chapter

12

Three-Phase Circuits

He who cannot forgive others breaks the bridge over which he must pass himself.

—G. Herbert

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.e), *"an ability to identify, formulate, and solve engineering problems."*

Developing and enhancing your "ability to identify, formulate, and solve engineering problems" is a primary focus of textbook. Following our six step problem-solving process is the best way to practice this skill. Our recommendation is that you use this process whenever possible. You may be pleased to learn that this process works well for nonengineering courses.

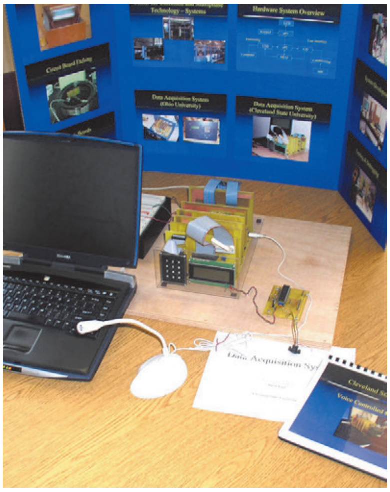

Photo by Charles Alexander

ABET EC 2000 criteria (f), *"an understanding of professional and ethical responsibility."*

"An understanding of professional and ethical responsibility" is required of every engineer. To some extent, this understanding is very personal for each of us. Let us identify some pointers to help you develop this understanding. One of my favorite examples is that an engineer has the responsibility to answer what I call the "unasked question." For instance, assume that you own a car that has a problem with the transmission. In the process of selling that car, the prospective buyer asks you if there is a problem in the right-front wheel bearing. You answer no. However, as an engineer, you are required to inform the buyer that there is a problem with the transmission without being asked.

Your responsibility both professionally and ethically is to perform in a manner that does not harm those around you and to whom you are responsible. Clearly, developing this capability will take time and maturity on your part. I recommend practicing this by looking for professional and ethical components in your day-to-day activities.

12.1 Introduction

So far in this text, we have dealt with single-phase circuits. A single-phase ac power system consists of a generator connected through a pair of wires (a transmission line) to a load. Figure 12.1(a) depicts a single-phase two-wire system, where V_p is the rms magnitude of the source voltage and ϕ is the phase. What is more common in practice is a single-phase three-wire system, shown in Fig. 12.1(b). It contains two identical sources (equal magnitude and the same phase) that are connected to two loads by two outer wires and the neutral. For example, the normal household system is a single-phase three-wire system because the terminal voltages have the same magnitude and the same phase. Such a system allows the connection of both 120-V and 240-V appliances.

Historical note: Thomas Edison invented a *three-wire system*, using three wires instead of four.

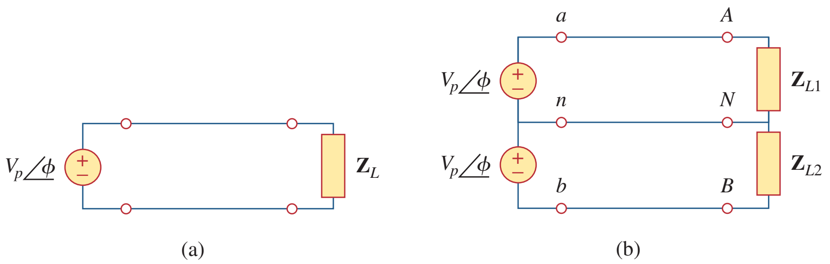

Figure 12.1
Single-phase systems: (a) two-wire type, (b) three-wire type.

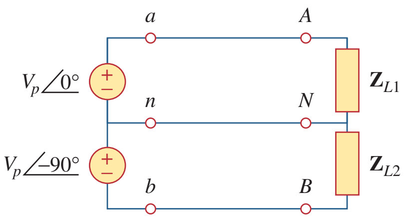

Figure 12.2
Two-phase three-wire system.

Circuits or systems in which the ac sources operate at the same frequency but different phases are known as *polyphase*. Figure 12.2 shows a two-phase three-wire system, and Fig. 12.3 shows a three-phase four-wire system. As distinct from a single-phase system, a two-phase system is produced by a generator consisting of two coils placed perpendicular to each other so that the voltage generated by one lags the other by 90°. By the same token, a three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°. Since the three-phase system is by far the most prevalent and most economical polyphase system, discussion in this chapter is mainly on three-phase systems.

Three-phase systems are important for at least three reasons. First, nearly all electric power is generated and distributed in three-phase, at the operating frequency of 60 Hz (or $\omega = 377$ rad/s) in the United States or 50 Hz (or $\omega = 314$ rad/s) in some other parts of the world. When one-phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently. Even when more than three phases are needed—such as in the aluminum industry, where 48 phases are required for melting purposes—they can be provided by manipulating the three phases supplied. Second, the instantaneous power in a three-phase system can be constant (not pulsating), as we will see in Section 12.7. This results in uniform power transmission and less vibration of three-phase machines. Third, for the same amount of power, the three-phase system is more economical than the single-phase. The amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

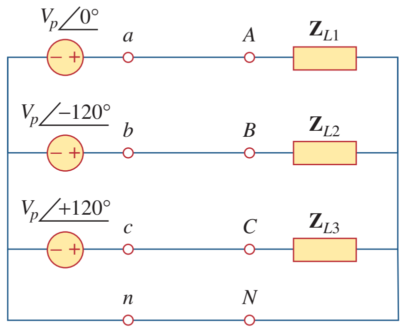

Figure 12.3
Three-phase four-wire system.

Historical

Nikola Tesla (1856–1943) was a Croatian-American engineer whose inventions—among them the induction motor and the first polyphase ac power system—greatly influenced the settlement of the ac versus dc debate in favor of ac. He was also responsible for the adoption of 60 Hz as the standard for ac power systems in the United States.

Born in Austria-Hungary (now Croatia), to a clergyman, Tesla had an incredible memory and a keen affinity for mathematics. He moved to the United States in 1884 and first worked for Thomas Edison. At that time, the country was in the "battle of the currents" with George Westinghouse (1846–1914) promoting ac and Thomas Edison rigidly leading the dc forces. Tesla left Edison and joined Westinghouse because of his interest in ac. Through Westinghouse, Tesla gained the reputation and acceptance of his polyphase ac generation, transmission, and distribution system. He held 700 patents in his lifetime. His other inventions include high-voltage apparatus (the tesla coil) and a wireless transmission system. The unit of magnetic flux density, the tesla, was named in honor of him.

Courtesy Smithsonian Institution

We begin with a discussion of balanced three-phase voltages. Then we analyze each of the four possible configurations of balanced three-phase systems. We also discuss the analysis of unbalanced three-phase systems. We learn how to use *PSpice for Windows* to analyze a balanced or unbalanced three-phase system. Finally, we apply the concepts developed in this chapter to three-phase power measurement and residential electrical wiring.

12.2 Balanced Three-Phase Voltages

Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross-sectional view is shown in Fig. 12.4. The generator basically consists of a rotating magnet (called the *rotor*) surrounded by a stationary winding (called the *stator*). Three separate

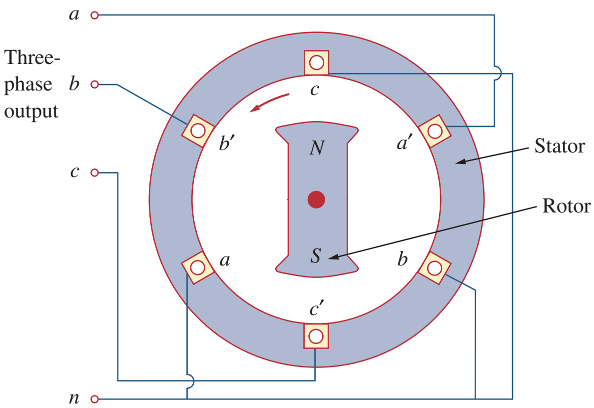

Figure 12.4
A three-phase generator.

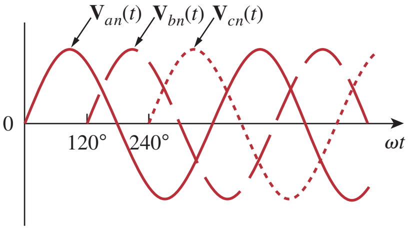

Figure 12.5
The generated voltages are 120° apart from each other.

windings or coils with terminals *a-a′*, *b-b′*, and *c-c′* are physically placed 120° apart around the stator. Terminals *a* and *a′*, for example, stand for one of the ends of coils going into and the other end coming out of the page. As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils. Because the coils are placed 120° apart, the induced voltages in the coils are equal in magnitude but out of phase by 120° (Fig. 12.5). Since each coil can be regarded as a single-phase generator by itself, the three-phase generator can supply power to both single-phase and three-phase loads.

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). (Three-phase current sources are very scarce.) A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either wye-connected as shown in Fig. 12.6(a) or delta-connected as in Fig. 12.6(b).

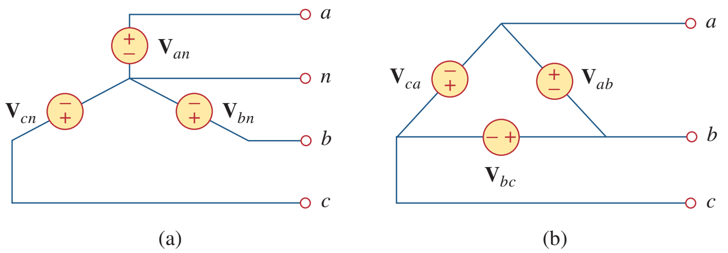

Figure 12.6
Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source.

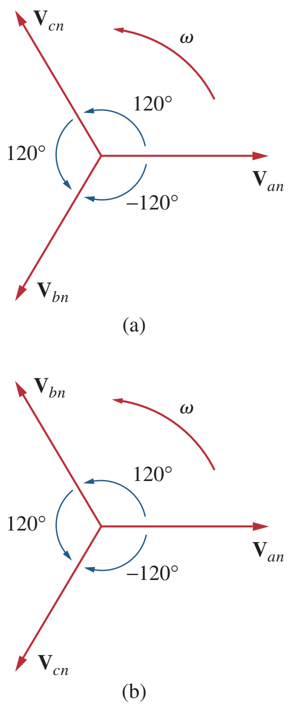

Figure 12.7
Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

Let us consider the wye-connected voltages in Fig. 12.6(a) for now. The voltages $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$ are respectively between lines *a*, *b*, and *c*, and the neutral line *n*. These voltages are called *phase voltages*. If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120°, the voltages are said to be *balanced*. This implies that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0 \qquad \textbf{(12.1)}$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \qquad \textbf{(12.2)}$$

Thus,

> **Balanced phase voltages** are equal in magnitude and are out of phase with each other by 120°.

Since the three-phase voltages are 120° out of phase with each other, there are two possible combinations. One possibility is shown in Fig. 12.7(a) and expressed mathematically as

$$\begin{aligned} \mathbf{V}_{an} &= V_p\underline{/0^\circ} \\ \mathbf{V}_{bn} &= V_p\underline{/-120^\circ} \\ \mathbf{V}_{cn} &= V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ} \end{aligned} \qquad \textbf{(12.3)}$$

where V_p is the effective or rms value of the phase voltages. This is known as the *abc sequence* or *positive sequence*. In this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$, which in turn leads $\mathbf{V}_{cn}$. This sequence is produced when the rotor in Fig. 12.4 rotates counterclockwise. The other possibility is shown in Fig. 12.7(b) and is given by

As a common tradition in power systems, voltage and current in this chapter are in rms values unless otherwise stated.

$$\begin{aligned}\mathbf{V}_{an} &= V_p\underline{/0^\circ}\\ \mathbf{V}_{cn} &= V_p\underline{/-120^\circ}\\ \mathbf{V}_{bn} &= V_p\underline{/-240^\circ} = V_p\underline{/+120^\circ}\end{aligned} \tag{12.4}$$

This is called the *acb sequence* or *negative sequence*. For this phase sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$, which in turn leads $\mathbf{V}_{bn}$. The *acb* sequence is produced when the rotor in Fig. 12.4 rotates in the clockwise direction. It is easy to show that the voltages in Eqs. (12.3) or (12.4) satisfy Eqs. (12.1) and (12.2). For example, from Eq. (12.3),

$$\begin{aligned}\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} &= V_p\underline{/0^\circ} + V_p\underline{/-120^\circ} + V_p\underline{/+120^\circ}\\ &= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866)\\ &= 0\end{aligned} \tag{12.5}$$

The **phase sequence** is the time order in which the voltages pass through their respective maximum values.

The phase sequence may also be regarded as the order in which the phase voltages reach their peak (or maximum) values with respect to time.

The phase sequence is determined by the order in which the phasors pass through a fixed point in the phase diagram.

In Fig. 12.7(a), as the phasors rotate in the counterclockwise direction with frequency ω, they pass through the horizontal axis in a sequence *abcabca* . . . . Thus, the sequence is *abc* or *bca* or *cab*. Similarly, for the phasors in Fig. 12.7(b), as they rotate in the counterclockwise direction, they pass the horizontal axis in a sequence *acbacba* . . . . This describes the *acb* sequence. The phase sequence is important in three-phase power distribution. It determines the direction of the rotation of a motor connected to the power source, for example.

Reminder: As time increases, each phasor (or sinor) rotates at an angular velocity ω.

Like the generator connections, a three-phase load can be either wye-connected or delta-connected, depending on the end application. Figure 12.8(a) shows a wye-connected load, and Fig. 12.8(b) shows a delta-connected load. The neutral line in Fig. 12.8(a) may or may not be there, depending on whether the system is four- or three-wire. (And, of course, a neutral connection is topologically impossible for a delta connection.) A wye- or delta-connected load is said to be *unbalanced* if the phase impedances are not equal in magnitude or phase.

A **balanced load** is one in which the phase impedances are equal in magnitude and in phase.

For a *balanced* wye-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y \tag{12.6}$$

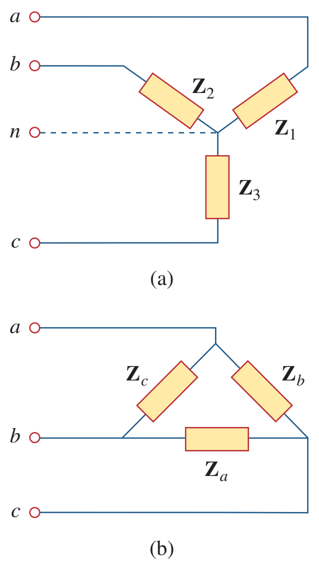

Figure 12.8
Two possible three-phase load configurations: (a) a Y-connected load, (b) a Δ-connected load.

Reminder: A Y-connected load consists of three impedances connected to a neutral node, while a Δ-connected load consists of three impedances connected around a loop. The load is balanced when the three impedances are equal in either case.

where $\mathbf{Z}_Y$ is the load impedance per phase. For a *balanced* delta-connected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta \tag{12.7}$$

where $\mathbf{Z}_\Delta$ is the load impedance per phase in this case. We recall from Eq. (9.69) that

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta \tag{12.8}$$

so we know that a wye-connected load can be transformed into a delta-connected load, or vice versa, using Eq. (12.8).

Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

- Y-Y connection (i.e., Y-connected source with a Y-connected load).
- Y-Δ connection.
- Δ-Δ connection.
- Δ-Y connection.

In subsequent sections, we will consider each of these possible configurations.

It is appropriate to mention here that a balanced delta-connected load is more common than a balanced wye-connected load. This is due to the ease with which loads may be added or removed from each phase of a delta-connected load. This is very difficult with a wye-connected load because the neutral may not be accessible. On the other hand, delta-connected sources are not common in practice because of the circulating current that will result in the delta-mesh if the three-phase voltages are slightly unbalanced.

Example 12.1

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$
$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:
The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}\text{ V}, \qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}\text{ V}, \qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}\text{ V}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

Practice Problem 12.1

Given that $\mathbf{V}_{bn} = 110\underline{/30^\circ}$ V, find $\mathbf{V}_{an}$ and $\mathbf{V}_{cn}$, assuming a positive (*abc*) sequence.

Answer: $110\underline{/150^\circ}$ V, $110\underline{/-90^\circ}$ V.