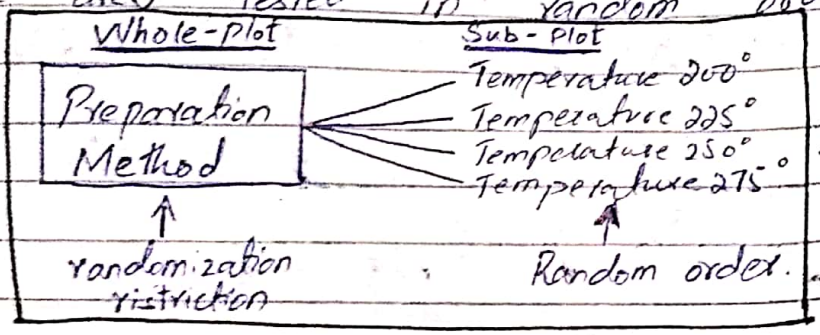
Split-Split-Plot Design

Previously, we read split-plot design in which we have only one level of randomization restriction within the experiment.

From example: The pulp preparation method is selected randomly for a replication, then the four temperature are tested in random order.



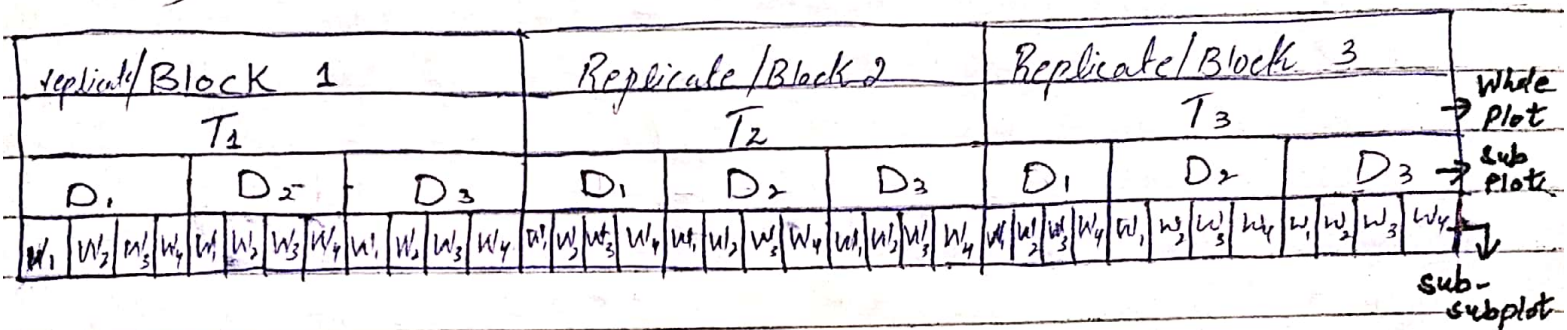
The concept of split-plot designs can be extended to situation in which randomization restrictions may occur at any number of levels within the experiment.

If there are two levels of randomization restrictions, the layout is called a **Split-split-plot design**. The following example illustrates such a design.

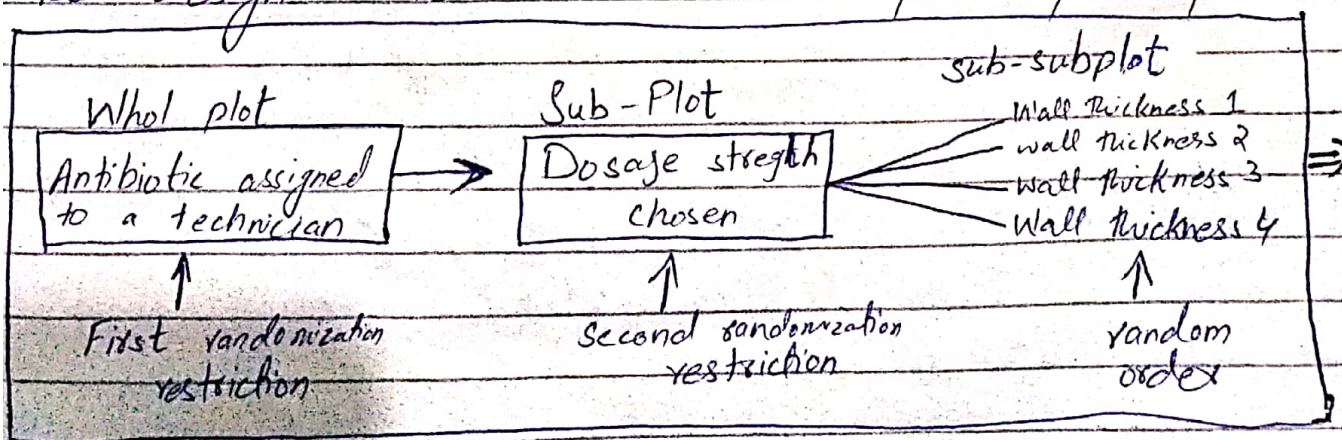
Example:

A researcher is studing the absorption times of a particular type of antibiotic capsule. There are three technicians, three dosage strengths and four capsule wall thicknesses of interest to the researcher. Each replication of a factorial experiment would require 36 observations ($3 \times 3 \times 4 = 36$). The experimentor has decided on three replicates and it is necessary to run each replicate on a different day. Note that the days can be consider as blocks. Within a replicate (or a block) (day), the experiment is performed by assigning a unit of antibiotic to a technician who conducts the experiment on the three dosage

strengths and the four wall thicknesses. Once a particular dosage strength is formulated, all four wall thicknesses are tested at that strength. Then another dosage strength is selected and all four wall thicknesses are tested. Finally, the third dosage strength and the four wall thicknesses are tested. Meanwhile, two other laboratory technicians also follow this plan, each starting with a unit of antibiotic.



Note that there are two randomization restrictions within each replicate (or block): technician and dosage strength. The whole plots correspond to the technician. The order in which the technicians are assigned the units of antibiotic is randomly determined. The dosage strengths form three sub-plots. Dosage strength may be randomly assigned to a sub-plot. Finally, within a particular dosage strength, the four capsule wall thicknesses are tested in random orders, forming four sub-subtreatments. Because there are two randomization restrictions in the experiment (some authors say two "splits" in the design) the design is called a split-split-plot design.



General ANOVA Table

SoV	d.f	S.S	MS
Block (B)	$b-1$	$\sum B_k^2/tdw - C.F$	$SSB/b-1$
Technician (T)	$t-1$	$\sum T_i^2/bdw - C.F$	$SST/t-1$
BXT (whole-plot error A)	$(b-1)(t-1)$	$\sum(B_k T_i)^2/dw - C.F - SSB - SST$	$SS(IN.P.E(A))/(b-1)(t-1)$
Dosage (D)	$d-1$	$\sum D_j^2/btw - C.F$	$SSD/d-1$
BXD (whole-plot error B)	$(b-1)(d-1)$	$\sum(B_k D_j)^2/tw - C.F - SSB - SSD$	$SS(W.P.E(B))/(b-1)(d-1)$
TXD (Interaction)	$(t-1)(d-1)$	$\sum(T_i D_j)^2/bw - C.F - SST - SSD$	$SS(TD)/(t-1)(d-1)$
BXTXD (Sub-plot error)	$(b-1)(t-1)(d-1)$	$\sum(B_k T_i D_j)^2/w - C.F - SSB - SST - SSD$	$SS(S.P.E)/(b-1)(t-1)(d-1)$
Wall Thickness (W)	$(w-1)$	$\sum W_k^2/btd - C.F$	$SSW/(w-1)$
BXW (Interaction)	$(b-1)(w-1)$	$\sum(B_k W_k)^2/td - C.F - SSB - SSW$	$SS(BW)/(b-1)(w-1)$
TXW (Interaction)	$(t-1)(w-1)$	$\sum(T_i W_k)^2/bd - C.F - SST - SSW$	$SS(TW)/(t-1)(w-1)$
BXTXW (Interaction)	$(b-1)(t-1)(w-1)$	$\sum(B_k T_i W_k)^2/d - C.F - SSB - SST - SSW$	$SS(BTW)/(b-1)(t-1)(w-1)$
DXW (Interaction)	$(d-1)(w-1)$	$\sum(D_j W_k)^2/bt - C.F - SSD - SSW$	$SS(DW)/(d-1)(w-1)$
BXDXW (Interaction)	$(b-1)(d-1)(w-1)$	$\sum(B_k D_j W_k)^2/t - C.F - SSB - SSD - SSW$	$SS(BDW)/(b-1)(d-1)(w-1)$
TXDXW (Interaction)	$(t-1)(d-1)(w-1)$	$\sum(T_i D_j W_k)^2/b - C.F - SST - SSD - SSW$	$SS(TDW)/(t-1)(d-1)(w-1)$
BXTXDXW (sub-subplot error)	By subtraction	By Subtraction	$SS(S.SP.E)/$ its d.f
Total	$btdw-1$	$\sum(B_k T_i D_j W_k)^2 - C.F$	

where $C.F = \dfrac{(T_{\ldots})^2}{btdw}$

SoV	F_{cal}	F_{tab}
B		
T (Whole plot error A)	$MST/MS(IN.P.E(A))$	$F_{\alpha,[(t-1),(b-1)(t-1)]}$
D (whole-plot error B)	$MSD/MS(IN.P.E(B))$	$F_{\alpha,[(d-1),(b-1)(d-1)]}$
TD (Sub-plot error)	$MS(TD)/MS(S.P.E)$	$F_{\alpha,[(t-1)(d-1),(b-1)(t-1)(d-1)]}$
W	$MSW/MS(S.SP.E)$	$F_{\alpha,[w-1),(S.SP.df)]}$
BW		
TW	$MS(TW)/MS(S.SP.E)$	$F_{\alpha,[(t-1)(w-1),(S.SP.df)]}$
BTW		
DW	$MS(DW)/MS(S.SP.E)$	$F_{\alpha,[(d-1)(w-1),(S.SP.df)]}$
BDW		
TDW (Sub-subplot error)	$MS(TDW)/MS(S.SP.E)$	$F_{\alpha,[(t-1)(d-1)(w-1),(S.SP.df)]}$
Total		

Numerical:

		Technicians								
		1			2			3		
Block	Dosage strength (Wall thickness)	1	2	3	1	2	3	1	2	3
1	1	95	71	108	96	70	108	95	70	100
	2	104	82	115	99	84	100	102	81	106
	3	101	85	117	95	83	105	105	84	113
	4	108	85	116	97	85	109	107	87	115
2	1	95	78	110	100	72	104	92	69	101
	2	106	84	109	101	79	102	100	76	104
	3	103	86	116	99	80	108	101	80	109
	4	109	84	110	112	86	109	108	86	113
3	1	96	70	107	94	66	100	90	73	98
	2	105	81	106	100	84	101	97	75	100
	3	106	88	112	104	87	109	100	82	104
	4	113	90	117	121	90	117	110	91	112

Solve This Question