Split Plot Design

In some multifactor factorial experiments, we may be unable to completely randomize the order of the runs. This often results in a generalization of the factorial design called a split-plot design.

In factorial experiment, there are certain situations where it is not possible to conduct the experiment. These conditions are as follows:

1- Nature of the factors may be such that the levels of one factor required greater precision as compaire to the levels of other factors. The factor for which the greater precision is required is applied to the sub-plots the other factors is applied to the main plot.

for example: If the two factors are: potassium and nitrogen And the greater precision is required for nitrogen then the nitrogen will be applied to the sub-plots and potassium is applied to the main plot/Whole plot.

P_0		P_1		→ main plots / whole plots
N_0	N_1	N_0	N_1	→ sub-plots

2- It might possible that the level of one factor required large experimental unit as compair to the level of other factors.

for example: If two factors are Sowing method and fertilizer then the sowing method required machinary hence the large experimental units are required for Sowing method and small for the fertilizer.

Sowing Method (1)		Sowing Method (2)		Sowing Method (3)	
F_1	F_2	F_1	F_2	F_1	F_2

3. It might possible that a new factor is introduced in an experiment which is already in progress.
for example: seeds of two compines are being studied suddenly experimenter wants to studied the fertilizer of three compines by using the same experimental material.

A company (seed)			B company (seed)		
F_1	F_2	F_3	F_1	F_2	F_3

:: Analysis ::

: General procedure :

Consider an example of a paper manufacturer who is interested in three different pulp preparation methods (the methods differ in the amount of hard word in the pulp mixture) and four different cooking temperatures for the pulp and who wishes to study the effect of these two factors on the tensile strength of the paper. Each replicate of a factorial experiment requires 12 observations and the experimenter has decided to run three replicates. However, the pilot plant is only capable of making 12 runs per day, so the experimenter decides to run one replicate on each of the three days and to consider the days or replicates as Blocks.

On any day, he conducts the experiment as follows. A batch of pulp is produced by one of the three methods under study. Then this batch is divided into four samples and each

Sample is cooked at one of the four temperatures. Then a second batch of pulp is made up using another of the three methods. This second batch is also divided into four samples that are tested at the four temperatures. The process is then repeated, using a batch of pulp produced by the third method.

	M1				M2				M3			
replication (or Block) 1	T_{200}	T_{225}	T_{250}	T_{275}	T_{200}	T_{225}	T_{250}	T_{275}	T_{200}	T_{225}	T_{250}	T_{275}
Replicate (or Block) 2	T_{200}	T_{225}	T_{250}	T_{275}	T_{200}	T_{225}	T_{250}	T_{275}	T_{200}	T_{225}	T_{250}	T_{275}
Replicate (or Block) 3	T_{200}	T_{225}	T_{250}	T_{275}	T_{200}	T_{225}	T_{250}	T_{275}	T_{200}	T_{225}	T_{250}	T_{275}

① Formulation of Hypothesis:

(a) H_0: The effect of all the preperation methods are same.
H_1: Not all the preperation Method's effects are same.

(b) H_0: The effect of all the temperatures are same.
H_1: Not all the temperature's effect are same.

(c) H_0: There is no interaction between preperation method and temperature.
H_1: There is interaction between preperation method and temperature.

② Level of Significance: $\alpha = ?$

③ Test Statistic:

(a) $F_1 = \dfrac{MS(\text{method})}{MS(\text{main plot error})}$ (b) $F_2 = \dfrac{MS(\text{Temp})}{MS(\text{Block} \times \text{Temp})}$ (c) $F_3 = \dfrac{MS(\text{Method} \times \text{Temp})}{MS(\text{Sub-plot error})}$

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④ Calculation:

ANOVA Table

SoV	d.f	SS	MS	F
Block (B)	$b-1$	$\sum B_K^2/mt - (T..)^2/bmt$	$SSB/b-1$	$MS(M)/MS(WPE)$
Method (M)	$m-1$	$\sum M_i^2/bt - (T..)^2/bmt$	$SSM/m-1$	
Whole-plot error	$(b-1)(m-1)$	$\sum (B_K M_i)^2/t - (T..)^2/bmt - SSB - SSM$	$SS(WPE)/(b-1)(m-1)$	
Temperature (T)	$t-1$	$\sum T_j^2/bm - (T..)^2/bmt$	$SST/t-1$	$MS(T)/MS(BT)$
BXT	$(b-1)(t-1)$	$\sum (B_K T_j)^2/m - (T..)^2/bmt - SSB - SST$	$SS(BT)/(b-1)(t-1)$	
MXT (interaction)	$(m-1)(t-1)$	$\sum (M_i T_j)^2/b - (T..)^2/bmt - SSM - SS$	$SS(MT)/(m-1)(t-1)$	$MS(MT)/MS(SPE)$
Sub-plot error	By subtraction	By subtraction	$SS(SPE)/$its d.f	
Total	$bmt-1$	$\sum (B_K M_i T_j)^2 - (T..)^2/bmt$		

⑤ Critical Region:

If $F_{cal} \geq F_{\alpha, v_1, v_2}$ then reject H_0 otherwise don't reject H_0.

(a) $v_1 = m-1$, $v_2 = (b-1)(m-1)$

(b) $v_1 = t-1$, $v_2 = (b-1)(t-1)$

(c) $v_1 = (m-1)(t-1)$, $v_2 =$ sub-plot error d.f

⑥ Decision.

Numerial:

Data of the previous example (the experiment on the Tensile strength of Paper) are shown in following table.

Temperature (°F)	Replicate/Block 1			Replicate/Block 2			Replicate/Block 3			
Pulp Preparation Method	1	2	3	1	2	3	1	2	3	Total (T_j)
200	30	34	29	28	31	31	31	35	32	281
225	35	41	26	32	36	30	37	40	34	311
250	37	38	33	40	42	32	41	39	39	341
275	36	42	36	41	40	40	40	44	45	364
Total (B_K)	417			423			457			1297 → $T..$

$$C.F = \frac{(T..)^2}{bmt} = \frac{(1297)^2}{3 \times 3 \times 4} = 46728.03$$

$$SSB = \frac{\Sigma B_K^2}{mt} - C.F$$

$$= \frac{(417)^2 + (423)^2 + (457)^2}{3 \times 4} - 46728.03$$

$$= \frac{561,667}{12} - 46728.03$$

$$\boxed{SSB = 77.55}$$

	M1	M2	M3	
B_1	138	155	124	417
B_2	141	149	133	423
B_3	149	158	150	457
	428	462	407	1297

→ you can also use this calculation for SSB.

$$SSM = \frac{\Sigma M_i^2}{bt} - C.F$$

$$= \frac{(428)^2 + (462)^2 + (407)^2}{3 \times 4} - 46728.03$$

$$= \frac{562277}{12} - 46728.03$$

$$\boxed{SSM = 128.39}$$

$$SS(W\text{-}P.E) = \frac{\Sigma (B_K M_i)^2}{t} - C.F - SSB - SSM$$

$$= \frac{(138)^2 + (155)^2 + (124)^2 + (141)^2 + \cdots + (457)^2}{4} - 46728.03 - 77.55 - 128.39$$

$$\boxed{SS(W\text{-}P.E) = 36.28}$$

$$SST = \frac{\sum T_j^2}{bm} - C.F$$

$$= \frac{(281)^2 + (311)^2 + (341)^2 + (364)^2}{3 \times 3} - 46728.03$$

$$= \frac{424459}{9} - 46728.03$$

$$\boxed{SST = 434.08}$$

$$SS(BT) = \frac{\sum (B_k T_j)^2}{m} - C.F - SSB - SST$$

	T_1	T_2	T_3	T_4
B_1	93	102	108	114
B_2	90	98	114	121
B_3	98	111	119	129

$$= \frac{(93)^2 + (102)^2 + (108)^2 + (114)^2 + \cdots + (129)^2}{3} - 46728.03 - 77.55 - 434.08$$

$$= \frac{141781}{3} - 46728.03 - 77.55 - 434.08$$

$$\boxed{SS(BT) = 20.67}$$

$$SS(MT) = \frac{\sum (M_i T_j)^2}{b} - C.F - SSM - SST$$

	M_1	M_2	M_3
T_1	89	100	92
T_2	104	117	90
T_3	118	119	104
T_4	117	126	121

$$= \frac{(89)^2 + (100)^2 + (92)^2 + (104)^2 + \cdots + (121)^2}{3} - 46728.03 - 128.39 - 434.08$$

$= \frac{142097}{3} - 46728.03 - 128.39 - 434.08$

SS(M) = 75.17

$SS\ Total = \sum (B_K M_i T_j)^2 - C.F$

$= (30)^2 + (34)^2 + (29)^2 + (28)^2 + \cdots + (45)^2 - 46728.03$

SS Total = 822.97

ANOVA Table

SoV	d.f	SS	MS	F_{cal}	F_{tab}
Block (B)	3-1=2	77.55	38.78		
Preperation method (M)	3-1=2	128.39	64.20	7.08	$F_{0.05(2,4)} = 6.94$ (Reject H_0)
Whole-Plot Error	(3-1)(3-1)=4	36.28	9.07		
Temperature (T)	(4-1)=3	434.08	144.69	41.94	$F_{0.05(3,6)} = 4.76$ (Reject H_0)
B X T	(3-1)(4-1)=6	20.67	3.45		
M X T	(3-1)(4-1)=6	75.17	12.53	2.96	$F_{0.05(6,12)} = 3.00$ (Don't reject H_0)
Sub-Plot Error	12	50.83	4.24		
Total	((3)(3)(4))-1=35	822.97			