chapter

11

AC Power Analysis

Four things come not back: the spoken word; the sped arrow; time past; the neglected opportunity.

—Al Halif Omar Ibn

Enhancing Your Career

Career in Power Systems

The discovery of the principle of an ac generator by Michael Faraday in 1831 was a major breakthrough in engineering; it provided a convenient way of generating the electric power that is needed in every electronic, electrical, or electromechanical device we use now.

Electric power is obtained by converting energy from sources such as fossil fuels (gas, oil, and coal), nuclear fuel (uranium), hydro energy (water falling through a head), geothermal energy (hot water, steam), wind energy, tidal energy, and biomass energy (wastes). These various ways of generating electric power are studied in detail in the field of power engineering, which has become an indispensable subdiscipline of electrical engineering. An electrical engineer should be familiar with the analysis, generation, transmission, distribution, and cost of electric power.

A pole-type transformer with a low-voltage, three-wire distribution system.

The electric power industry is a very large employer of electrical engineers. The industry includes thousands of electric utility systems ranging from large, interconnected systems serving large regional areas to small power companies serving individual communities or factories. Due to the complexity of the power industry, there are numerous electrical engineering jobs in different areas of the industry: power plant (generation), transmission and distribution, maintenance, research, data acquisition and flow control, and management. Since electric power is used everywhere, electric utility companies are everywhere, offering exciting training and steady employment for men and women in thousands of communities throughout the world.

11.1 Introduction

Our effort in ac circuit analysis so far has been focused mainly on calculating voltage and current. Our major concern in this chapter is power analysis.

Power analysis is of paramount importance. Power is the most important quantity in electric utilities, electronic, and communication systems, because such systems involve transmission of power from one point to another. Also, every industrial and household electrical device—every fan, motor, lamp, pressing iron, TV, personal computer—has a power rating that indicates how much power the equipment requires; exceeding the power rating can do permanent damage to an appliance. The most common form of electric power is 50- or 60-Hz ac power. The choice of ac over dc allowed high-voltage power transmission from the power generating plant to the consumer.

We will begin by defining and deriving *instantaneous power* and *average power*. We will then introduce other power concepts. As practical applications of these concepts, we will discuss how power is measured and reconsider how electric utility companies charge their customers.

11.2 Instantaneous and Average Power

As mentioned in Chapter 2, the *instantaneous power* $p(t)$ absorbed by an element is the product of the instantaneous voltage $v(t)$ across the element and the instantaneous current $i(t)$ through it. Assuming the passive sign convention,

$$p(t) = v(t)i(t) \tag{11.1}$$

The **instantaneous power** (in watts) is the power at any instant of time.

We can also think of the instantaneous power as the power absorbed by the element at a specific instant of time. Instantaneous quantities are denoted by lowercase letters.

It is the rate at which an element absorbs energy.

Consider the general case of instantaneous power absorbed by an arbitrary combination of circuit elements under sinusoidal excitation, as shown in Fig. 11.1. Let the voltage and current at the terminals of the circuit be

$$v(t) = V_m \cos(\omega t + \theta_v) \tag{11.2a}$$

$$i(t) = I_m \cos(\omega t + \theta_i) \tag{11.2b}$$

where V_m and I_m are the amplitudes (or peak values), and θ_v and θ_i are the phase angles of the voltage and current, respectively. The instantaneous power absorbed by the circuit is

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \tag{11.3}$$

We apply the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)] \tag{11.4}$$

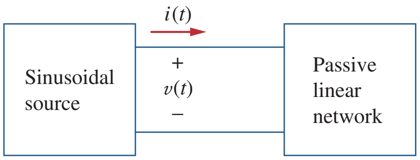

Figure 11.1
Sinusoidal source and passive linear circuit.

and express Eq. (11.3) as

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i) \qquad \textbf{(11.5)}$$

This shows us that the instantaneous power has two parts. The first part is constant or time independent. Its value depends on the phase difference between the voltage and the current. The second part is a sinusoidal function whose frequency is 2ω, which is twice the angular frequency of the voltage or current.

A sketch of $p(t)$ in Eq. (11.5) is shown in Fig. 11.2, where $T = 2\pi/\omega$ is the period of voltage or current. We observe that $p(t)$ is periodic, $p(t) = p(t + T_0)$, and has a period of $T_0 = T/2$, since its frequency is twice that of voltage or current. We also observe that $p(t)$ is positive for some part of each cycle and negative for the rest of the cycle. When $p(t)$ is positive, power is absorbed by the circuit. When $p(t)$ is negative, power is absorbed by the source; that is, power is transferred from the circuit to the source. This is possible because of the storage elements (capacitors and inductors) in the circuit.

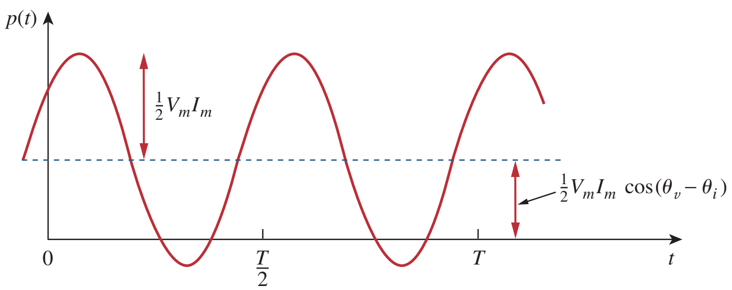

Figure 11.2
The instantaneous power $p(t)$ entering a circuit.

The instantaneous power changes with time and is therefore difficult to measure. The *average* power is more convenient to measure. In fact, the wattmeter, the instrument for measuring power, responds to average power.

The **average power**, in watts, is the average of the instantaneous power over one period.

Thus, the average power is given by

$$P = \frac{1}{T}\int_0^T p(t)\, dt \qquad \textbf{(11.6)}$$

Although Eq. (11.6) shows the averaging done over T, we would get the same result if we performed the integration over the actual period of $p(t)$ which is $T_0 = T/2$.

Substituting $p(t)$ in Eq. (11.5) into Eq. (11.6) gives

$$\begin{aligned} P &= \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\, dt \\ &\quad + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\, dt \\ &= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt \\ &\quad + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\, dt \end{aligned} \tag{11.7}$$

The first integrand is constant, and the average of a constant is the same constant. The second integrand is a sinusoid. We know that the average of a sinusoid over its period is zero because the area under the sinusoid during a positive half-cycle is canceled by the area under it during the following negative half-cycle. Thus, the second term in Eq. (11.7) vanishes and the average power becomes

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) \tag{11.8}$$

Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, what is important is the difference in the phases of the voltage and current.

Note that $p(t)$ is time-varying while P does not depend on time. To find the instantaneous power, we must necessarily have $v(t)$ and $i(t)$ in the time domain. But we can find the average power when voltage and current are expressed in the time domain, as in Eq. (11.8), or when they are expressed in the frequency domain. The phasor forms of $v(t)$ and $i(t)$ in Eq. (11.2) are $\mathbf{V} = V_m\underline{/\theta_v}$ and $\mathbf{I} = I_m\underline{/\theta_i}$, respectively. P is calculated using Eq. (11.8) or using phasors $\mathbf{V}$ and $\mathbf{I}$. To use phasors, we notice that

$$\begin{aligned} \frac{1}{2}\mathbf{VI}^* &= \frac{1}{2}V_m I_m\underline{/\theta_v - \theta_i} \\ &= \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)] \end{aligned} \tag{11.9}$$

We recognize the real part of this expression as the average power P according to Eq. (11.8). Thus,

$$P = \frac{1}{2}\text{Re}[\mathbf{VI}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) \tag{11.10}$$

Consider two special cases of Eq. (11.10). When $\theta_v = \theta_i$, the voltage and current are in phase. This implies a purely resistive circuit or resistive load R, and

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R \tag{11.11}$$

where $|\mathbf{I}|^2 = \mathbf{I} \times \mathbf{I}^*$. Equation (11.11) shows that a purely resistive circuit absorbs power at all times. When $\theta_v - \theta_i = \pm 90°$, we have a purely reactive circuit, and

$$P = \frac{1}{2}V_m I_m \cos 90° = 0 \tag{11.12}$$

showing that a purely reactive circuit absorbs no average power. In summary,

> A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

Example 11.1

Given that

$$v(t) = 120 \cos(377t + 45°) \text{ V} \quad \text{and} \quad i(t) = 10 \cos(377t - 10°) \text{ A}$$

find the instantaneous power and the average power absorbed by the passive linear network of Fig. 11.1.

Solution:
The instantaneous power is given by

$$p = vi = 1200 \cos(377t + 45°) \cos(377t - 10°)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

gives

$$p = 600[\cos(754t + 35°) + \cos 55°]$$

or

$$p(t) = 344.2 + 600 \cos(754t + 35°) \text{ W}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10) \cos[45° - (-10°)]$$
$$= 600 \cos 55° = 344.2 \text{ W}$$

which is the constant part of $p(t)$ above.

Practice Problem 11.1

Calculate the instantaneous power and average power absorbed by the passive linear network of Fig. 11.1 if

$$v(t) = 330 \cos(10t + 20°) \text{ V} \quad \text{and} \quad i(t) = 33 \sin(10t + 60°) \text{ A}$$

Answer: $3.5 + 5.445 \cos(20t - 10°)$ kW, 3.5 kW.

Example 11.2

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\underline{/0°}$ is applied across it.

Solution:
The current through the impedance is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\underline{/0°}}{30 - j70} = \frac{120\underline{/0°}}{76.16\underline{/-66.8°}} = 1.576\underline{/66.8°} \text{ A}$$

The average power is

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8°) = 37.24 \text{ W}$$

Practice Problem 11.2

A current $\mathbf{I} = 33\angle 30°$ A flows through an impedance $\mathbf{Z} = 40\angle -22°\ \Omega$. Find the average power delivered to the impedance.

Answer: 20.19 kW.

Example 11.3

For the circuit shown in Fig. 11.3, find the average power supplied by the source and the average power absorbed by the resistor.

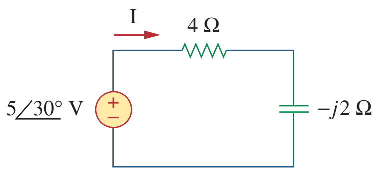

Figure 11.3
For Example 11.3.

Solution:
The current **I** is given by

$$\mathbf{I} = \frac{5\angle 30°}{4 - j2} = \frac{5\angle 30°}{4.472\angle -26.57°} = 1.118\angle 56.57° \text{ A}$$

The average power supplied by the voltage source is

$$P = \frac{1}{2}(5)(1.118)\cos(30° - 56.57°) = 2.5 \text{ W}$$

The current through the resistor is

$$\mathbf{I}_R = \mathbf{I} = 1.118\angle 56.57° \text{ A}$$

and the voltage across it is

$$\mathbf{V}_R = 4\mathbf{I}_R = 4.472\angle 56.57° \text{ V}$$

The average power absorbed by the resistor is

$$P = \frac{1}{2}(4.472)(1.118) = 2.5 \text{ W}$$

which is the same as the average power supplied. Zero average power is absorbed by the capacitor.

Practice Problem 11.3

In the circuit of Fig. 11.4, calculate the average power absorbed by the resistor and inductor. Find the average power supplied by the voltage source.

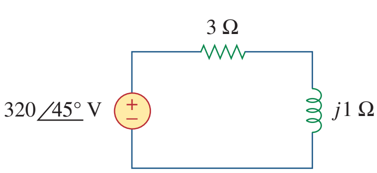

Figure 11.4
For Practice Prob. 11.3.

Answer: 15.361 kW, 0 W, 15.361 kW.

Example 11.4

Determine the average power generated by each source and the average power absorbed by each passive element in the circuit of Fig. 11.5(a).

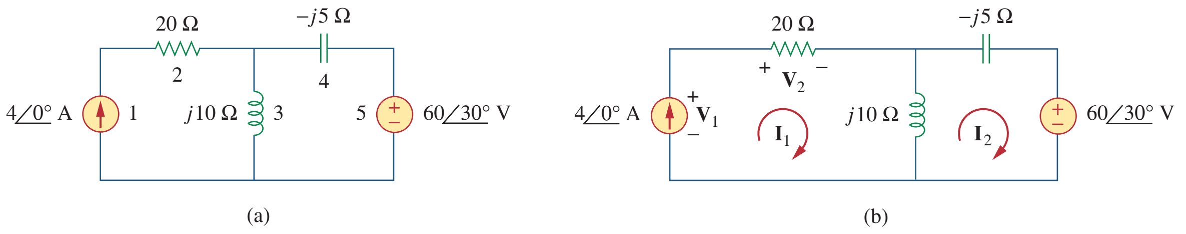

Figure 11.5
For Example 11.4.

Solution:
We apply mesh analysis as shown in Fig. 11.5(b). For mesh 1,

$$\mathbf{I}_1 = 4\text{ A}$$

For mesh 2,

$$(j10 - j5)\mathbf{I}_2 - j10\mathbf{I}_1 + 60\underline{/30^\circ} = 0, \qquad \mathbf{I}_1 = 4\text{ A}$$

or

$$j5\mathbf{I}_2 = -60\underline{/30^\circ} + j40 \qquad \Rightarrow \qquad \mathbf{I}_2 = -12\underline{/-60^\circ} + 8 = 10.58\underline{/79.1^\circ}\text{ A}$$

For the voltage source, the current flowing from it is $\mathbf{I}_2 = 10.58\underline{/79.1^\circ}$ A and the voltage across it is $60\underline{/30^\circ}$ V, so that the average power is

$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ) = 207.8\text{ W}$$

Following the passive sign convention (see Fig. 1.8), this average power is absorbed by the source, in view of the direction of $\mathbf{I}_2$ and the polarity of the voltage source. That is, the circuit is delivering average power to the voltage source.

For the current source, the current through it is $\mathbf{I}_1 = 4\underline{/0^\circ}$ and the voltage across it is

$$\mathbf{V}_1 = 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 80 + j10(4 - 2 - j10.39) = 183.9 + j20 = 184.984\underline{/6.21^\circ}\text{ V}$$

The average power supplied by the current source is

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0) = -367.8\text{ W}$$

It is negative according to the passive sign convention, meaning that the current source is supplying power to the circuit.

For the resistor, the current through it is $\mathbf{I}_1 = 4\underline{/0^\circ}$ and the voltage across it is $20\mathbf{I}_1 = 80\underline{/0^\circ}$, so that the power absorbed by the resistor is

$$P_2 = \frac{1}{2}(80)(4) = 160\text{ W}$$

For the capacitor, the current through it is $\mathbf{I}_2 = 10.58\underline{/79.1^\circ}$ and the voltage across it is $-j5\mathbf{I}_2 = (5\underline{/-90^\circ})(10.58\underline{/79.1^\circ}) = 52.9\underline{/79.1^\circ - 90^\circ}$. The average power absorbed by the capacitor is

$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90^\circ) = 0$$

For the inductor, the current through it is $\mathbf{I}_1 - \mathbf{I}_2 = 2 - j10.39 = 10.58\underline{/-79.1^\circ}$. The voltage across it is $j10(\mathbf{I}_1 - \mathbf{I}_2) = 10.58\underline{/-79.1^\circ + 90^\circ}$. Hence, the average power absorbed by the inductor is

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos 90^\circ = 0$$

Notice that the inductor and the capacitor absorb zero average power and that the total power supplied by the current source equals the power absorbed by the resistor and the voltage source, or

$$P_1 + P_2 + P_3 + P_4 + P_5 = -367.8 + 160 + 0 + 0 + 207.8 = 0$$

indicating that power is conserved.

Practice Problem 11.4

Calculate the average power absorbed by each of the five elements in the circuit of Fig. 11.6.

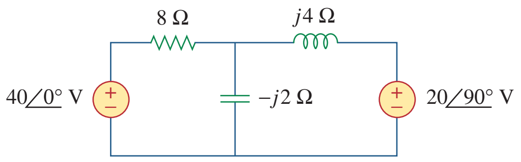

Figure 11.6
For Practice Prob. 11.4.

Answer: 40-V Voltage source: −60 W; j20-V Voltage source: −40 W; resistor: 100 W; others: 0 W.

11.3 Maximum Average Power Transfer

In Section 4.8 we solved the problem of maximizing the power delivered by a power-supplying resistive network to a load R_L. Representing the circuit by its Thevenin equivalent, we proved that the maximum power would be delivered to the load if the load resistance is equal to the Thevenin resistance $R_L = R_{\text{Th}}$. We now extend that result to ac circuits.

Consider the circuit in Fig. 11.7, where an ac circuit is connected to a load $\mathbf{Z}_L$ and is represented by its Thevenin equivalent. The load is usually represented by an impedance, which may model an electric

Example 11.6

In the circuit in Fig. 11.11, find the value of R_L that will absorb the maximum average power. Calculate that power.

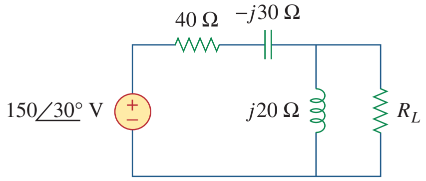

Figure 11.11
For Example 11.6.

Solution:
We first find the Thevenin equivalent at the terminals of R_L.

$$\mathbf{Z}_{\text{Th}} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

By voltage division,

$$\mathbf{V}_{\text{Th}} = \frac{j20}{j20 + 40 - j30}(150\underline{/30^\circ}) = 72.76\underline{/134^\circ}\ \text{V}$$

The value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_{\text{Th}}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + R_L} = \frac{72.76\underline{/134^\circ}}{33.66 + j22.35} = 1.8\underline{/100.42^\circ}\ \text{A}$$

The maximum average power absorbed by R_L is

$$P_{\text{max}} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29\ \text{W}$$

Practice Problem 11.6

In Fig. 11.12, the resistor R_L is adjusted until it absorbs the maximum average power. Calculate R_L and the maximum average power absorbed by it.

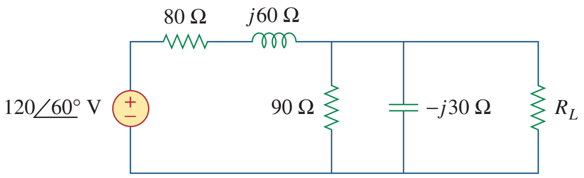

Figure 11.12
For Practice Prob. 11.6.

Answer: 30 Ω, 6.863 W.

11.4 Effective or RMS Value

The idea of *effective value* arises from the need to measure the effectiveness of a voltage or current source in delivering power to a resistive load.

> The **effective value** of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

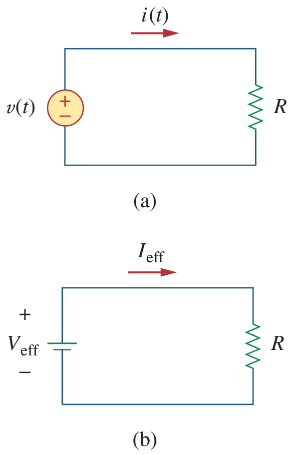

Figure 11.13
Finding the effective current: (a) ac circuit, (b) dc circuit.

In Fig. 11.13, the circuit in (a) is ac while that of (b) is dc. Our objective is to find I_{eff} that will transfer the same power to resistor R as the sinusoid i. The average power absorbed by the resistor in the ac circuit is

$$P = \frac{1}{T}\int_0^T i^2 R\, dt = \frac{R}{T}\int_0^T i^2\, dt \tag{11.22}$$

while the power absorbed by the resistor in the dc circuit is

$$P = I_{\text{eff}}^2 R \tag{11.23}$$

Equating the expressions in Eqs. (11.22) and (11.23) and solving for I_{eff}, we obtain

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} \tag{11.24}$$

The effective value of the voltage is found in the same way as current; that is,

$$V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt} \tag{11.25}$$

This indicates that the effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal. Thus, the effective value is often known as the *root-mean-square* value, or *rms* value for short; and we write

$$I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}} \tag{11.26}$$

For any periodic function $x(t)$ in general, the rms value is given by

$$X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\, dt} \tag{11.27}$$

The **effective value** of a periodic signal is its root mean square (rms) value.

Equation 11.27 states that to find the rms value of $x(t)$, we first find its *square* x^2 and then find the *mean* of that, or

$$\frac{1}{T}\int_0^T x^2\, dt$$

and then the square *root* ($\sqrt{\quad}$) of that mean. The rms value of a constant is the constant itself. For the sinusoid $i(t) = I_m \cos \omega t$, the effective or rms value is

$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t\, dt} \\ &= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}} \end{aligned} \tag{11.28}$$

Similarly, for $v(t) = V_m \cos \omega t$,

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \tag{11.29}$$

Keep in mind that Eqs. (11.28) and (11.29) are only valid for sinusoidal signals.

The average power in Eq. (11.8) can be written in terms of the rms values.

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \tag{11.30}$$

Similarly, the average power absorbed by a resistor R in Eq. (11.11) can be written as

$$P = I_{rms}^2 R = \frac{V_{rms}^2}{R} \tag{11.31}$$

When a sinusoidal voltage or current is specified, it is often in terms of its maximum (or peak) value or its rms value, since its average value is zero. The power industries specify phasor magnitudes in terms of their rms values rather than peak values. For instance, the 110 V available at every household is the rms value of the voltage from the power company. It is convenient in power analysis to express voltage and current in their rms values. Also, analog voltmeters and ammeters are designed to read directly the rms value of voltage and current, respectively.

Example 11.7

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a 2-Ω resistor, find the average power absorbed by the resistor.

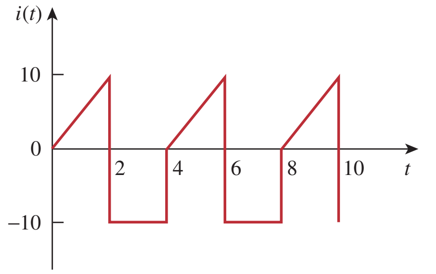

Figure 11.14
For Example 11.7.

Solution:
The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Big|_0^2 + 100t\Big|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165\text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{rms}^2 R = (8.165)^2(2) = 133.3\text{ W}$$

Practice Problem 11.7

Find the rms value of the current waveform of Fig. 11.15. If the current flows through a 9-Ω resistor, calculate the average power absorbed by the resistor.

Answer: 9.238 A, 768 W.

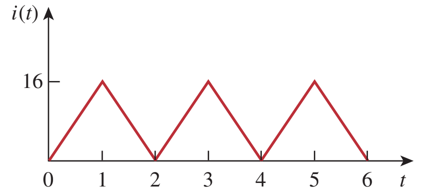

Figure 11.15
For Practice Prob. 11.7.

Example 11.8

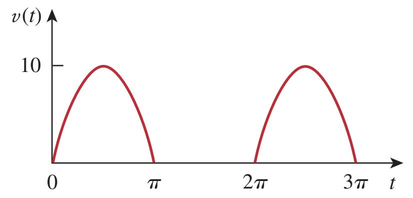

Figure 11.16
For Example 11.8.

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.

Solution:
The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\, dt = \frac{1}{2\pi}\left[\int_0^{\pi} (10 \sin t)^2\, dt + \int_{\pi}^{2\pi} 0^2\, dt\right]$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{\pi} \frac{100}{2}(1 - \cos 2t)\, dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Bigg|_0^{\pi}$$
$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5 \text{ V}$$

The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5 \text{ W}$$

Practice Problem 11.8

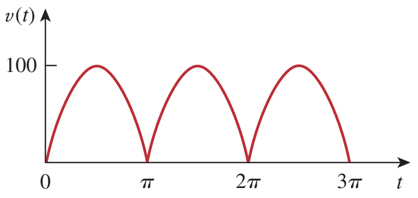

Figure 11.17
For Practice Prob. 11.8.

Find the rms value of the full-wave rectified sine wave in Fig. 11.17. Calculate the average power dissipated in a 6-Ω resistor.

Answer: 70.71 V, 833.3 W.

11.5 Apparent Power and Power Factor

In Section 11.2 we saw that if the voltage and current at the terminals of a circuit are

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad \text{and} \qquad i(t) = I_m \cos(\omega t + \theta_i) \tag{11.32}$$

or, in phasor form, $\mathbf{V} = V_m \angle \theta_v$ and $\mathbf{I} = I_m \angle \theta_i$, the average power is

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \tag{11.33}$$

In Section 11.4, we saw that

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i) \tag{11.34}$$

We have added a new term to the equation:

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}} \tag{11.35}$$