Lab Session 13

DRAW PHASORS DIAGRAMS FOR RL, RC AND RLC CIRCUITS.

OBJECTIVE:

1. To Study the AC response of Resistor, Inductor and Capacitor.
2. To study and draw the phasors diagram of the current and the voltage for Pure Resistive Circuits, RL , RC and RLC Circuits.

APPARATUS:

1. Resistor
2. Inductor
3. Capacitor
4. DMM
5. Connecting Wires
6. AC Power Supply
7. Oscilloscope

Theory

Introduction : 1. **Pure Resistive Circuit**

No Energy storing element present. AC circuits involving purely resistance, behave the same way as DC circuits. Both voltage and current have zero phase shift as shown in the diagram below:

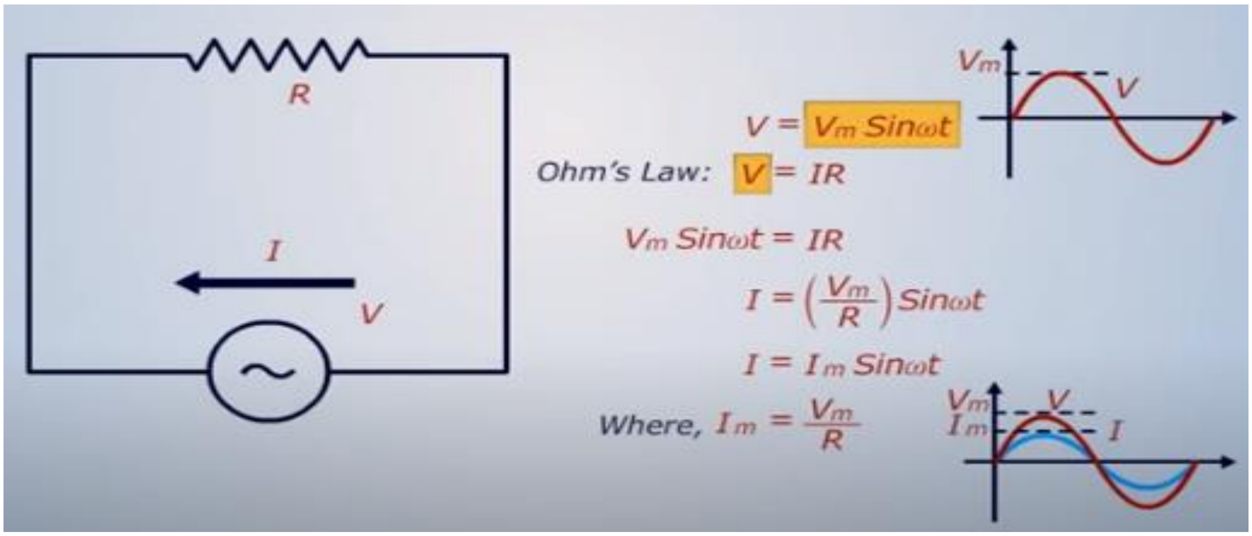


Figure 13.1 (Pure Resistive Circuit and wave form)

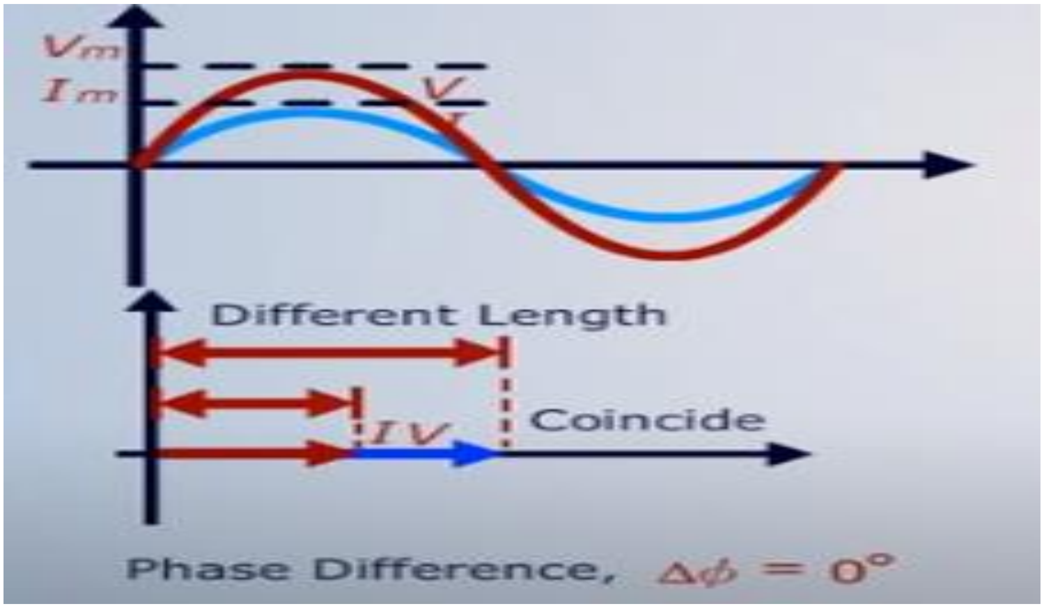
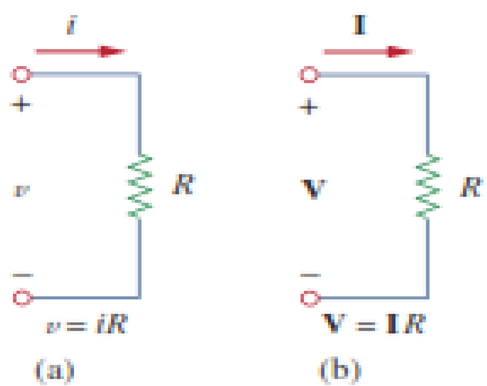


Figure 13.2 (Pure Resistive wave form &Pharos diagram)



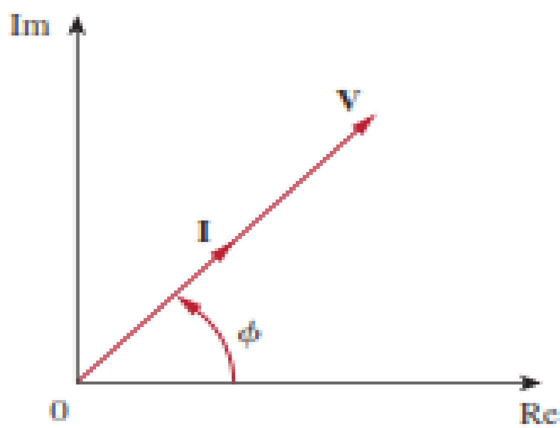


Figure 13.3 Voltage-current relations for a resistor in : (a) time domain, (b) frequency domain.

(Pharos diagram for the resistor)

Simulation results of Pure Resistive Circuit:

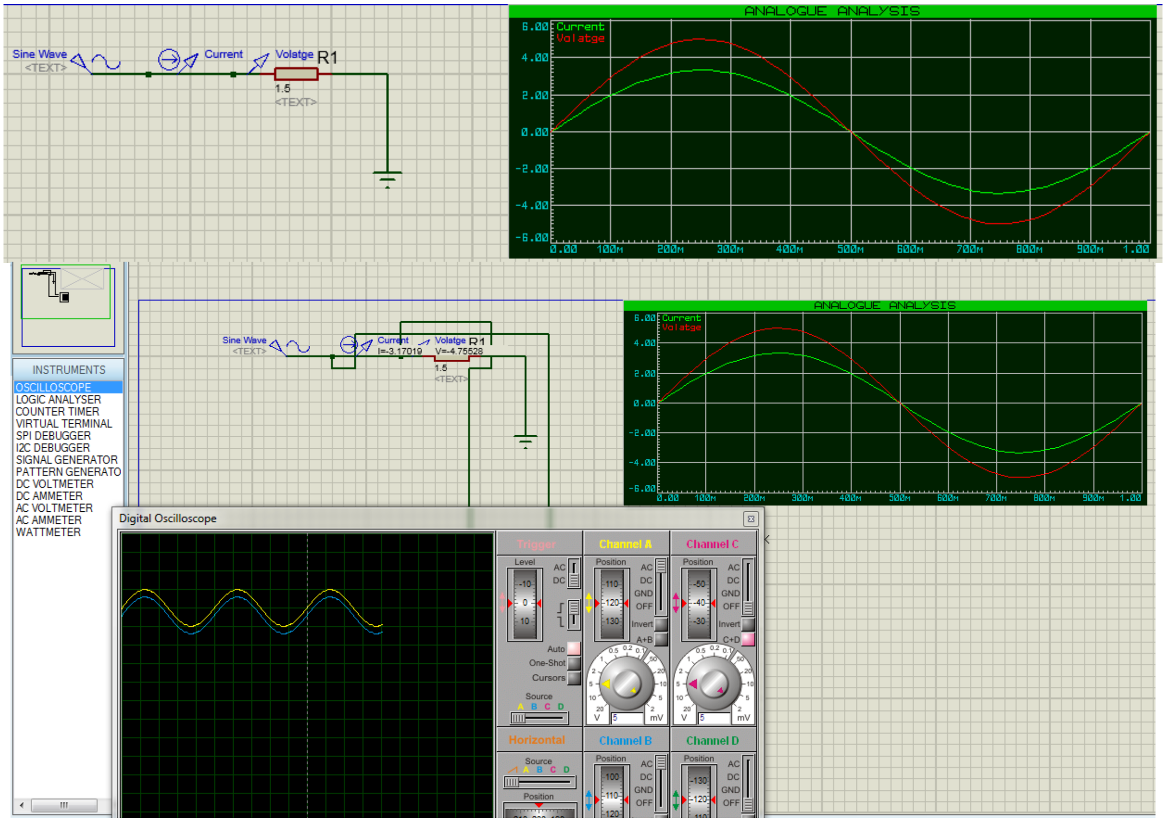
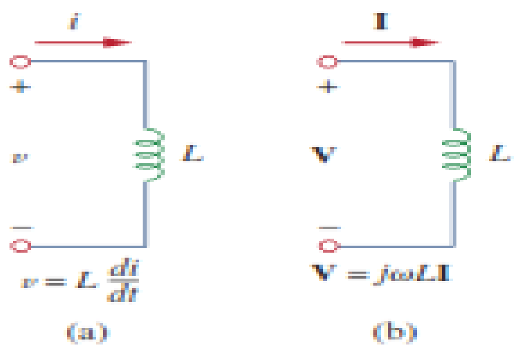


Figure 13.4 (Simulation results of Pure Resistive Circuits, no Phase difference Observed)

2. **Inductive Circuits**

The inductor presents a high impedance to AC current, and as the voltage continues to grow, the current builds up slowly. The current (blue) lags the voltage waveform (green) by 90° as shown in figure 13.6. The phase angle, symbol θ is shown on the diagram below:



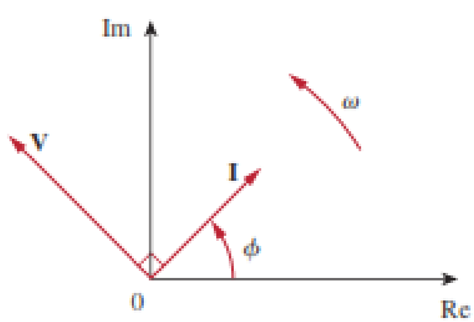


Figure 13.5 Voltage-current relations for an inductor in the: (a) time domain, (b) frequency domain.

(Pharos diagram for the inductor , **I** lags **V.**

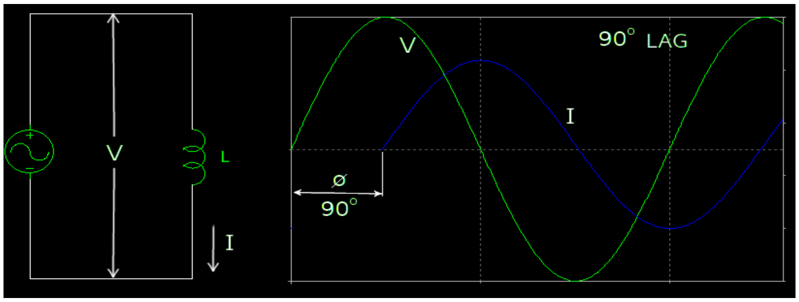
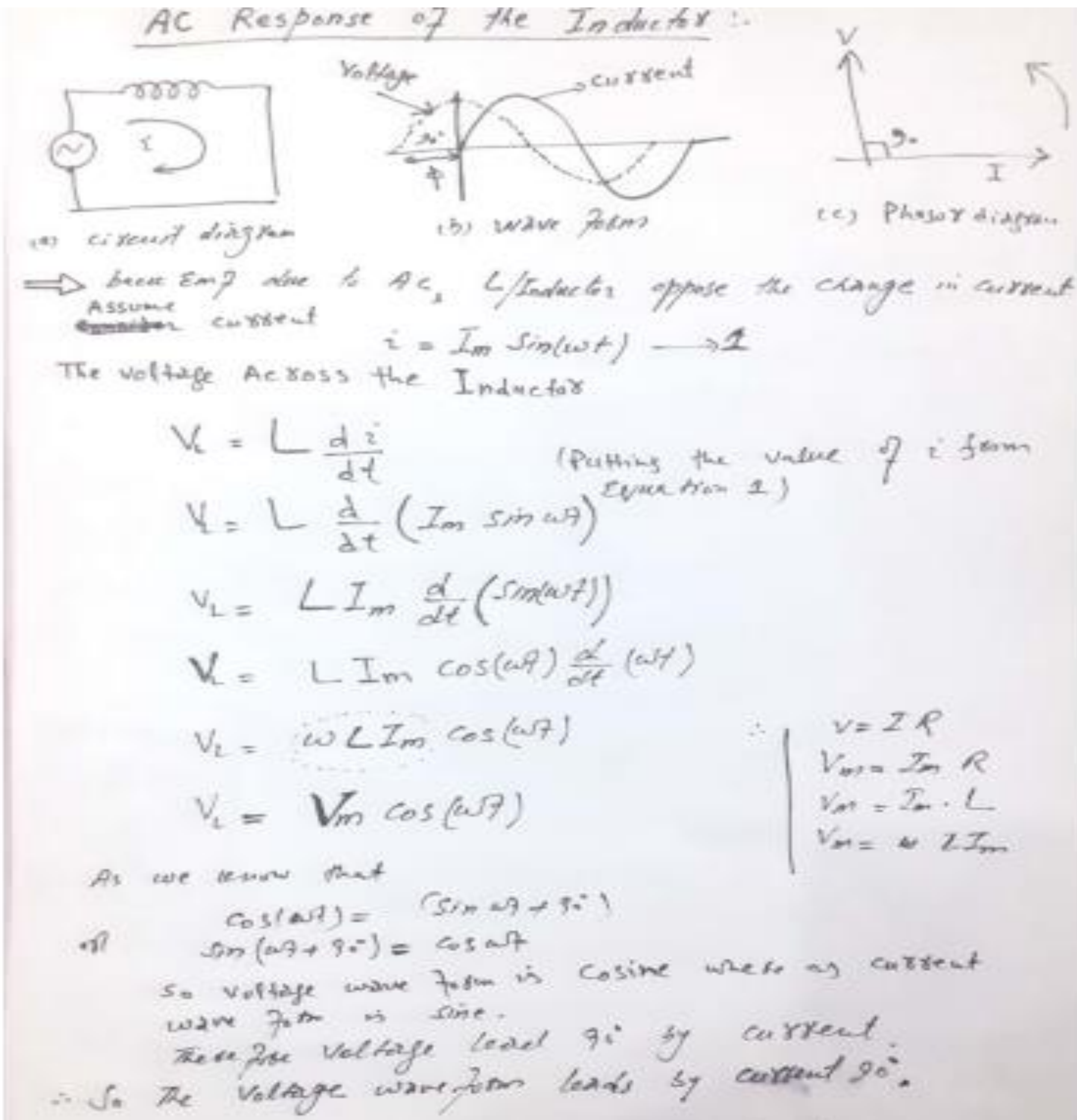


Figure 13.6 (Inductive circuit showing **I l**ags **V by 90º.**

The inductors resistance to AC is called inductive reactance, symbol X_L and is calculated with the formula is $X_L = 2\pi fL$.

<u>Theoretically Prove of I lags V</u>



AC Response of the Inductor :-

Voltage
current
90°
ɸ
(a) circuit diagram
(b) wave form
(c) Phasor diagram

⇒ Back Emf due to AC, L/Inductor oppose the change in current

Assume current

$i = I_m \sin(\omega t) \longrightarrow 1$

The voltage Across the Inductor

$V_L = L \frac{di}{dt}$ (Putting the value of i from Equation 1)

$V_L = L \frac{d}{dt}(I_m \sin \omega t)$

$V_L = L I_m \frac{d}{dt}(\sin(\omega t))$

$V_L = L I_m \cos(\omega t) \frac{d}{dt}(\omega t)$

$V_L = \omega L I_m \cos(\omega t)$

$V_L = V_m \cos(\omega t)$

∴ $V = IR$
$V_m = I_m R$
$V_m = I_m \cdot L$
$V_m = \omega L I_m$

As we know that

$\cos(\omega t) = (\sin \omega t + 90°)$

or $\sin(\omega t + 90°) = \cos \omega t$

So voltage wave form is Cosine where as current wave form is sine.

Therefore Voltage lead 90° by current.

∴ So The Voltage wave form leads by current 90°.

<u>Implementing RL Series Circuit</u>

All inductors have resistance as they are made from a coil of wire. Often additional series resistance will be added to shape the response of the circuit. A fixed resistor will offer the same resistance to an AC circuit at all frequencies, whereas the inductors resistance will change with frequency. The combined resistance is now an impedance, and again the current will lag the voltage through the inductor.

Consider a simple RL circuit as shown in figure 13.7 below in which resistor, R and inductor, L are connected in series with a voltage supply of V volts. Let us think the current flowing in the circuit is I (amp) and current through resistor and inductor is IR and IX_L respectively. Since both resistance and inductor are connected in series, so the current in both the elements and the circuit remains the same. i.e $IR = IX_L = I$.

Let V_R and V_L be the voltage drop across resistor and inductor.

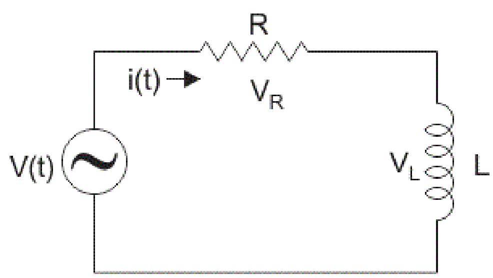


Fig 13.7: RL Series Circuit

Applying <u>Kirchhoff voltage law</u> (i.e sum of voltage drop must be equal to apply voltage) to this circuit we get,

$$V = V_R + V_L$$

Phasor Diagram for RL Circuit:

Before drawing the **phasors diagram of series RL circuit**, one should know the relationship between voltage and current in case of resistor and inductor.

1. In case of resistor, the voltage and the current are in same phase or we can say that the phase angle difference between voltage and current is zero.

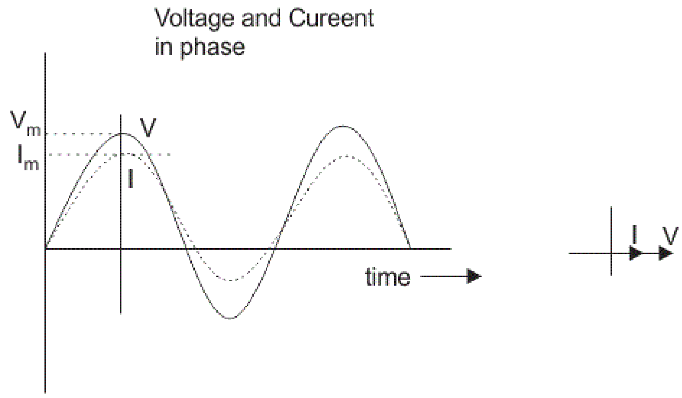


Fig. 13.8 (In phase Voltage & Current in Pure Resistive Circuit)

2. In inductor, the voltage and the current are not in phase. The voltage leads that of current by 90^o or in other words, voltage attains its maximum and zero value 90^o before the current attains it.

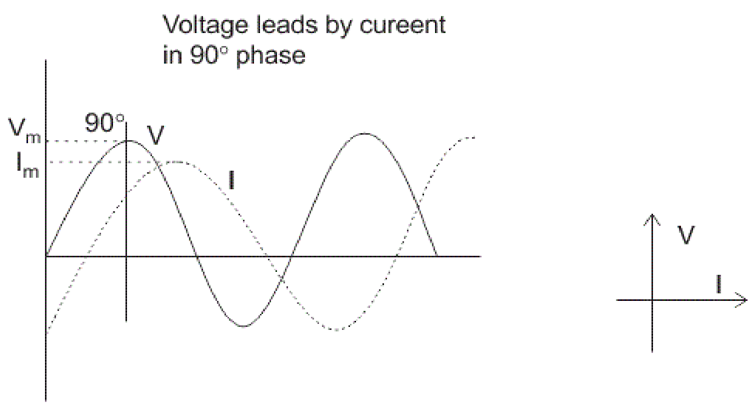


Fig. 13.9 (Voltage leads by Current in 90^o in Inductive Circuit)

3. For drawing the pharos diagram of series RL circuit; follow the following steps:

Step- I. In case of series RL circuit, resistor and inductor are connected in series, so current flowing in both the elements are same i.e $I_R = I_L = I$. So, take current pharos as reference and draw it on horizontal axis as shown in diagram.
Step- II. In case of resistor, both voltage and current are in same phase. So draw the voltage phasor, V_R along same axis or direction as that of current pharos. i.e V_R is in phase with I.
Step- III. We know that in inductor, voltage leads current by 90^o, so draw V_L (voltage drop across inductor) perpendicular to current pharos.
Step- IV. Now we have two voltages V_R and V_L. Draw the resultant vector (V_G) of these two voltages. Such as,

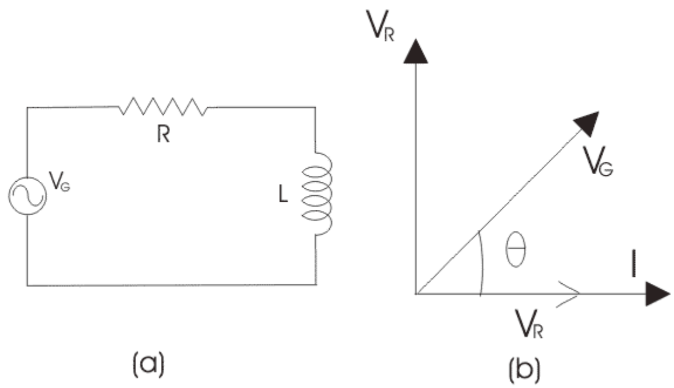


Figure 13.10 (Circuit and Pharos diagram of RL Circuit)

Impedance of Series RL Circuit:

The impedance of series RL circuit opposes the flow of alternating current. The impedance of series RL Circuit is nothing but the combine effect of resistance (R) and inductive reactance (X_L) of the circuit as a whole. The impedance Z in ohms is given by,

$$Z = \sqrt{R^2 + X_L^2}$$

and from right angle triangle, phase angle θ is

$$\theta = \tan^{-1}\frac{XL}{R}$$

Simulation results

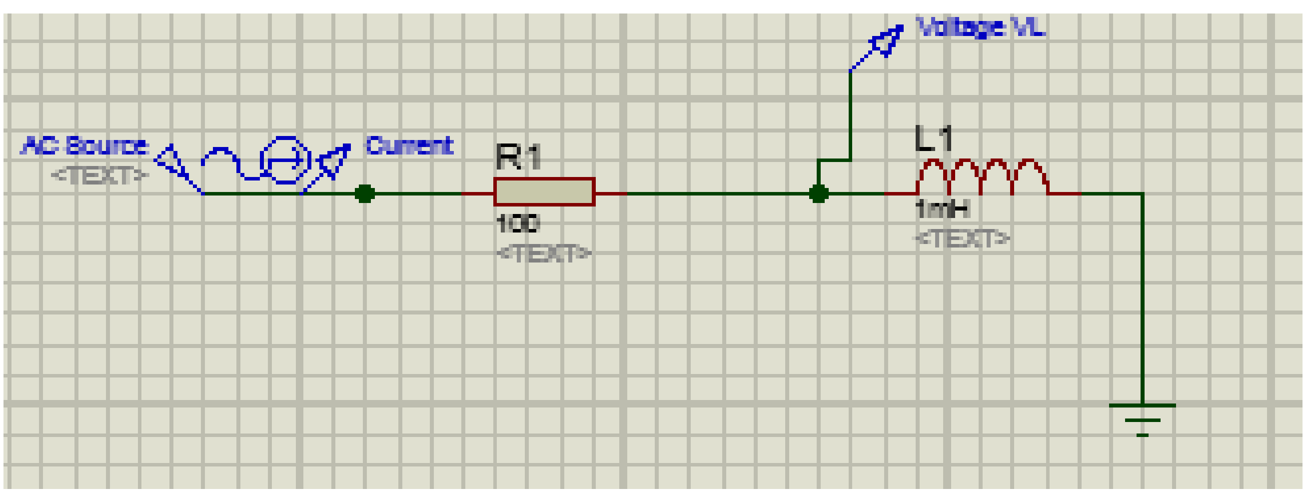


Figure 13.11 (Simulation of RL Series Circuit)

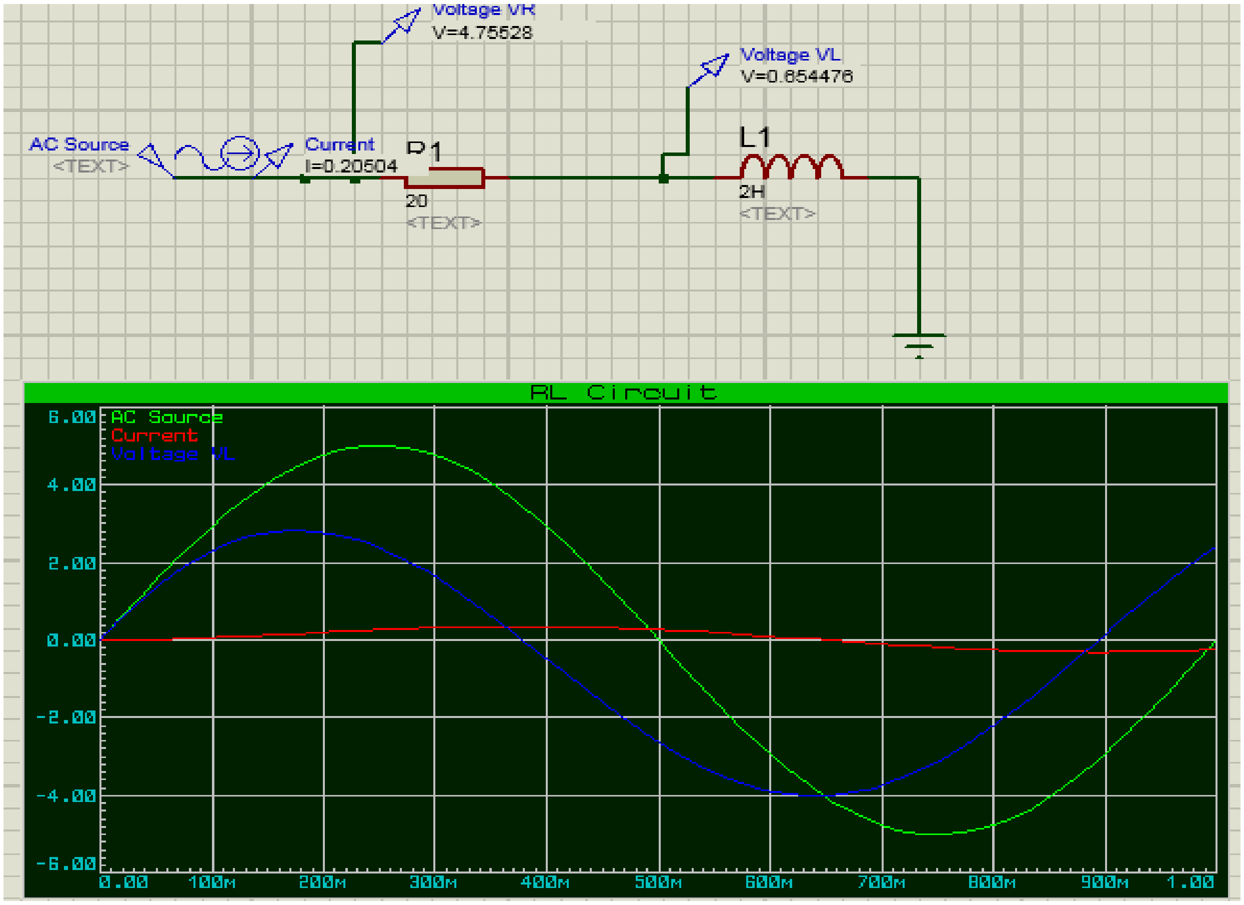


Figure 13.12 (Simulation Results of RL Series Circuit)

3. **<u>Capacitive Circuits</u>**

The capacitor initially presents a low impedance to AC current, decreasing as the voltage rises. The current (blue) leads the voltage waveform (green) by 90°. The phase angle, symbol θ is shown on the diagram below.

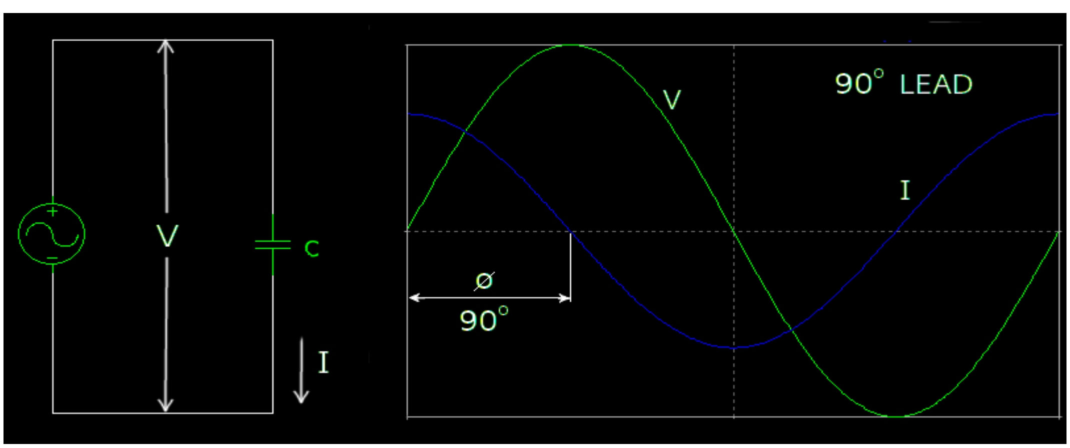


Figure 13.13 (Capacitive circuit I lead V)

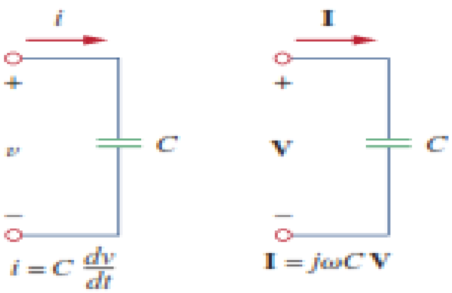
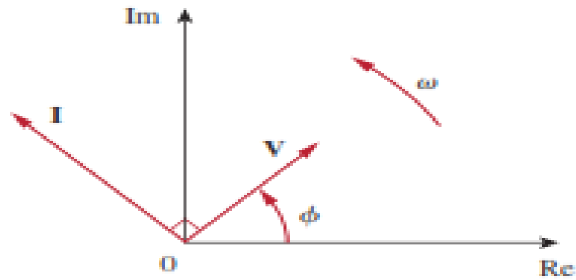


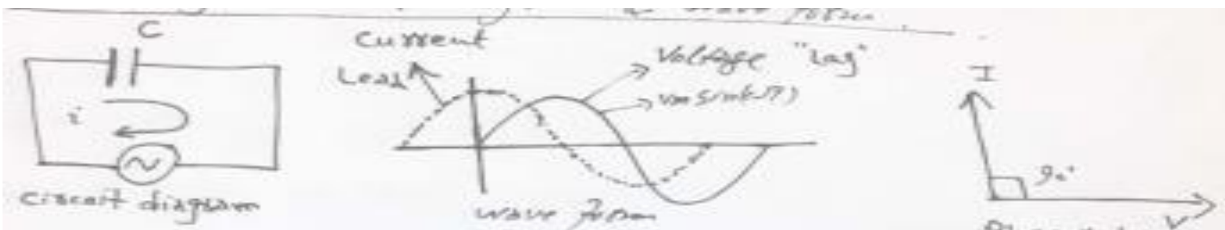
fig. 13.14 Voltage-current relations for a capacitor in the: (a) time domain, (b) frequency domain.



Pharos diagram , I leads V.

The capacitors resistance to AC is called capacitive reactance, symbol X_C and is calculated with the formula: Theoretically prove Current lead with Voltage in capacitor as given below:

$$X_C = \frac{1}{2\pi fC}$$



Capacitor don't like the abrupt change in voltage

$V(t) = V_m \sin(\omega t)$ $\quad \therefore V = V_m \sin(\omega t)$

The current through the capacitor is

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d}{dt}(V_m \sin(\omega t))$$

$$i = C V_m \frac{d}{dt}(\sin(\omega t))$$

$$i = C V_m \cos(\omega t)\frac{d}{dt}(\omega t)$$

$$i = \omega C V_m \cos(\omega t)$$

$$\downarrow$$

$$i = I_m \cos(\omega t)$$

ohm's Law

$\therefore i = V/R$

$i = \frac{V}{\frac{1}{C}}$

$i = CV$

As we know that,

$$\cos(\omega t) = \sin(\omega t + 90^\circ)$$

$$\therefore i = I_m \sin(\omega t + 90^\circ)$$

Result: we can say that the voltage wave form lags the current wave form by 90°.

"OR"

current Lead 90° by voltage

RC Series Circuit:

Consider a simple RC circuit as shown in fig.13.15 in which resistor, R and capacitor, C are connected in series with a voltage supply of V volts. Let us think the current flowing in the circuit is I (amp) and current through resistor and capacitor is I_R and I_c respectively. Since both resistance and capacitor are connected in series, so the current in both the elements and the circuit remains the same. i.e $I_R = I_c = I$. Let V_R and V_c be the voltage drop across resistor and capacitor. When the resistor is connected in series to the capacitor in AC circuit, this results the in-phase between the load current I and voltage V_R. While the current at capacitor is 90° leads applied Voltage $V_{C.}$

The total voltage is presented by the summation of Vectors as

$$V_T = V_R + V_{C.}$$

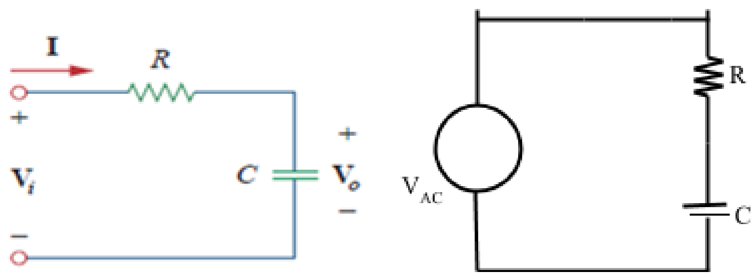


Fig. 13.15 RC Series Circuit

Pharos diagram for RC Circuit:

In capacitor, the voltage and the current are not in phase. The voltage lags that of current by 90° or in other words, voltage attains its maximum and zero value 90° after the current attains it.

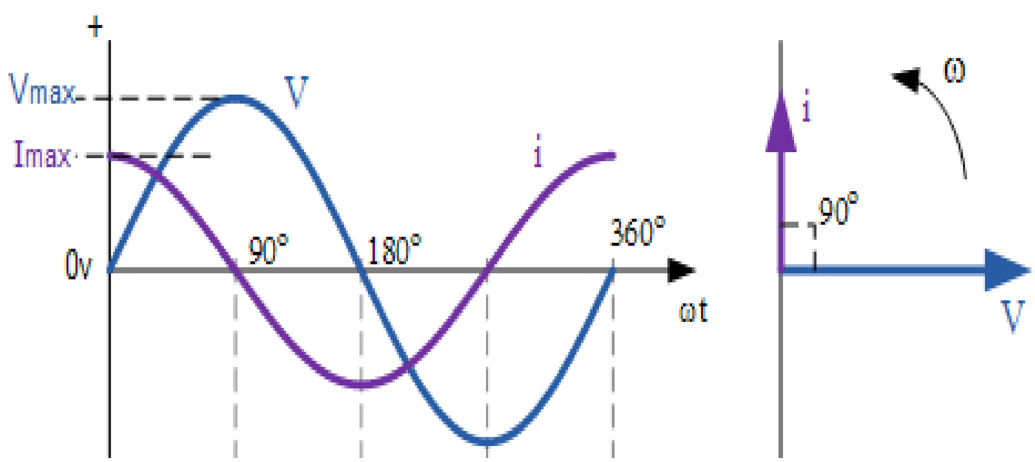


Fig. 13.16 Wave form and Pharos diagram of RC Circuit

Simulation Results

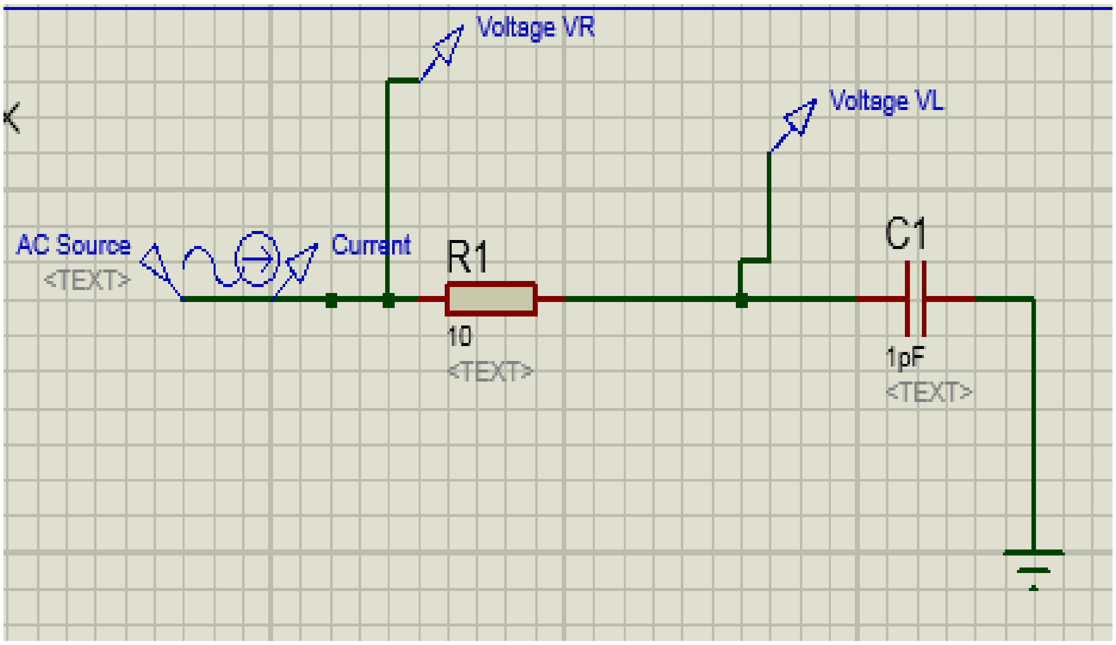


Figure16.17 (Simulation RC Circuit)

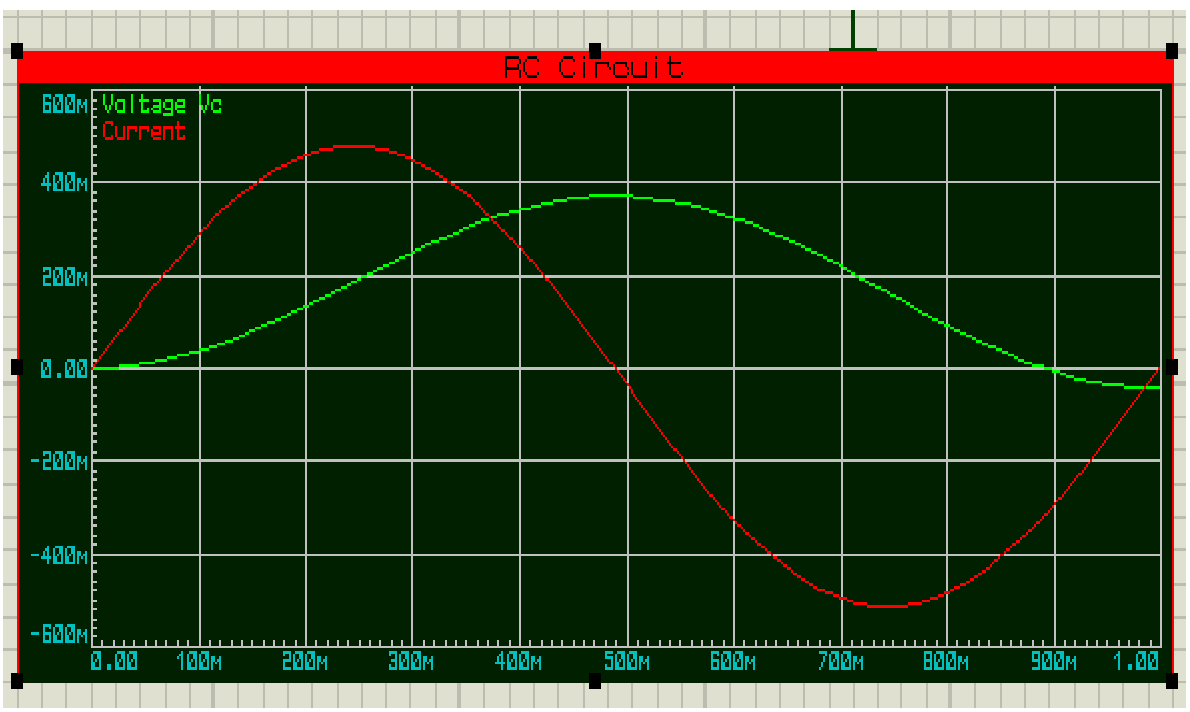


Fig. 13.18 (Pharos difference of RC Circuit)

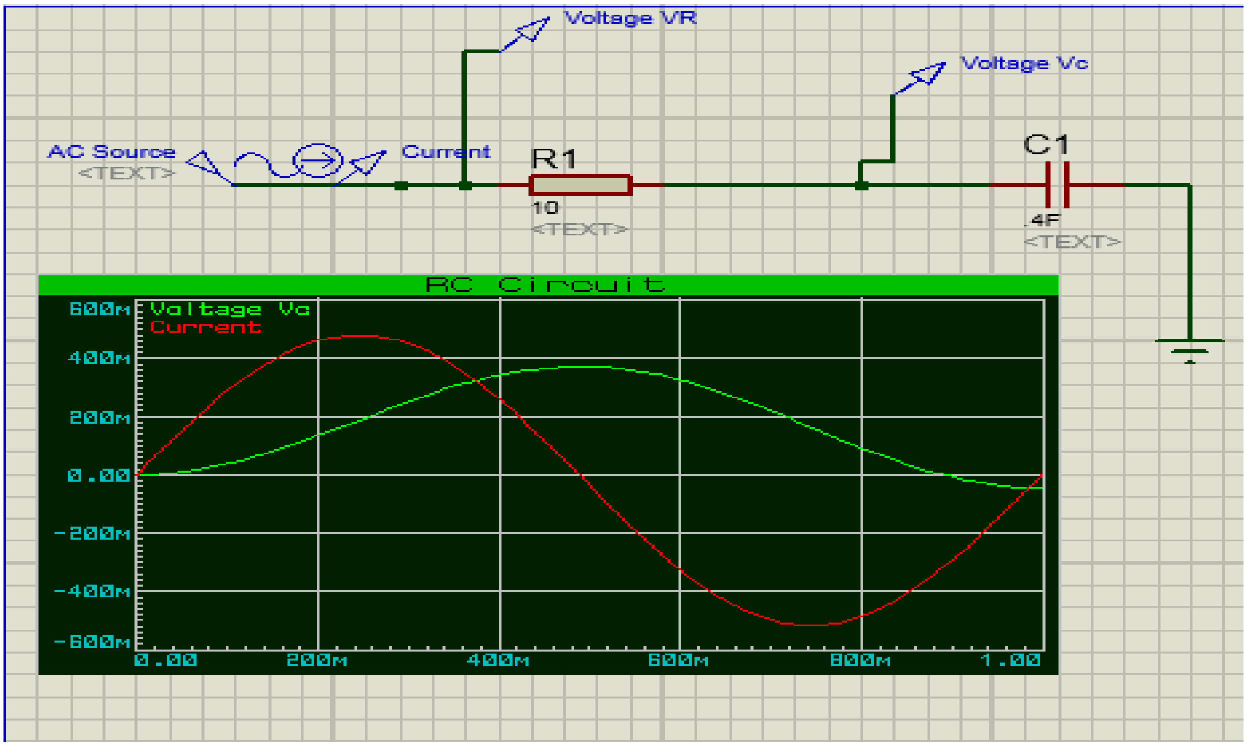


Fig. 13.19 (Pharos difference of RC Circuit)

Impedance of series RC circuit:

Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).

$$Z = \sqrt{R^2 + X_C^2}$$

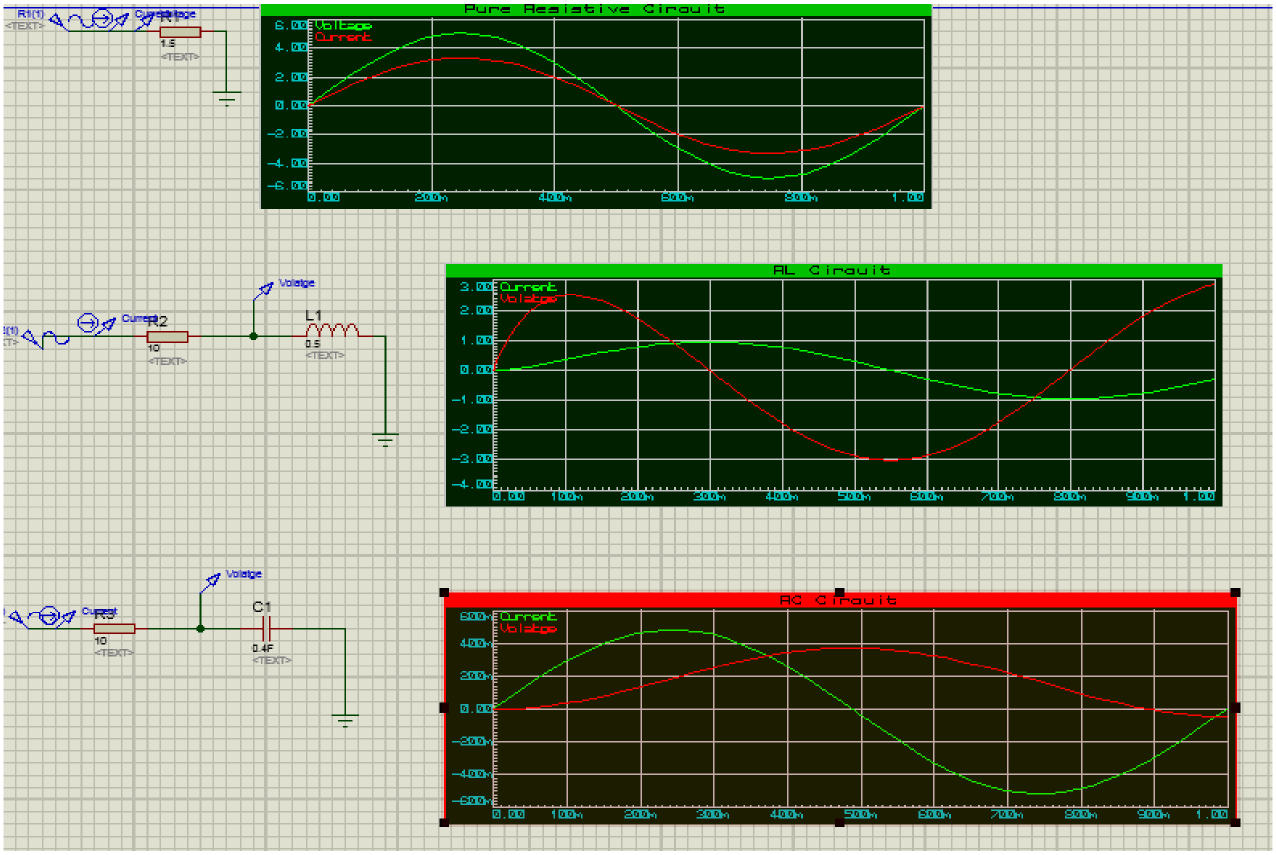
Phase angle:

From the pharos diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Over all Simulation examples



Series RLC Circuits

The combination of series resistor, inductor and capacitance is often used in filter circuits to adapt frequency response.

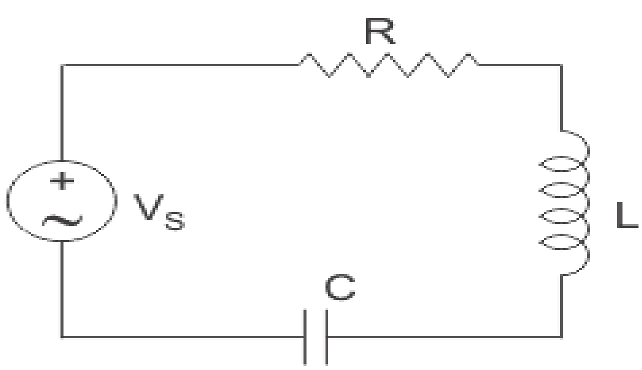


Fig. 13.19 (RLC Series Circuit)

The combined impedance is:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{Z}$$

$$X_L = 2\pi fL$$

$$X_C = \frac{1}{2\pi fC}$$

Current Lags if $X_L > X_C$

Current Leads if **$X_C > X_L$**

Phase angle θ given by:

$$\tan\theta = \frac{X_L - X_C}{R} \quad \text{or} \quad \frac{X_C - X_L}{R}$$

<u>Simulation of RLC</u>

Resistor = 100 Ohm , Inductor = 10mH and Capacitor = 1mF

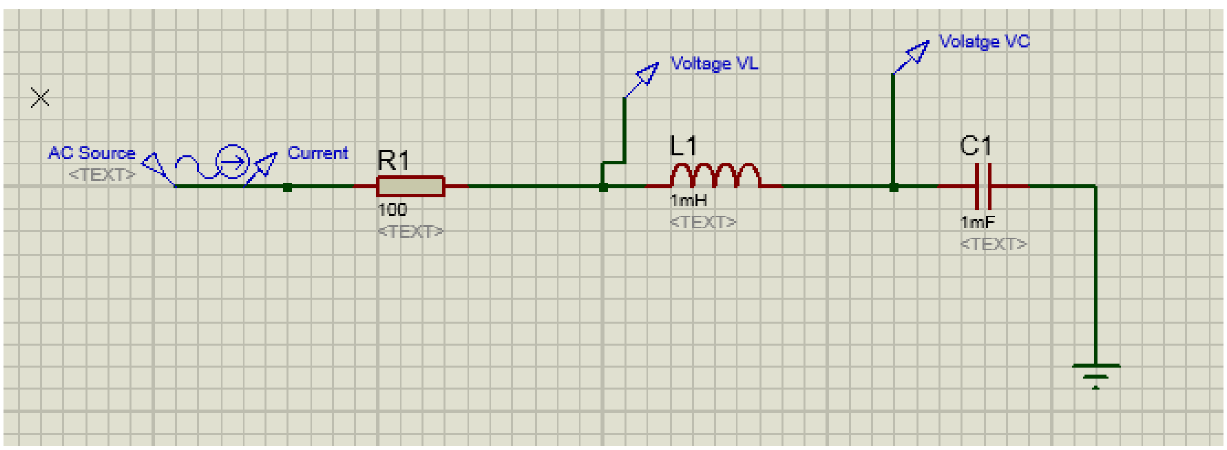
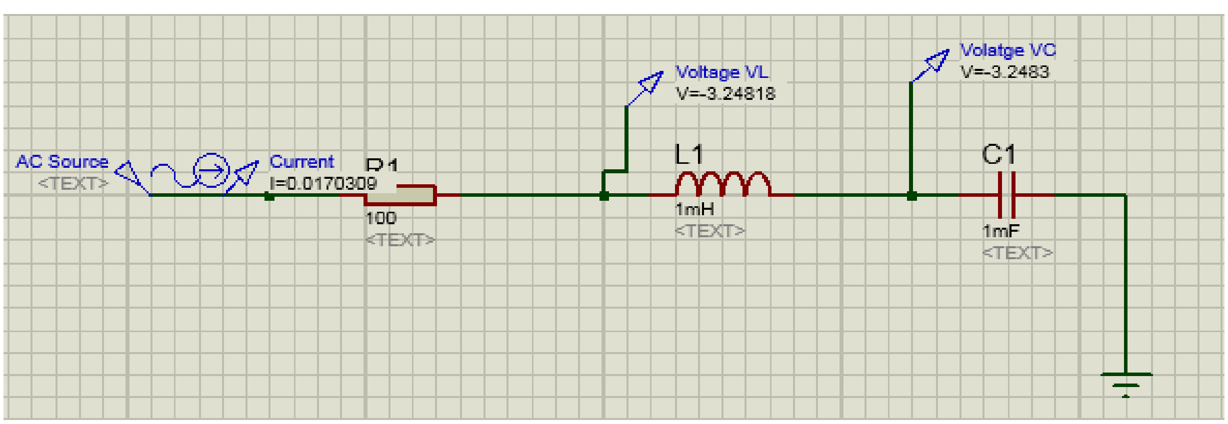


Figure 13.20 (Simulation of RLC Series -Circuits)



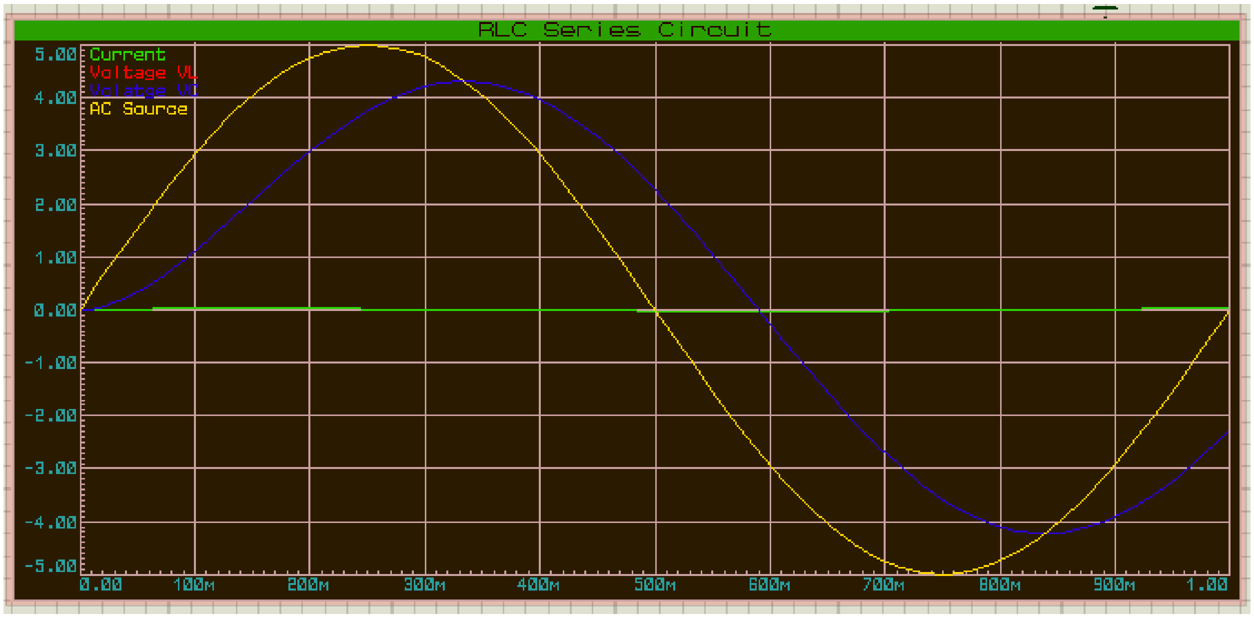
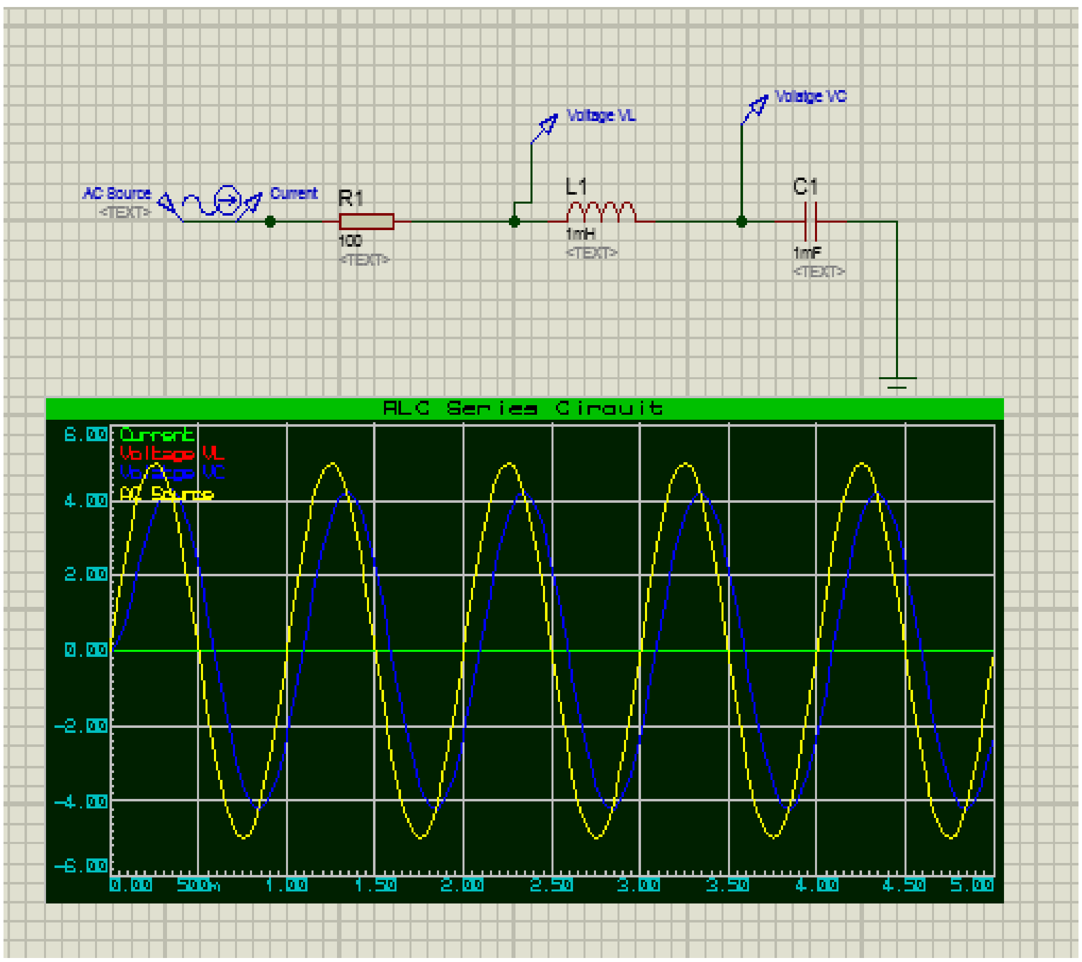


Figure 13.20 (Phase difference diagram of RLC)



Hint for Memorizing

1. ELI is ICE Man

2. CIVIL

Comments & Discussion