Quasi-Latin Square Designs:

In previous lectures we have discussed the confounding of 2^k and 3^k factorial design in blocks. Now we study the confounding of factorial design in rows and columns. One area of application for such designs is glasshouse experimentation. In which physical management operations performed during the course of the experiment are often done by rows or by columns.

Quasi-Latin square designs introduced by Yates for factorial experiments, are of the type required. Like a Latin square, the treatments are arranged such that no treatment occurs more than once in a row or a column and the treatments are applied to experimental units arranged in an equal number of rows and columns. A quasi-Latin square design may consists one or more quasi-Latin squares and each quasi-Latin square contains one or more complete set of treatments. This is the extension of the factorial designs that have treatment effects confounded with blocks to those that allow for two-way elimination of heterogeneity. They require the total confounding of some factorial effects with rows or columns within each replicate. Treatments must be equally replicated and the number of replicates is restricted.

e.g: consider a 2^3 factorial experiment. The eight treatments can be arranged in one or more 4x4 squares: the number of replicates for treatments must be a multiple of 2.

Cases:

There are four cases in the quasi-Latin square design.

1- When complete confounding is used row-wise as well as column-wise.

2- When complete confounding is used column-wise and Partial confounding is used row-wise.

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3- When complete confounding is used row-wise and partial confounding is used column-wise.

4- When partial confounding is used row-wise as well as column wise.

Case 1:

Give plan & ANOVA Table for 2^3 factorial experiment with two replications when ABC is confounded row-wise and AB is confounded column-wise.

$2^3 = 8 \rightarrow$ Possible combinations

1, a, b, ab, c, ac, bc, abc.

⇒ Row-wise confounding:

ABC

$L = X_1 + X_2 + X_3$

(1) : $L = 0+0+0 = 0$

a : $L = 1+0+0 = 1$

b : $L = 0+1+0 = 1$

ab : $L = 1+1+0 = 2$

c : $L = 0+0+1 = 1$

ac : $L = 1+0+1 = 2$

bc : $L = 0+1+1 = 2$

abc : $L = 1+1+1 = 3$

E	O
1	a
ab	b
ac	c
bc	abc

⇒ Column-wise confounding:

AB

$L = X_1 + X_2$

1 : $L = 0+0 = 0$

a : $L = 1+0 = 1$

b : $L = 0+1 = 1$

ab : $L = 1+1 = 2$

c : $L = 0+0 = 0$

ac : $L = 1+0 = 1$

bc : $L = 0+1 = 1$

abc : $L = 1+1 = 2$

E	O
1	a
ab	b
c	ac
abc	bc

Quasi-Latin Square Design:

		AB				
		E	O	E	O	rep = 2
ABC	E	1	ac	ab	bc	
	O	c	a	abc	b	
	E	ab	bc	1	ac	
	O	abc	b	c	a	

General ANOVA Table:

SOV	d.f	SS	MS	F
Row	$r-1$	$\sum R_i^2/n_r - (T_{...})^2/p^n(rep)$		
Column	$c-1$	$\sum C_j^2/n_c - (T_{...})^2/p^n(rep)$		
A	$P-1$	$(\text{effect of } A)^2/p^n(rep)$	$SSA/p-1$	MSA/MSE
B	$P-1$	$(\text{effect of } B)^2/p^n(rep)$	$SSB/p-1$	MSB/MSE
C	$P-1$	$(\text{effect of } C)^2/p^n(rep)$	$SSC/p-1$	MSC/MSE
AC	$P-1$	$(\text{effect of } AC)^2/p^n(rep)$	$SSAC/p-1$	$MS(AC)/MSE$
BC	$P-1$	$(\text{effect of } BC)^2/p^n(rep)$	$SSBC/p-1$	$MS(BC)/MSE$
Error	By subtraction	By subtraction	$SSE/\text{its d.f}$	
total	$P^n(rep)-1$	$\sum\sum\sum\sum Y^2 - \frac{(T_{....})^2}{P^n(rep)}$		

∴ Complete confounding so confounding terms not include in SOV

$n_r \rightarrow$ row size

$n_c \rightarrow$ column size.

Case 2:

Give plan of & ANOVA Table 2^3 factorial experiment with two replications when ABC is confounded column-wise, AB and AC is confounded row-wise.

Column-wise

ABC	
E	O
1	a
ab	b
ac	c
bc	abc

Row-wise

AB		AC	
E	O	E	O
1	a	1	a
ab	b	ac	c
c	ac	b	ab
abc	bc	abc	bc

Quasi-Latin Square design with rep=2

		ABC			
		E	O	E	O
AB	E	1	abc	ab	c
	O	bc	a	ac	b
AC	E	ac	b	1	abc
	O	ab	c	bc	a

General ANOVA Table

SOV	d.f	SS	MS	F
Row	$r-1$	$\sum R_i^2/n_r - (T_{..})^2/p^n(rep)$		
Column	$c-1$	$\sum C_j^2/n_c - (T_{..})^2/p^n(rep)$		
A	$P-1$	$(\text{effect of } A)^2/p^n(rep)$	$SSA/P-1$	MSA/MSE
B	$P-1$	$(\text{effect of } B)^2/p^n(rep)$	$SSB/P-1$	MSB/MSE
AB'	$P-1$	$(\text{effect of } AB')^2/p^n(rep-1)$	$SSAB'/P-1$	$MS(AB')/MSE$
C	$P-1$	$(\text{effect of } C)^2/p^n(rep)$	$SSC/P-1$	MSC/MSE
AC'	$P-1$	$(\text{effect of } AC')^2/p^n(rep-1)$	$SS(AC')/P-1$	$MS(AC')/MSE$
BC	$P-1$	$(\text{effect of } BC)^2/p^n(rep)$	$SS(BC)/P-1$	$MS(BC)/MSE$
Error	By sub	By Substraction	SSE / its d.f	
Total	$P^n(rep)-1$	$\sum\sum\sum\sum(Y)^2 - \frac{(T_{..})^2}{p^n(rep)}$		

Case 3:

Give plan and General ANOVA table for 2^3 factorial experiment with two replications when AC and BC is confounded column-wise while ABC is confounded row-wise.

Colum-wise

AC E	AC O	BC E	BC O
1	a	1	b
b	ab	a	ab
ac	c	bc	c
abc	bc	abc	ac

row-wise

ABC E	ABC O
1	a
ab	b
ac	c
bc	abc

Quasi-Latin Square design with two replicates

		AC		BC	
		E	O	E	O
	E	1	ab	bc	ac
ABC	O	b	a	abc	c
	E	ac	bc	1	ab
	O	abc	c	a	b

General ANOVA Table

SOV	d.f	SS	MS	F
Row	$r-1$	$\sum R_i^2/n_r - (T_{..})^2/p^n(rep)$		
Column	$c-1$	$\sum C_j^2/n_c - (T_{..})^2/p^n(rep)$		
A	$p-1$	$(\text{effect of } A)^2/p^n(rep)$	$SSA/(p-1)$	MSA/MSE
B	$p-1$	$(\text{effect of } B)^2/p^n(rep)$	$SSB/p-1$	MSB/MSE
AB	$p-1$	$(\text{effect of } AB)^2/p^n(rep)$	$SSAB/p-1$	$MS(AB)/MSE$
C	$p-1$	$(\text{effect of } C)^2/p^n(rep)$	$SSC/p-1$	MSC/MSE
AC′	$p-1$	$(\text{effect of } AC')^2/p^n(rep-1)$	$SS(AC')/p-1$	$MS(AC')/MSE$
BC′	$p-1$	$(\text{effect of } BC')^2/p^n(rep-1)$	$SS(BC')/p-1$	$MS(BC')/MSE$
Error	By sub.	By subtraction	SSE/its. d.f.	
Total	$p^n(rep)-1$	$\sum\sum\sum(Y)^2 - \dfrac{(T_{..})^2}{p^n(rep)}$		

Case 4:

Give Plan and ANOVA Table for 2^3 factorial experiment with two replicates when AC & BC are confounded column-wise, and ABC & AB are confounded row-wise.

column-wise

AC

E	O
1	a
b	ab
ac	c
abc	bc

BC

E	O
1	b
a	ab
bc	c
abc	ac

row-wise

ABC

E	O
1	a
ab	b
ac	c
bc	abc

AB

E	O
1	a
ab	b
c	ac
abc	bc

Quasi-Latin Square design for 2^3 with two replicates

		(AC) E	(AC) O	(BC) E	(BC) O
(ABC)	E	1	ab	bc	ac
	O	b	a	abc	c
(AB)	E	abc	c	1	ab
	O	ac	bc	a	b

ANOVA table for 2^3

SOV	d.f	SS	MS	F
Row	3	$\sum R_i^2/4 - (T_{...})^2/16$		
Column	3	$\sum C_j^2/4 - (T_{...})^2/16$		
A	1	(effect of A)^2/16	SSA	MSA/MSE
B	1	(effect of B)^2/16	SSB	MSB/MSE
AB′	1	(effect of AB)^2/8	SSAB′	MS(AB)′/MSE
C	1	(effect of C)^2/16	SSC	MSC/MSE
AC′	1	(effect of AC′)/8	SSAC′	MS AC′/MSE
BC′	1	(effect of BC′)/8	SSBC′	MS BC′/MSE
ABC′	1	(effect of ABC′)/8	SS ABC′	MS ABC′/MSE
Error	2	By subtraction	SSE/2	
Total	15	$\sum\sum\sum\sum Y^2 - \frac{(T_{...})^2}{16}$		