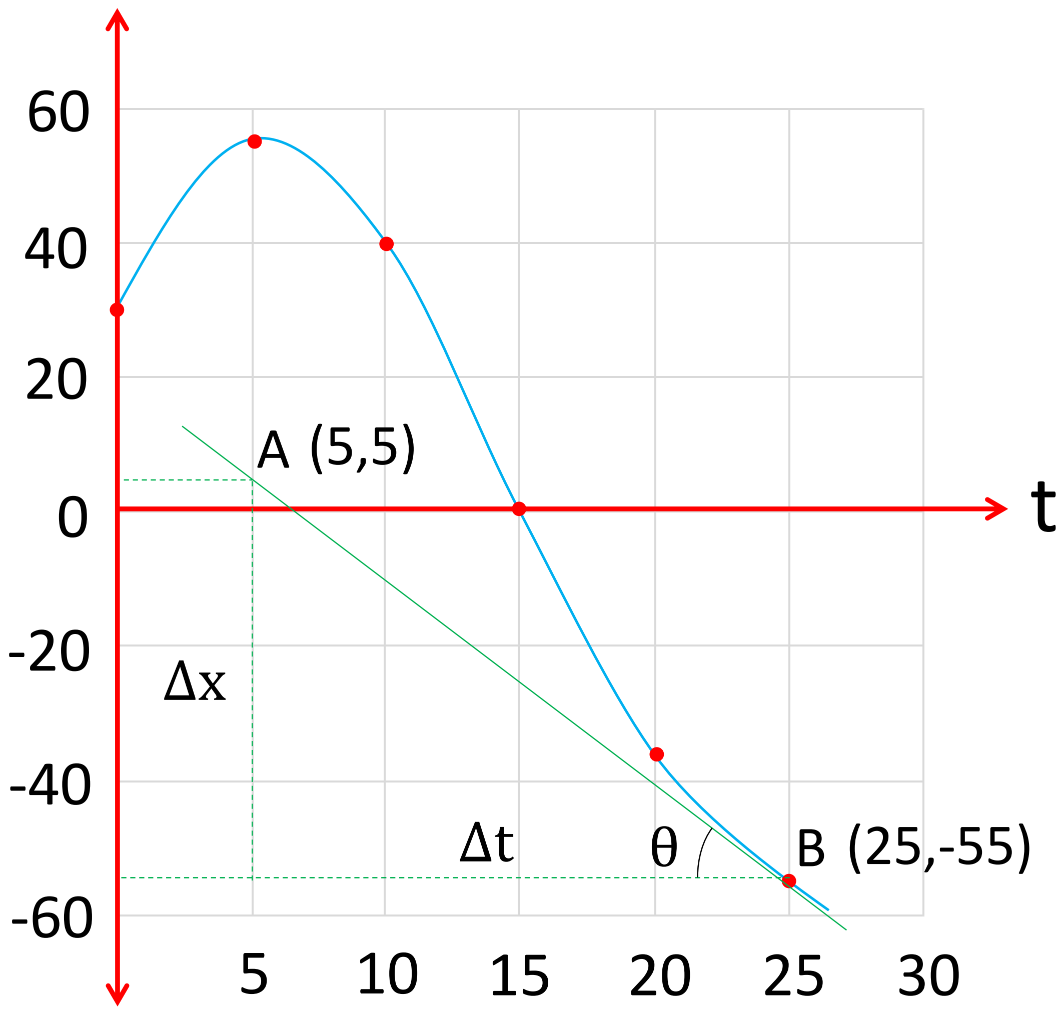
$$\text{slope of line AB} = \tan\theta$$

$$\tan\theta = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = v_{avg}$$

$$v_{avg(AB)} = \frac{-55 - 55}{25 - 5} = \frac{-110}{20}$$

$$v_{avg(AB)} = -5.5\ \mathrm{m/s}$$

Instantaneous velocity from position-time graph



Slope of tangent at point B gives instantaneous velocity at point B.

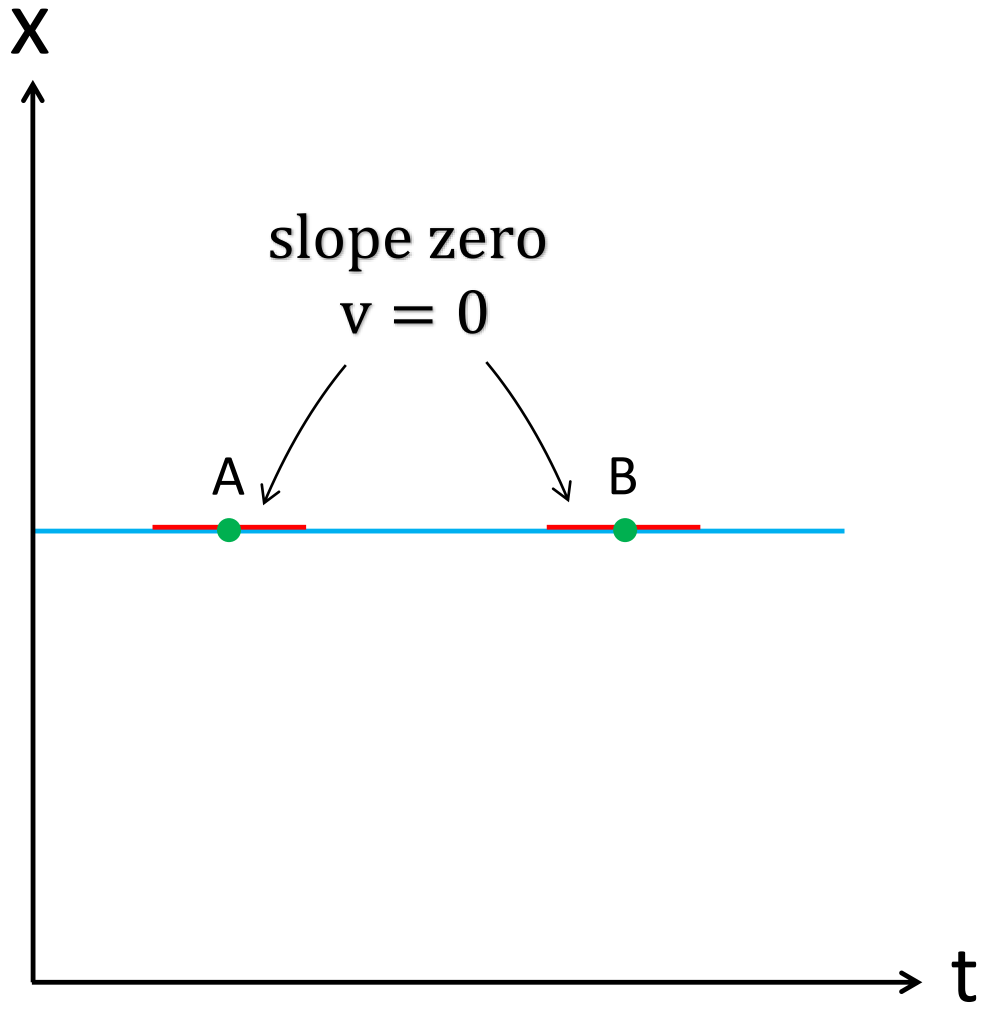
$$\text{slope of tangent} = \tan\theta$$

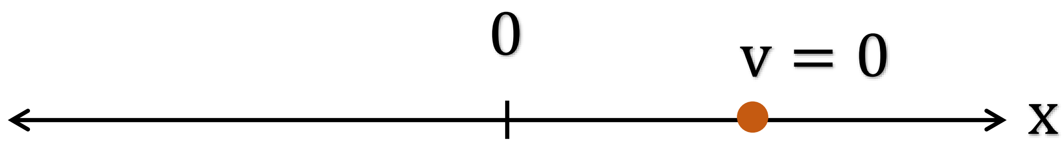
$$\tan\theta = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = v$$

$$v_B = \frac{-55 - 5}{25 - 5} = \frac{-60}{20}$$

$$v_B = -3 \text{ m/s}$$

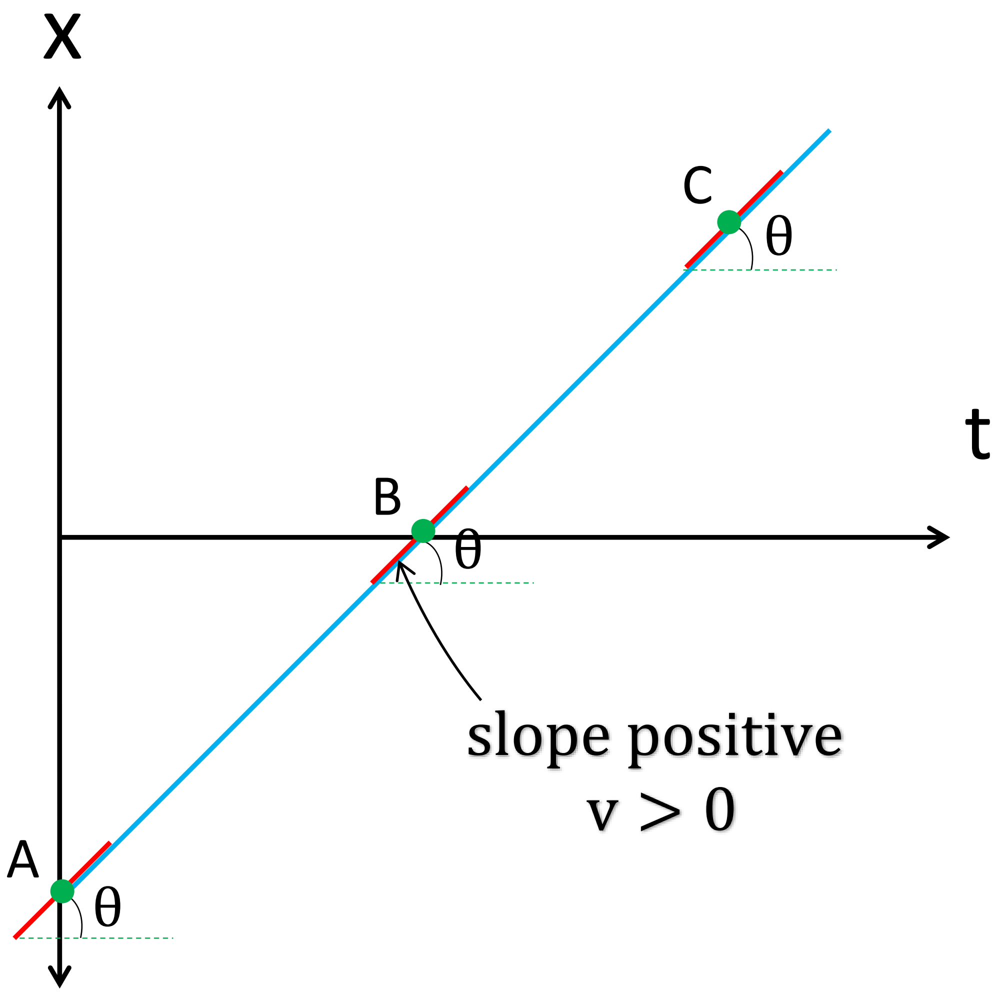
Analyzing nature of motion through position-time graph



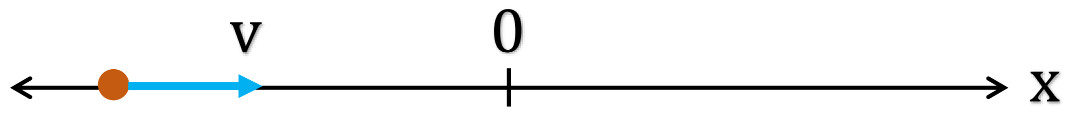


The particle is at rest on positive side of origin.

Analyzing nature of motion through position-time graph

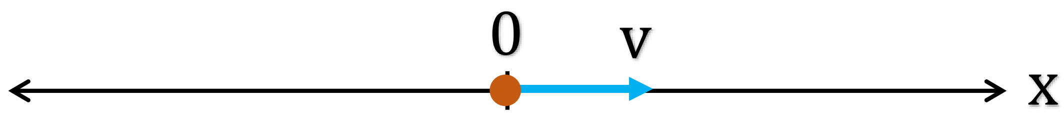


For portion AB:



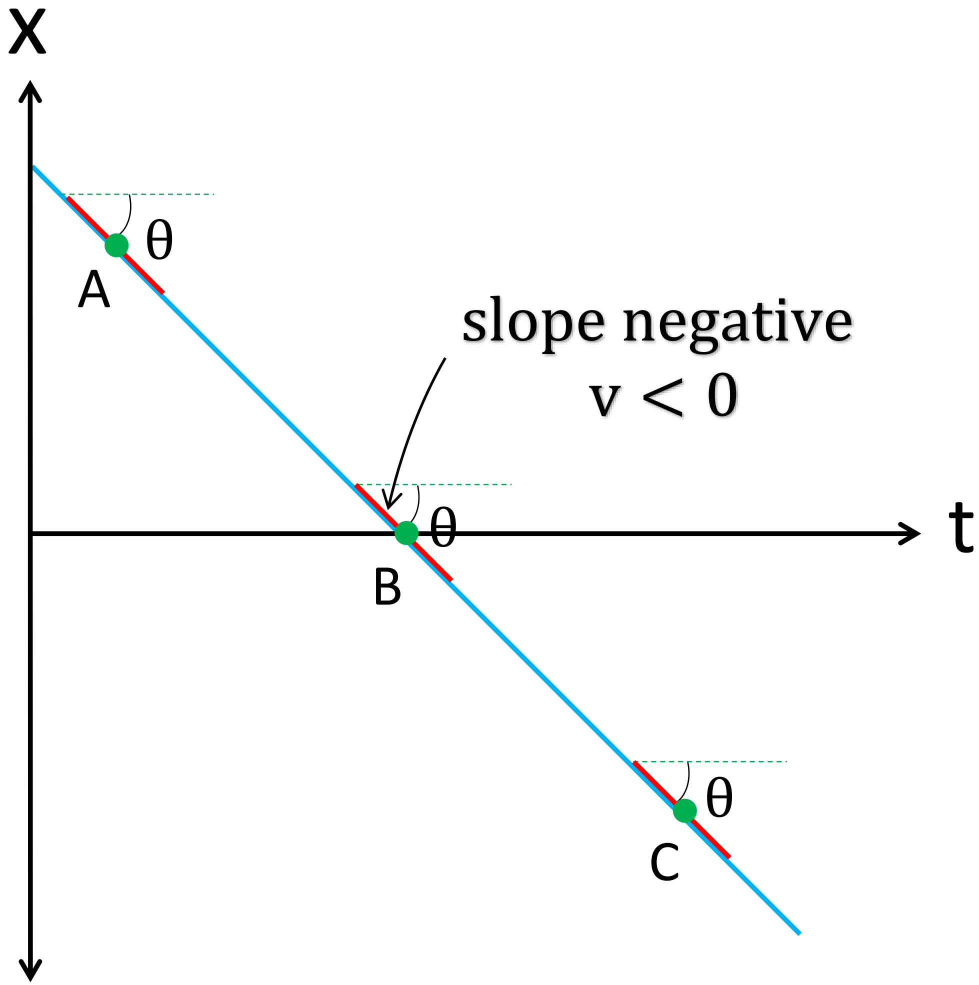
The particle is at negative side of origin and moving with constant velocity towards origin.

For portion BC:

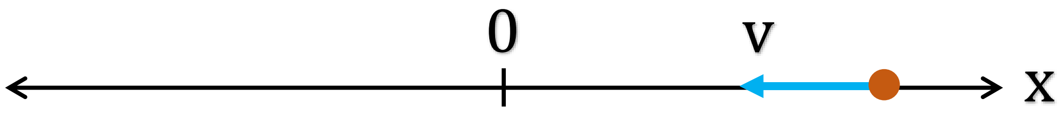


The particle is at origin and moving with constant velocity towards right side of origin.

Analyzing nature of motion through position-time graph

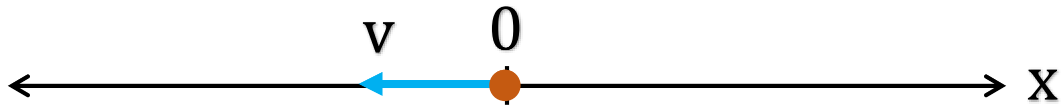


For portion AB:



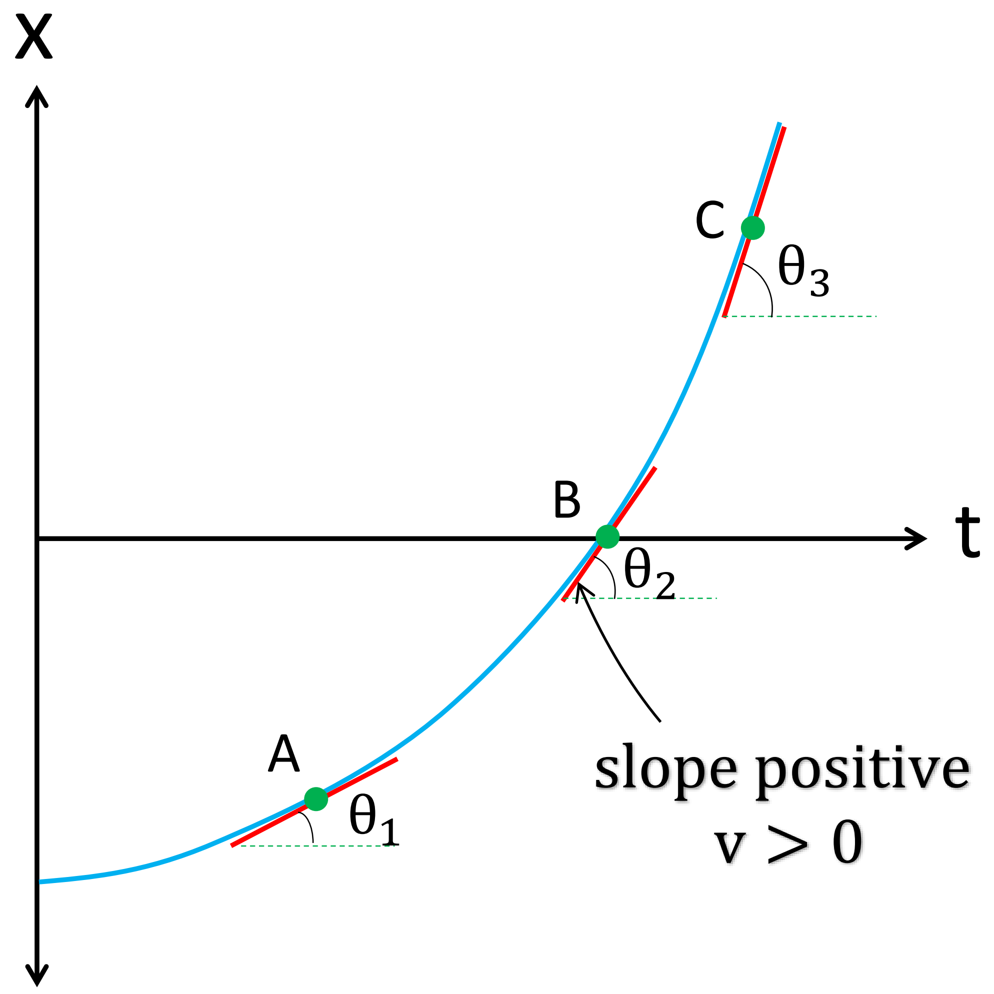
The particle is at positive side of origin and moving with constant velocity towards origin.

For portion BC:

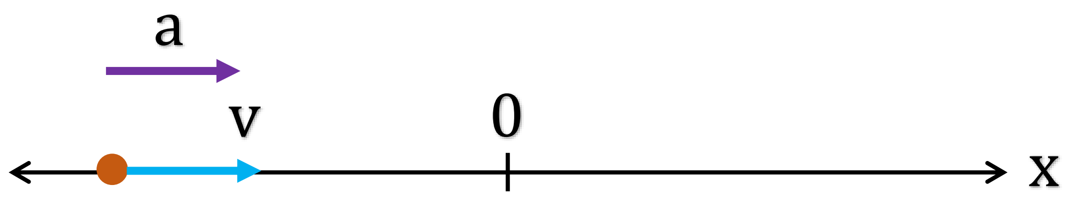


The particle is at origin and moving with constant velocity towards left side of origin.

Analyzing nature of motion through position-time graph

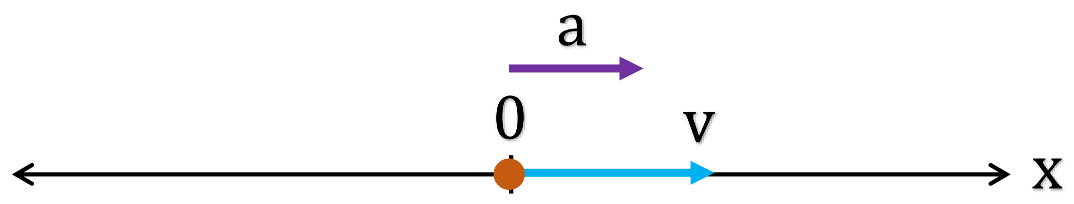


For portion AB:



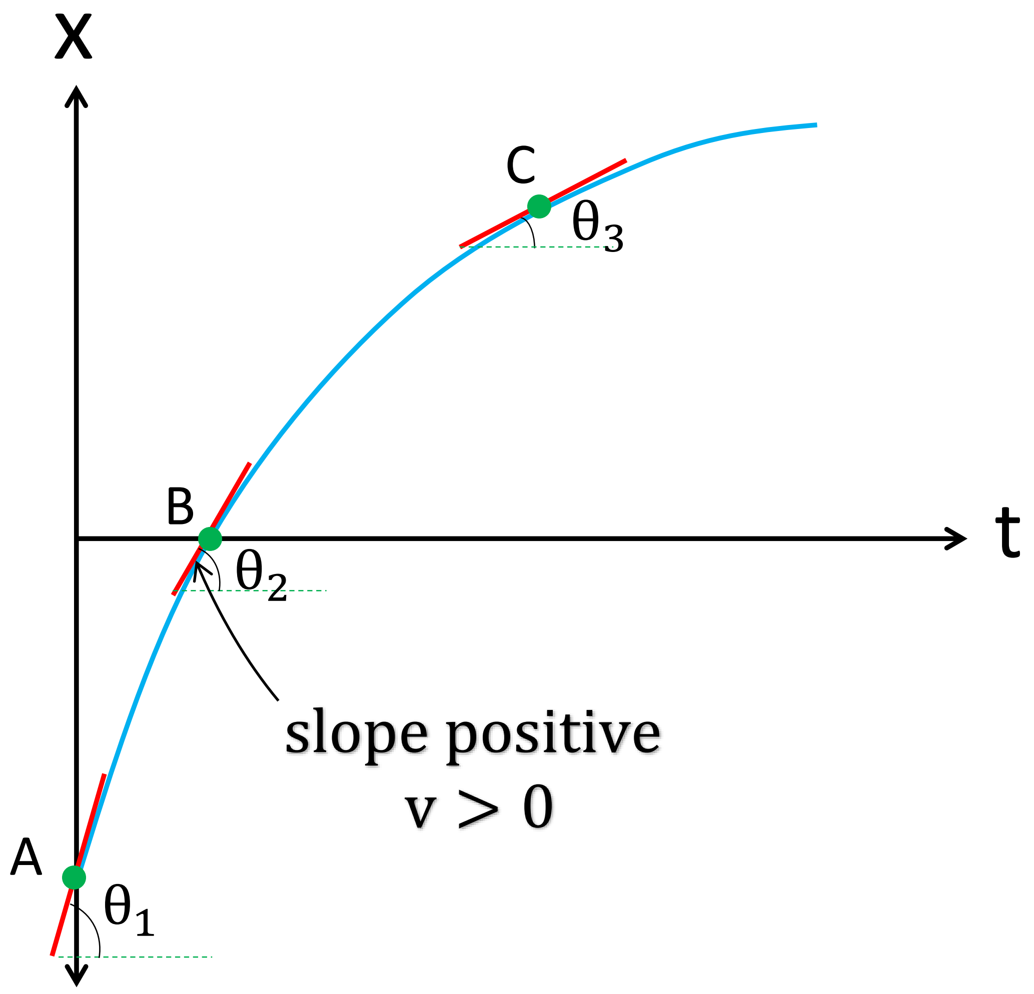
The particle is at negative side of origin and moving with increasing velocity towards origin.

For portion BC:

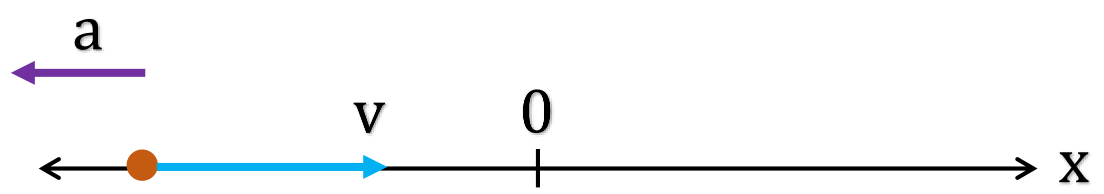


The particle is at origin and moving with increasing velocity towards right side of origin.

Analyzing nature of motion through position-time graph

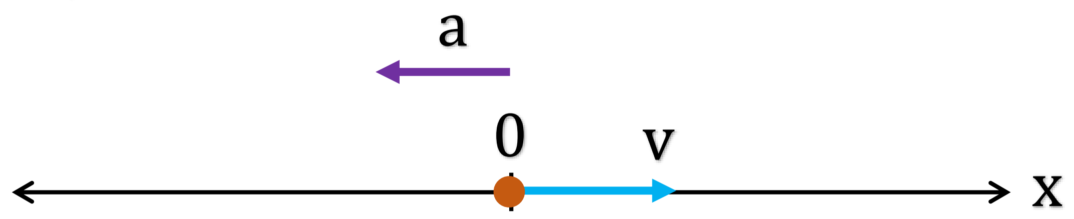


For portion AB:



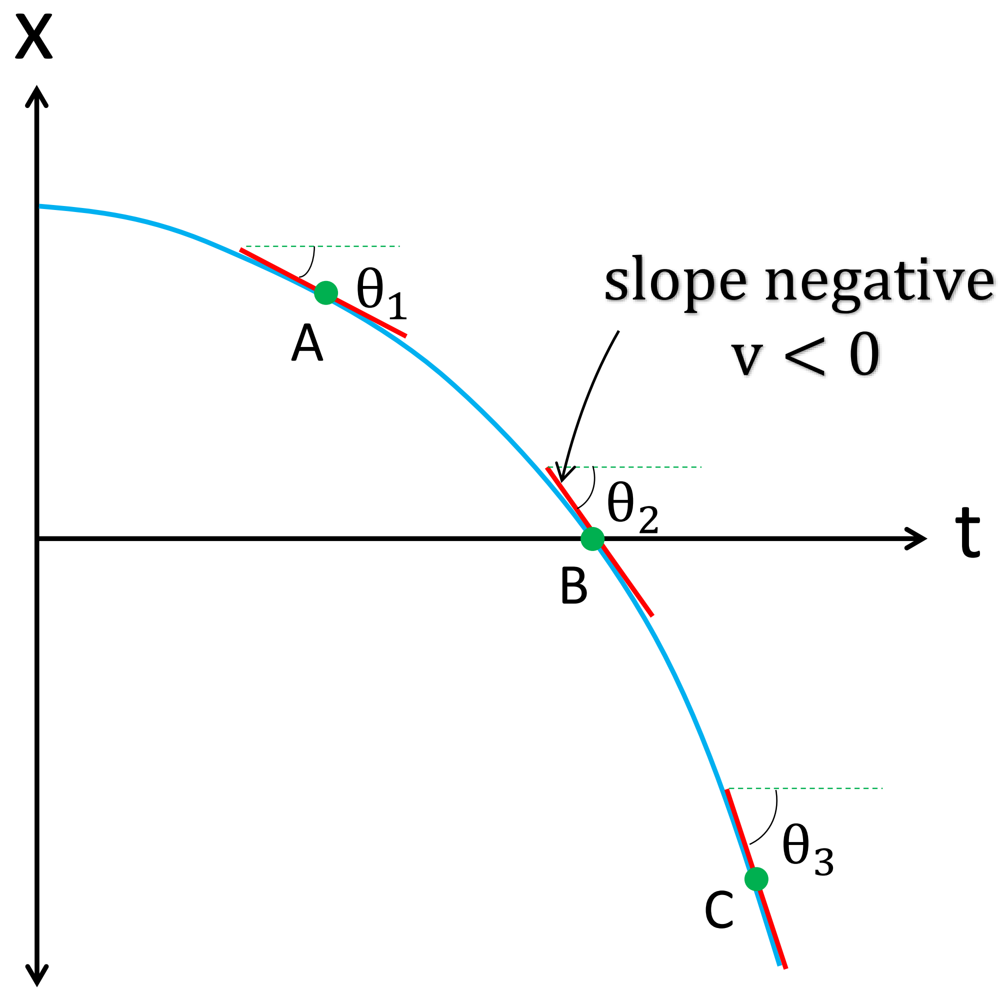
The particle is at negative side of origin and moving with decreasing velocity towards origin.

For portion BC:

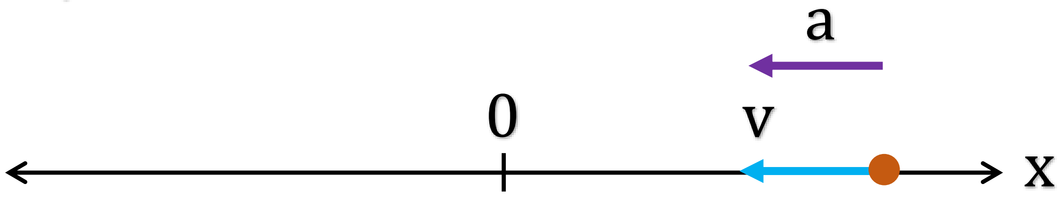


The particle is at origin and moving with decreasing velocity towards right side of origin.

Analyzing nature of motion through position-time graph

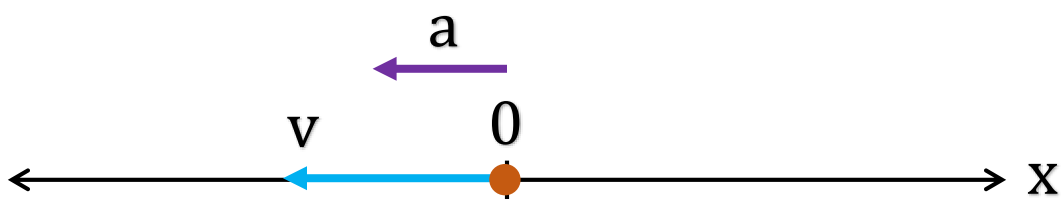


For portion AB:



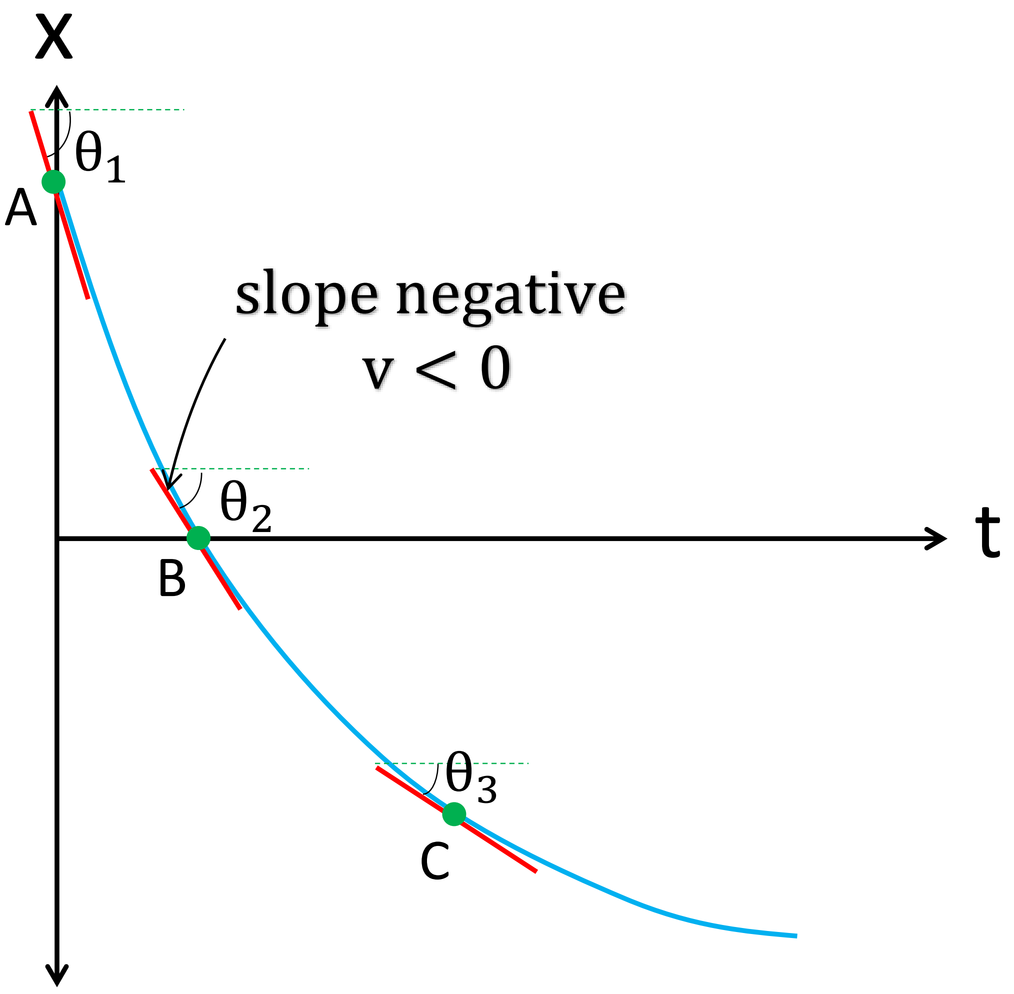
The particle is at positive side of origin and moving with increasing velocity towards origin.

For portion BC:

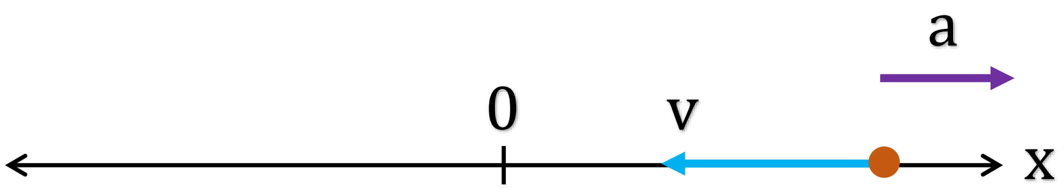


The particle is at origin and moving with increasing velocity towards left side of origin.

Analyzing nature of motion through position-time graph

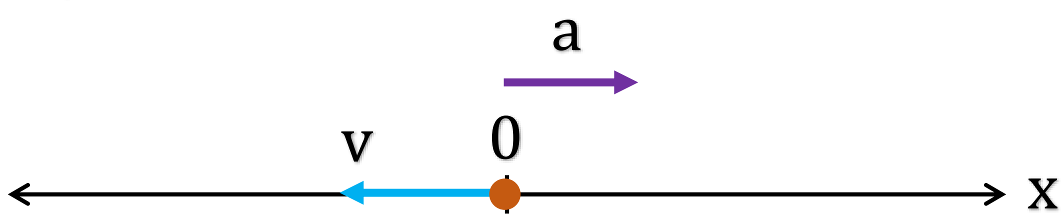


For portion AB:



The particle is at positive side of origin and moving with decreasing velocity towards origin.

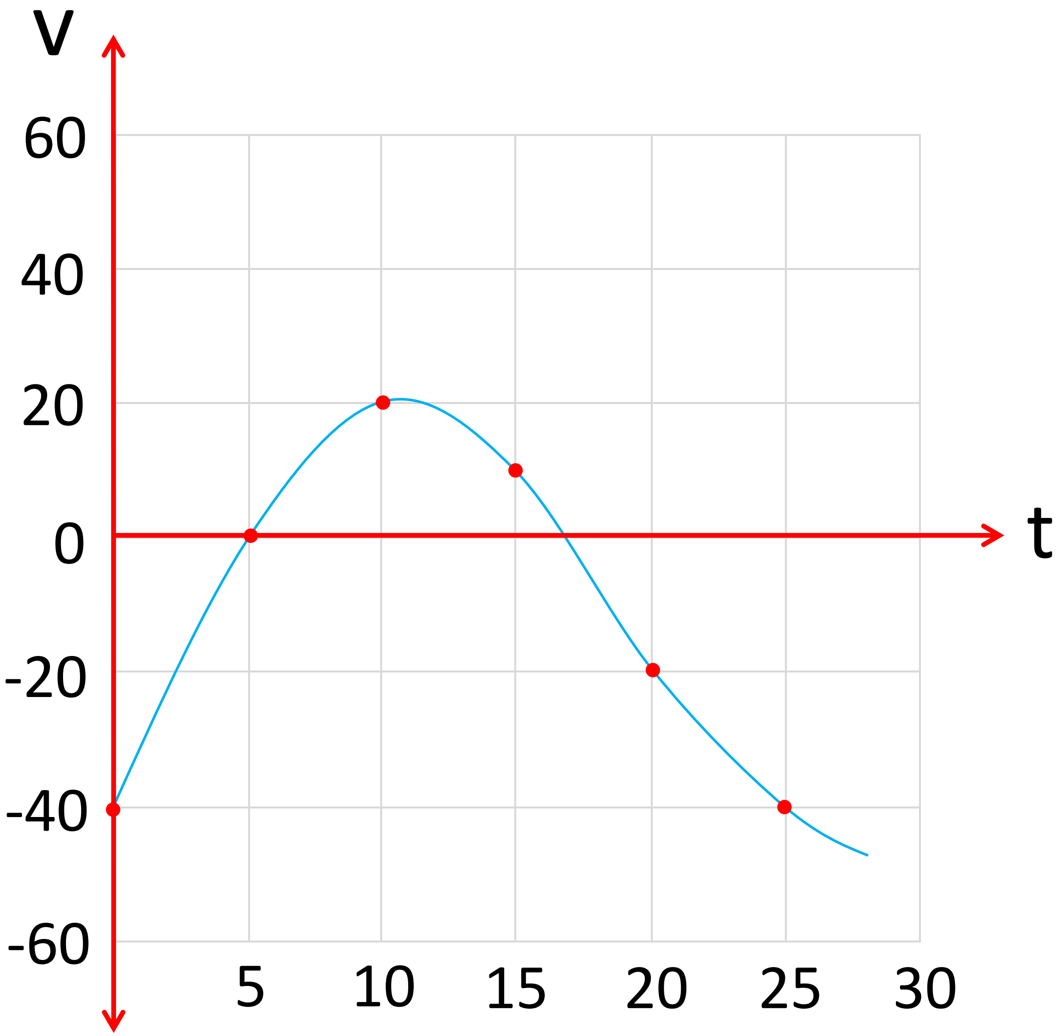
For portion BC:



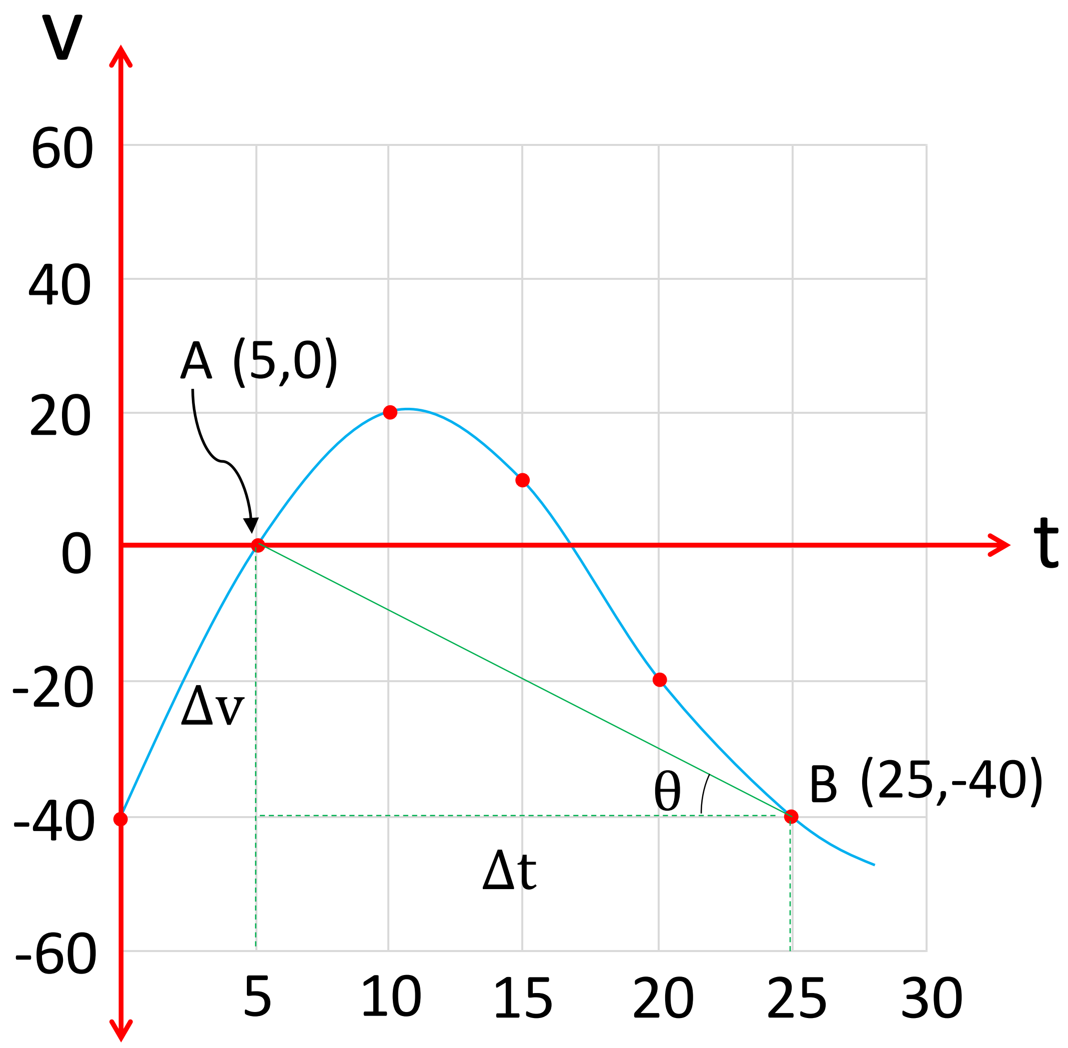
The particle is at origin and moving with decreasing velocity towards left side of origin.

Velocity-time graph

Time (s)	velocity (m/s)
0	-40
5	0
10	20
15	10
20	-20
25	-40



Average acceleration from velocity-time graph



Slope of line AB gives average acceleration between points A & B.

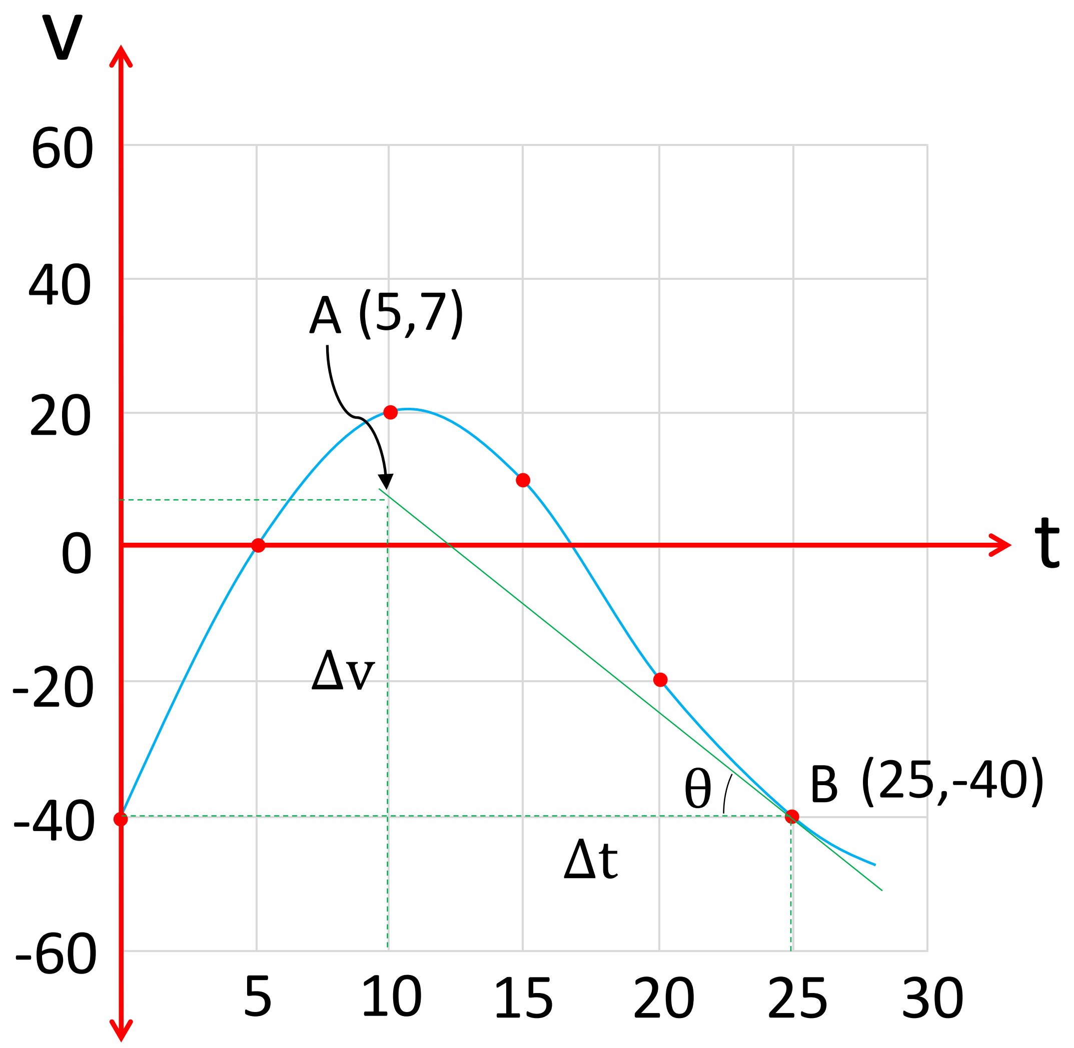
$$\text{slope of line AB} = \tan\theta$$

$$\tan\theta = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = a_{avg}$$

$$a_{avg(AB)} = \frac{-40 - 0}{25 - 5} = \frac{-40}{20}$$

$$a_{avg(AB)} = -2\ \mathrm{m/s^2}$$

Instantaneous acceleration from velocity-time graph



Slope of line AB gives instantaneous acceleration between points A & B.

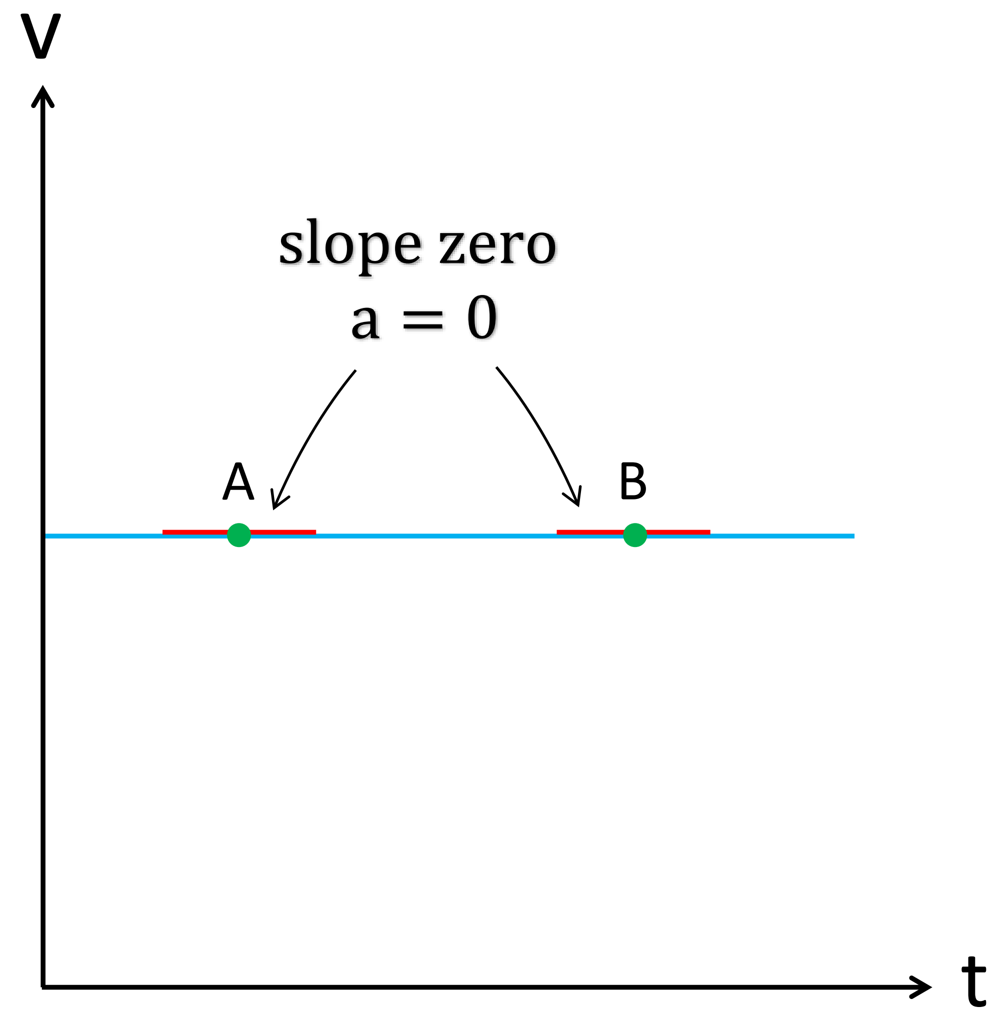
$$\text{slope of tangent} = \tan\theta$$

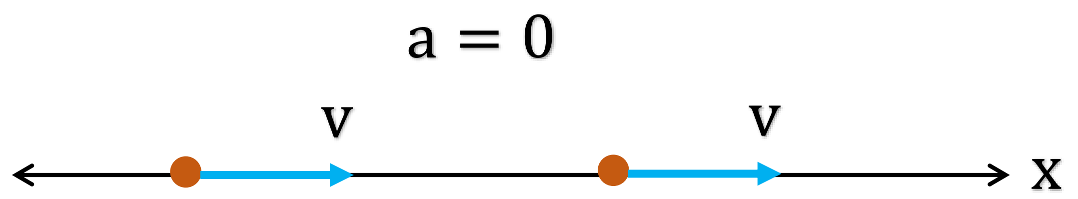
$$\tan\theta = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = a$$

$$a_B = \frac{-40 - 7}{25 - 10} = \frac{-47}{15}$$

$$a_B = -3.14 \text{ m/s}^2$$

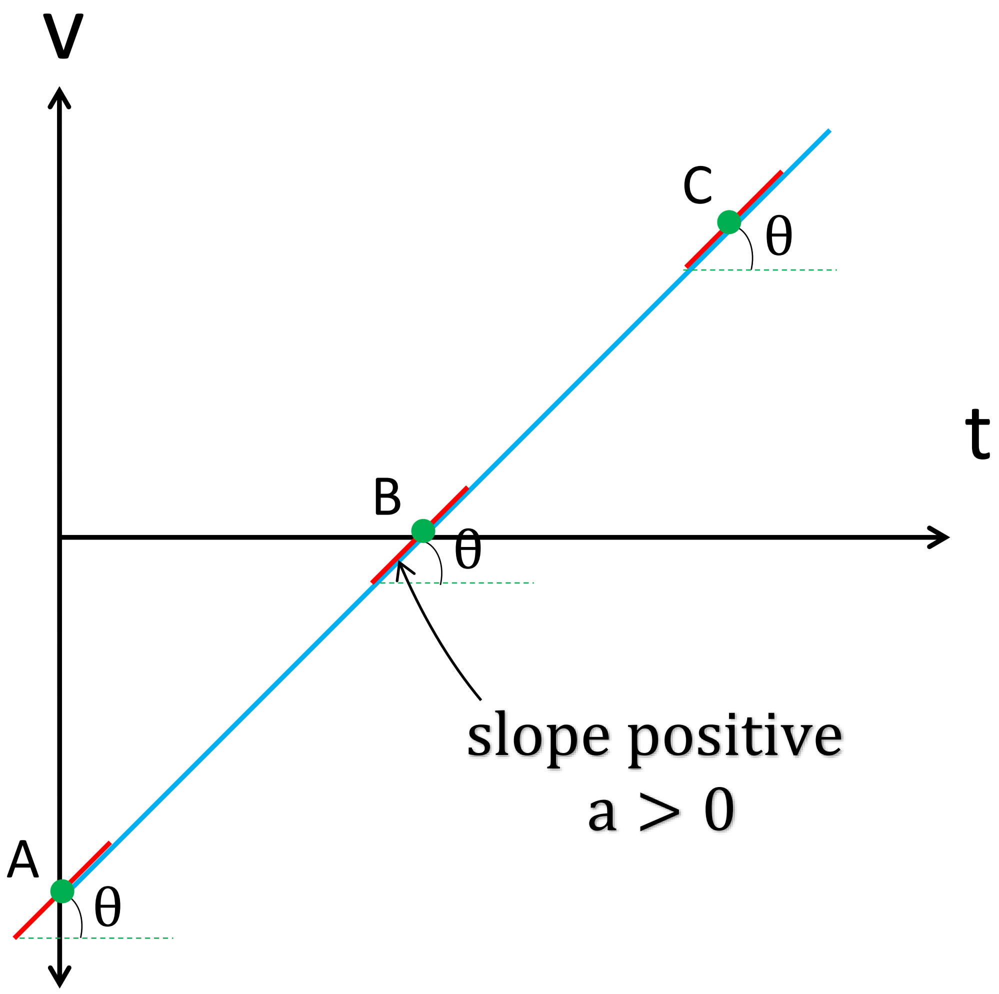
Analyzing nature of motion through velocity-time graph



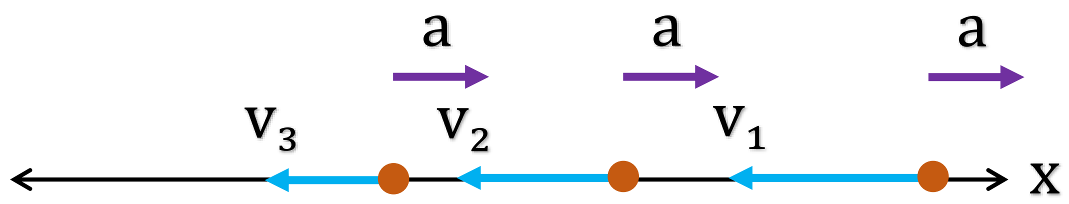


The particle is moving with constant velocity towards right.

Analyzing nature of motion through velocity-time graph

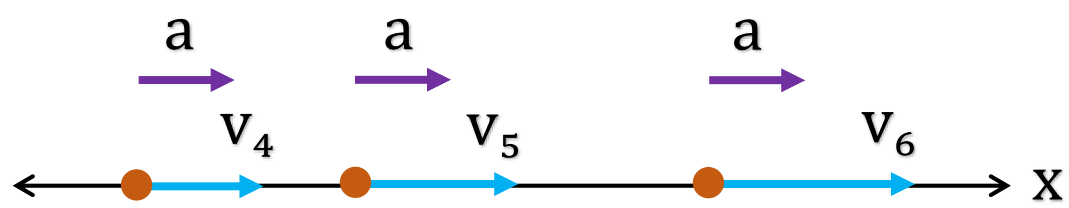


For portion AB:



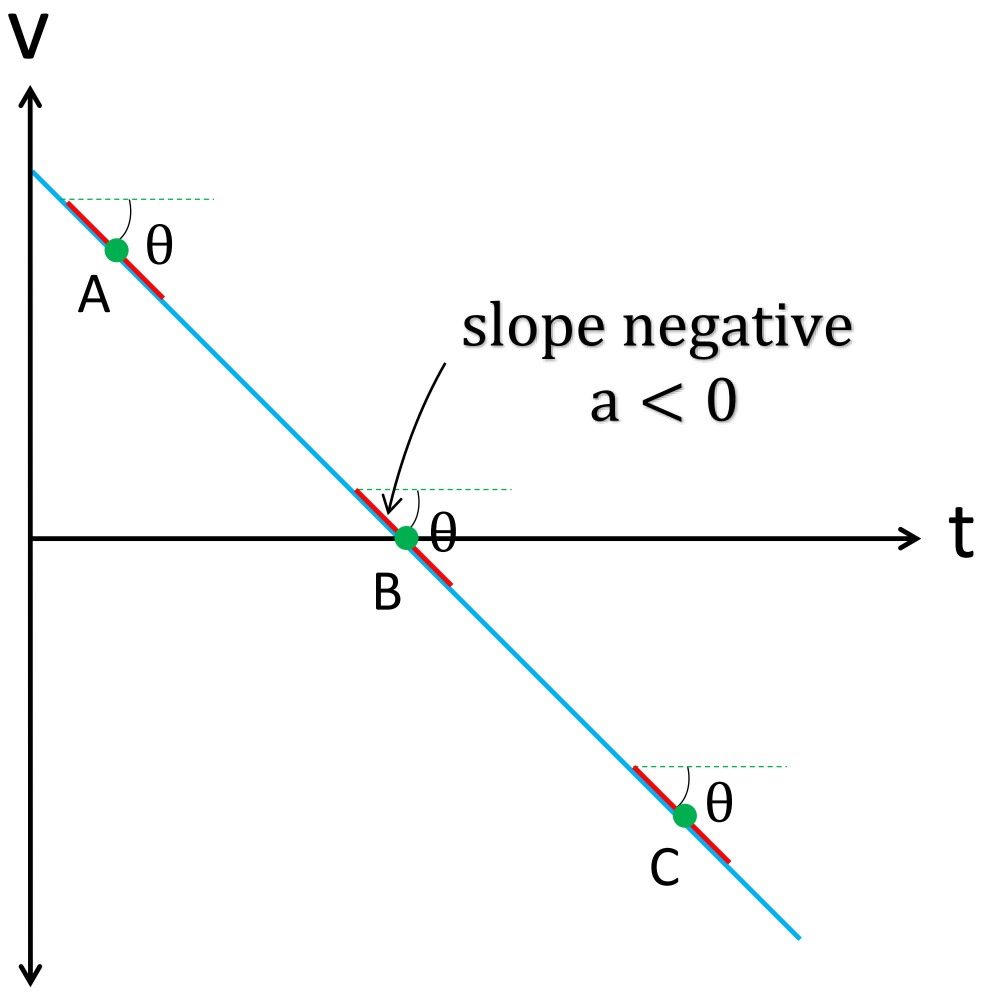
The particle is moving towards left and its velocity is uniformly decreasing.

For portion BC:

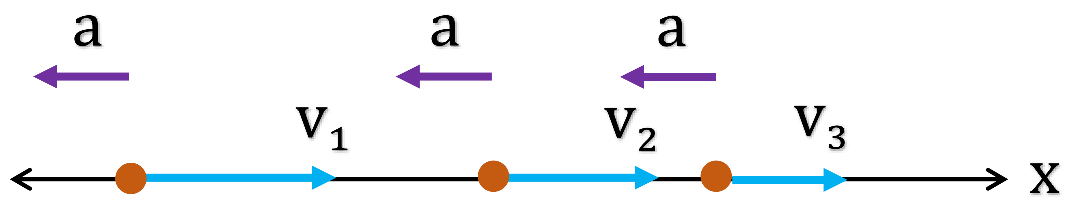


The particle is moving towards right and its velocity is uniformly increasing.

Analyzing nature of motion through velocity-time graph

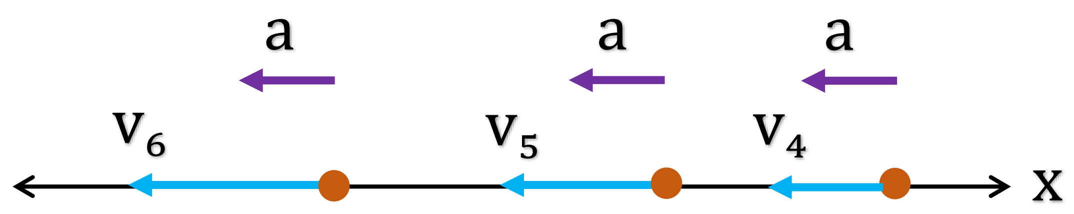


For portion AB:



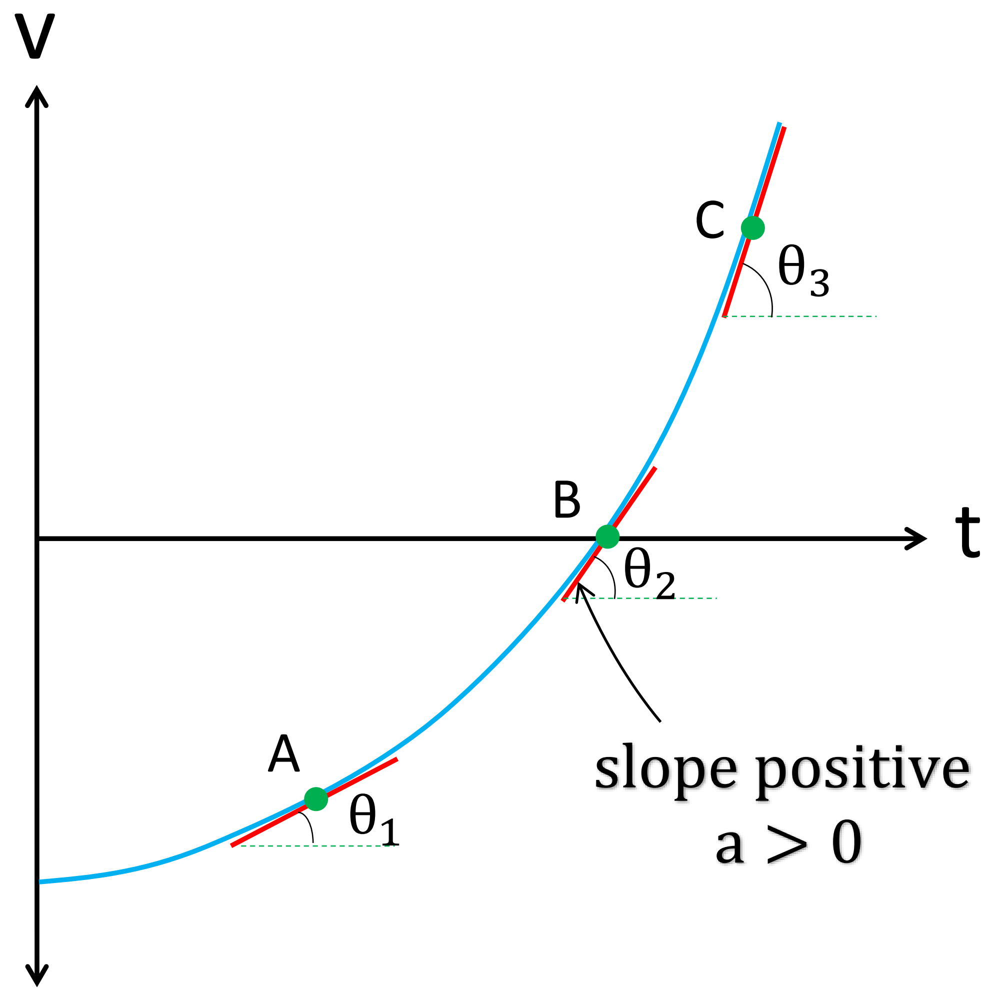
The particle is moving towards right and its velocity is uniformly decreasing.

For portion BC:

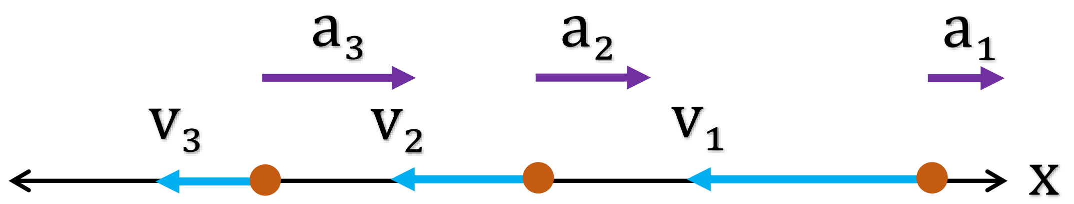


The particle is moving towards left and its velocity is uniformly increasing.

Analyzing nature of motion through velocity-time graph

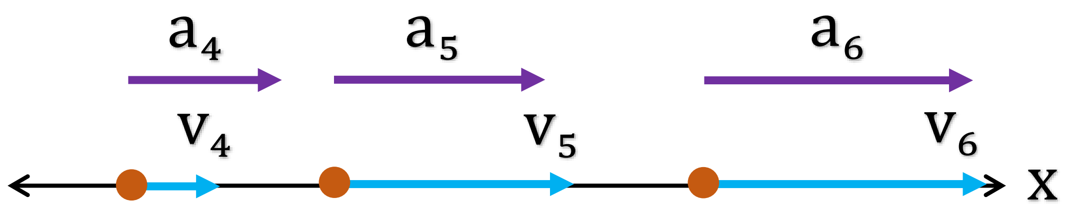


For portion AB:



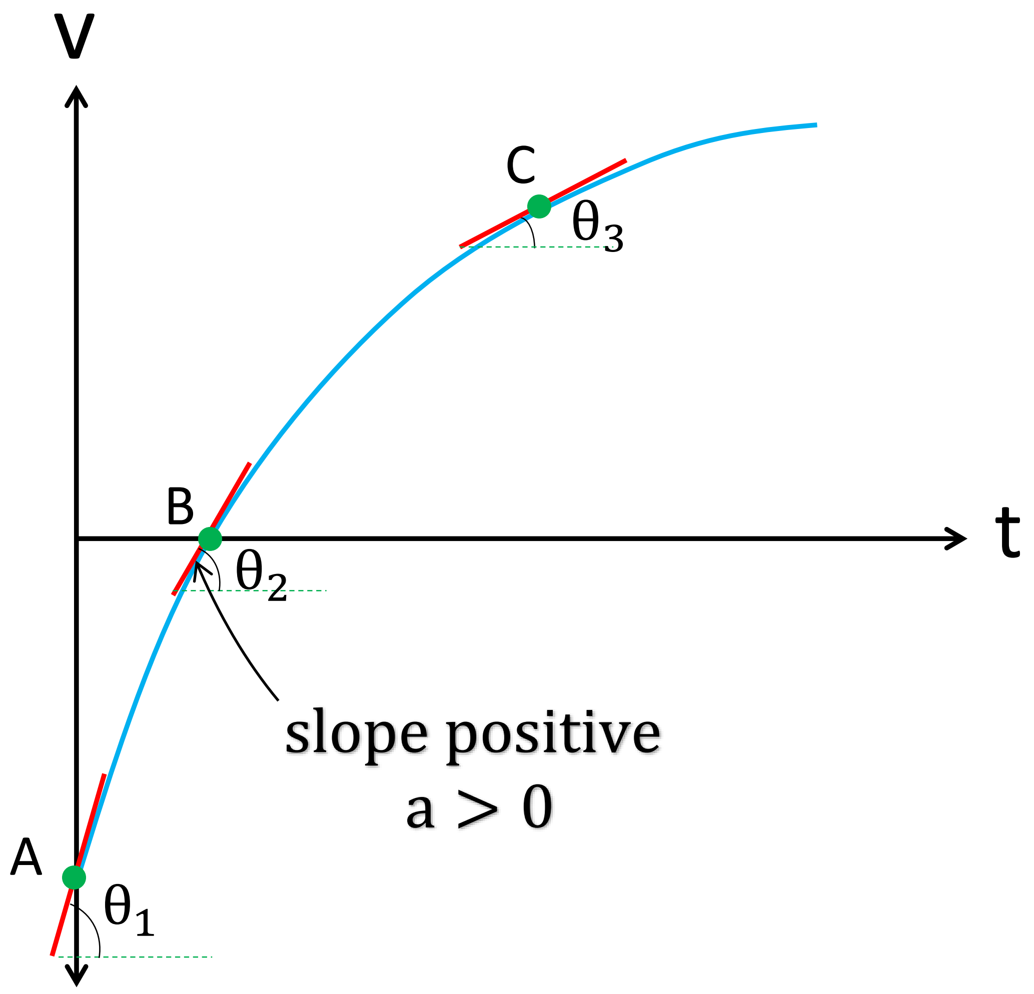
The particle is moving towards left and its velocity is decreasing slowly at non-uniform rate.

For portion BC:

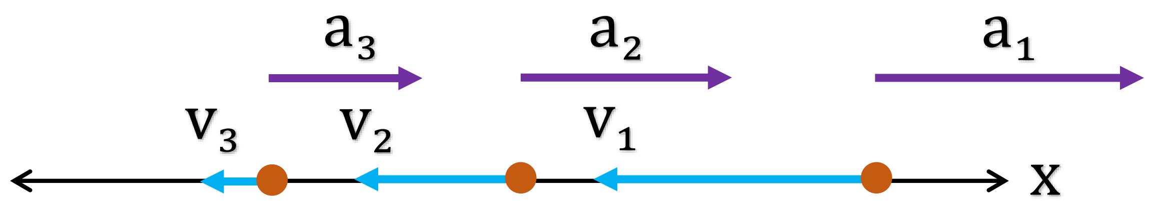


The particle is moving towards right and its velocity is increasing rapidly at non-uniform rate.

Analyzing nature of motion through velocity-time graph

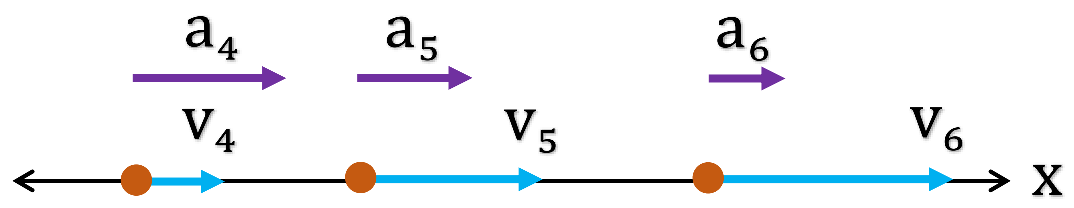


For portion AB:



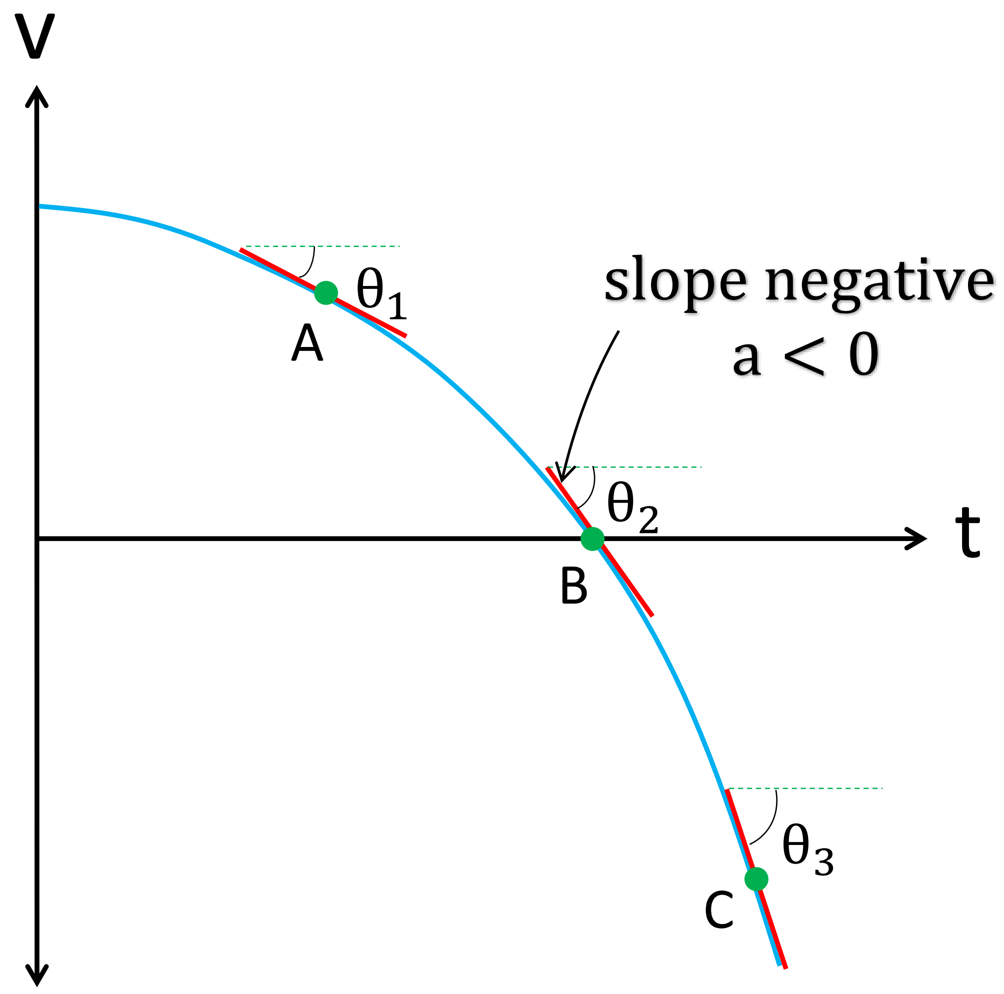
The particle is moving towards left and its velocity is decreasing rapidly at non-uniform rate.

For portion BC:

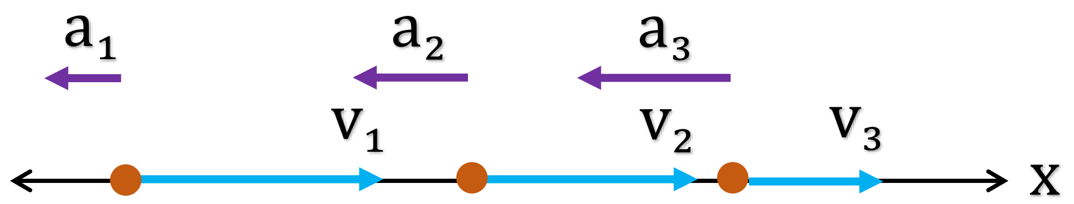


The particle is moving towards right and its velocity is increasing slowly at non-uniform rate.

Analyzing nature of motion through position-time graph

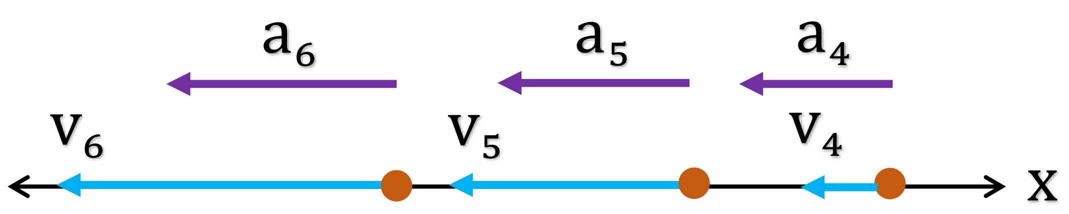


For portion AB:



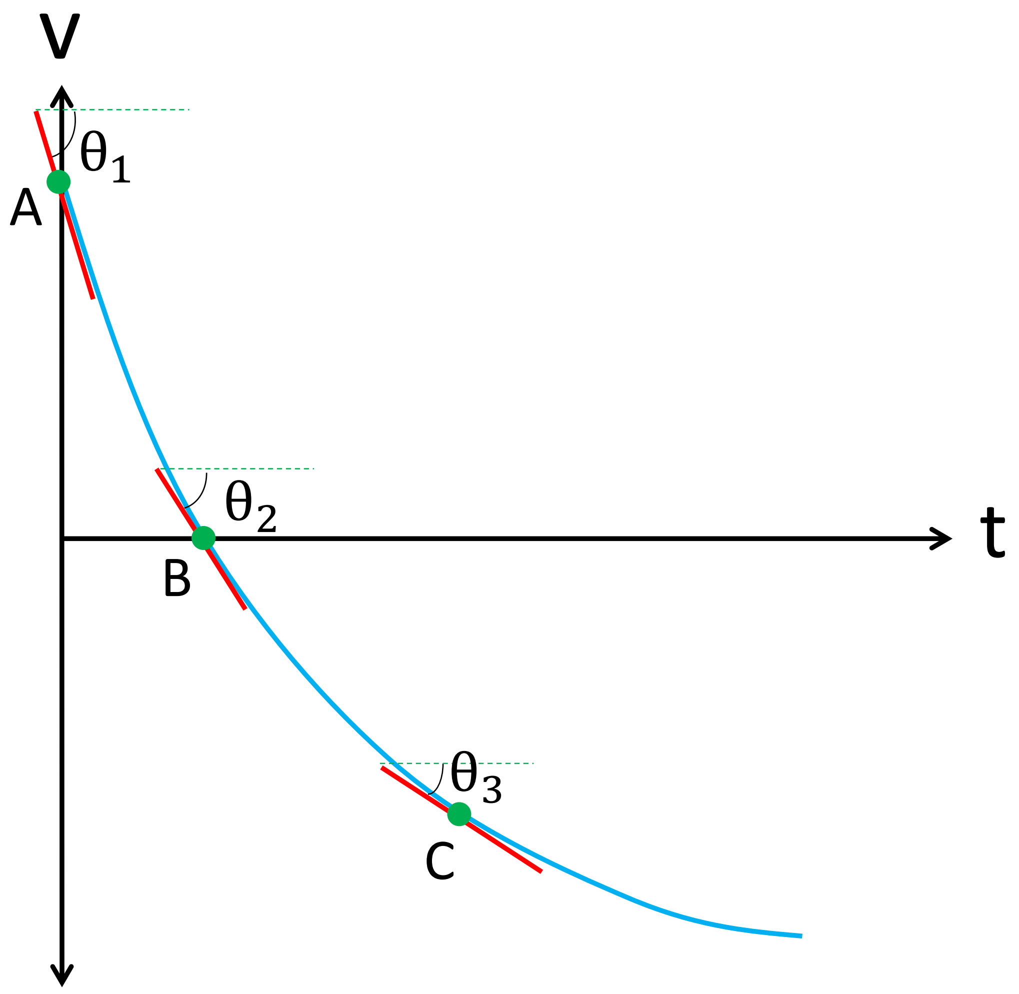
The particle is moving towards right and its velocity is decreasing slowly at non-uniform rate.

For portion BC:

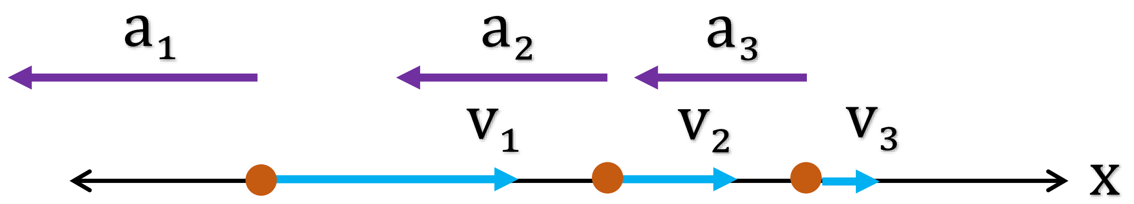


The particle is moving towards left and its velocity is increasing rapidly at non-uniform rate.

Analyzing nature of motion through position-time graph

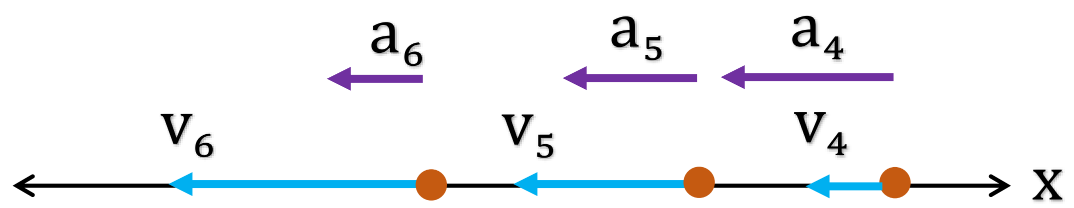


For portion AB:



The particle is moving towards right and its velocity is decreasing rapidly at non-uniform rate.

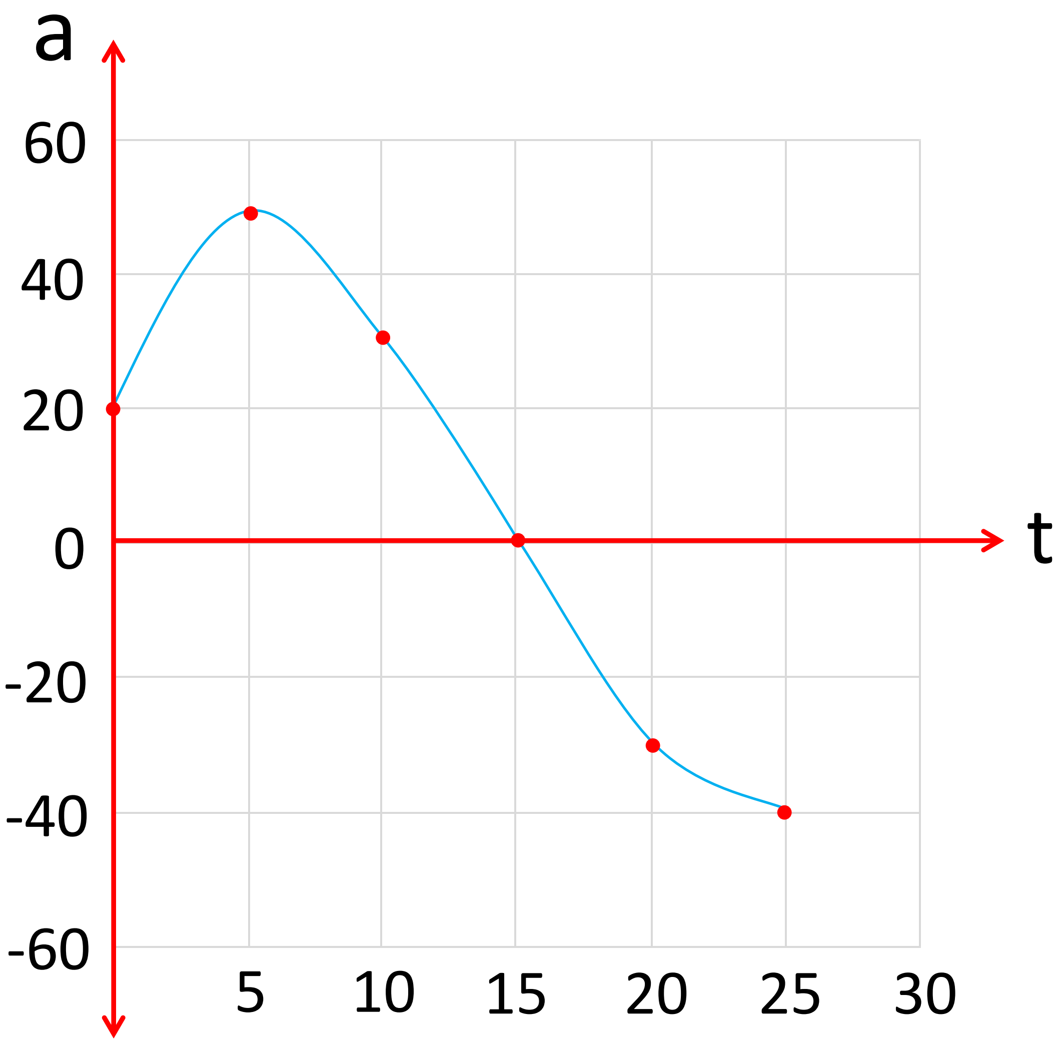
For portion BC:



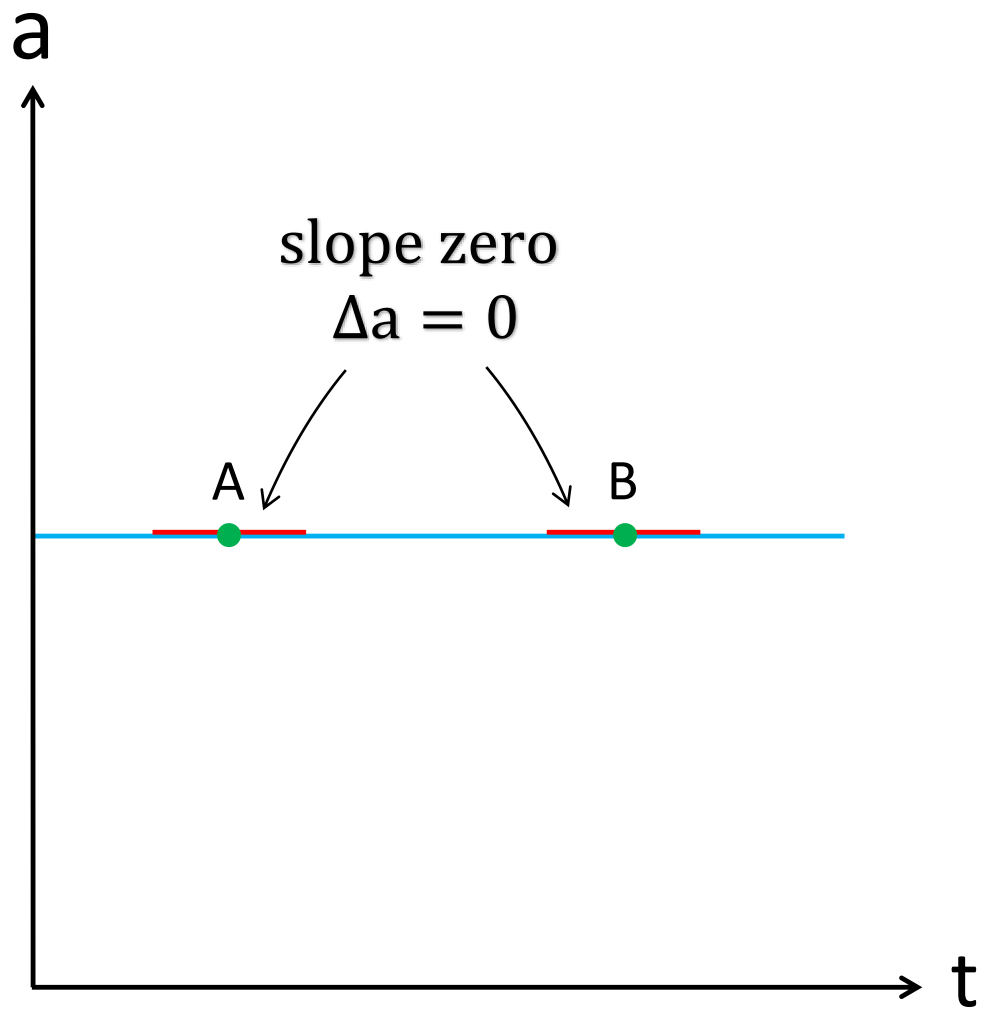
The particle is moving towards left and its velocity is increasing slowly at non-uniform rate.

Acceleration-time graph

Time (s)	acceleration (m/s^2)
0	20
5	50
10	30
15	0
20	-30
25	-40

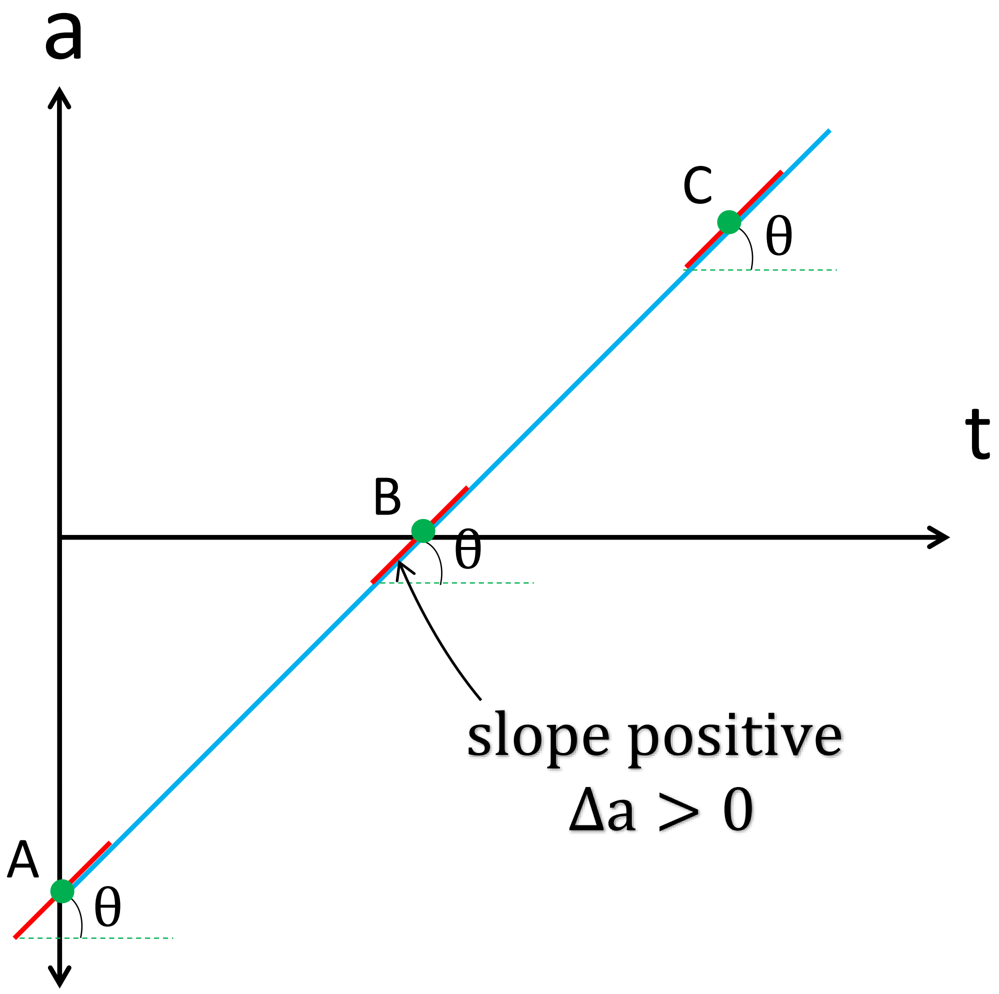


Analyzing nature of motion through acceleration-time graph

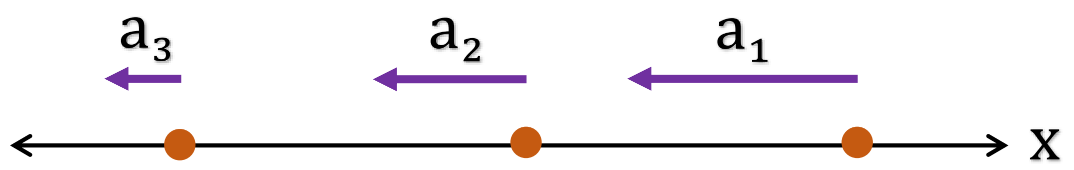


The particle is moving with constant acceleration.

Analyzing nature of motion through acceleration-time graph

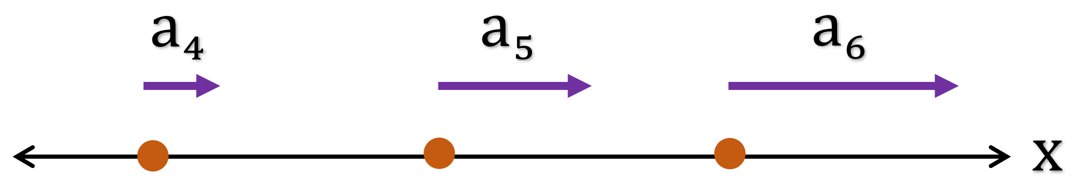


For portion AB:



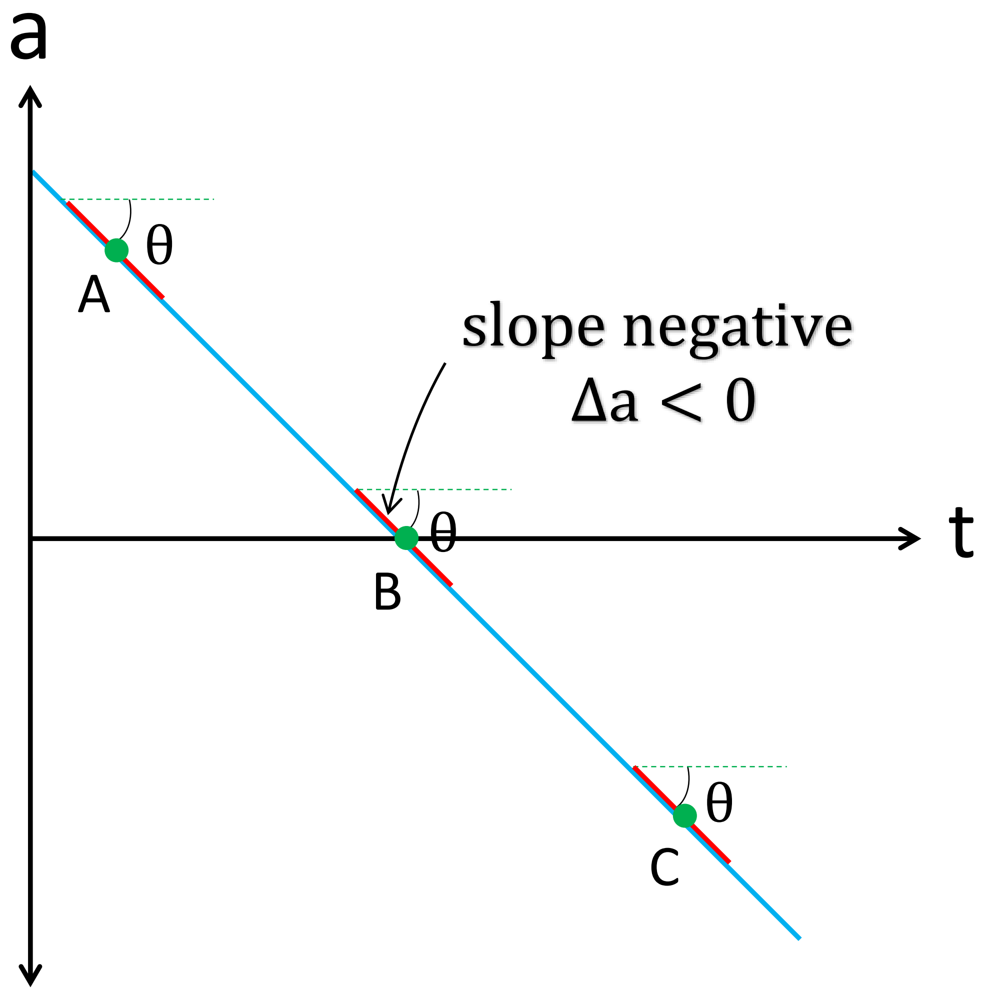
The acceleration of particle is uniformly decreasing towards left.

For portion BC:

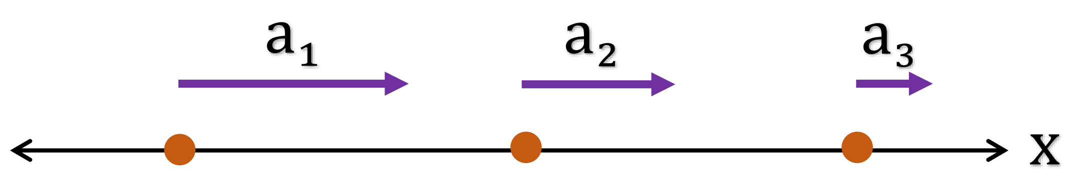


The acceleration of particle is uniformly increasing towards right.

Analyzing nature of motion through acceleration-time graph

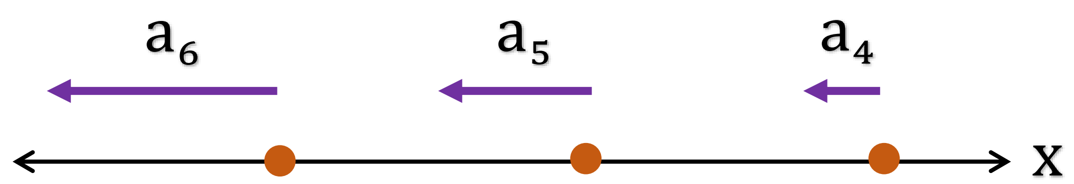


For portion AB:



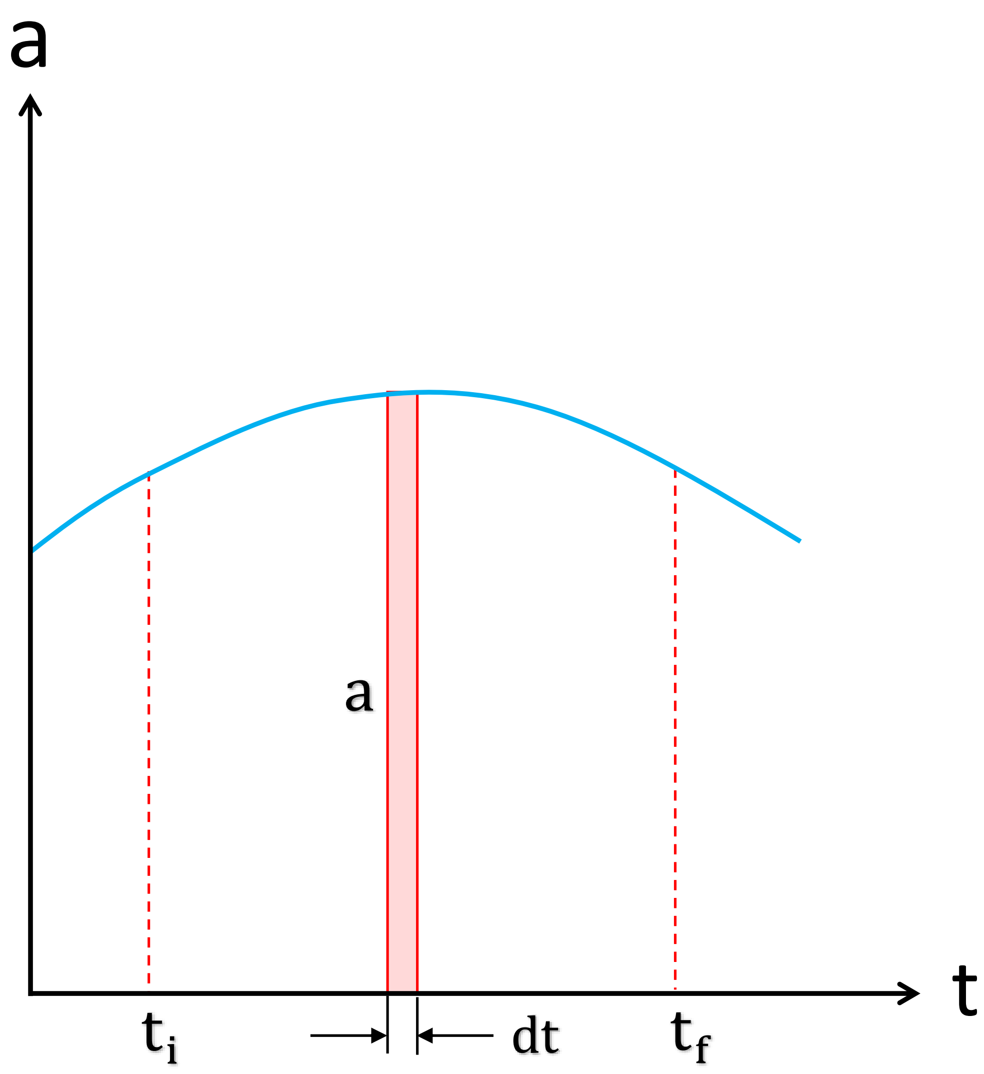
The acceleration of particle is uniformly decreasing towards right.

For portion BC:



The acceleration of particle is uniformly increasing towards left.

Graphical integration in motion analysis



From graph,

$$\text{area of strip} = a\,dt$$

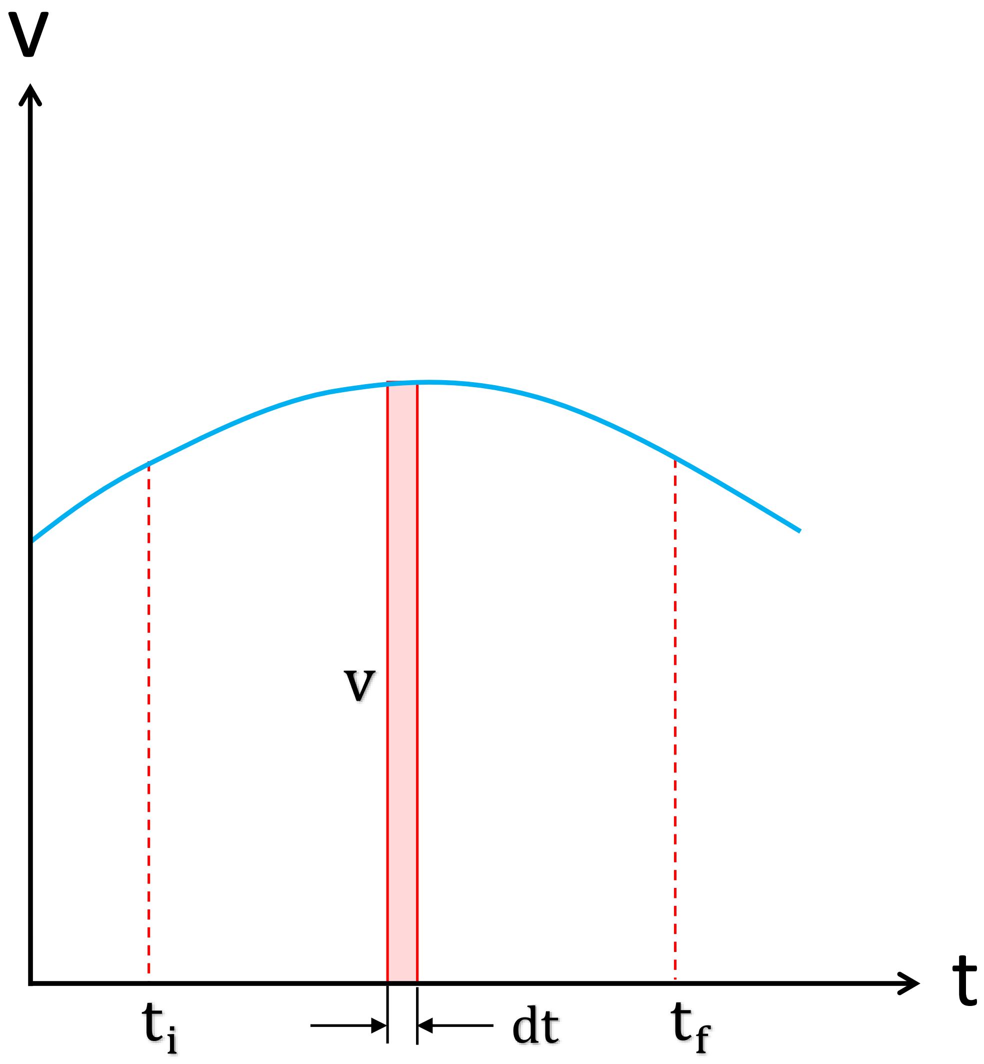
By definition,

$$dv = a\,dt$$

Net change in velocity from time t_i to t_f ,

$$\int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt$$

$$v_f - v_i = \left(\begin{array}{c}\text{area between acceleration curve} \\ \text{and time axis, from } t_i \text{ to } t_f\end{array}\right)$$



From graph,

$$\text{area of strip} = v\,dt$$

By definition,

$$dx = v\,dt$$

Net change in position from time t_i to t_f ,

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v\,dt$$

$$x_f - x_i = \left(\begin{array}{c}\text{area between velocity curve} \\ \text{and time axis, from } t_i \text{ to } t_f\end{array}\right)$$

Thank you