MAXIMIZATION OF UTILITY

It is the natural desire of the consumer to utilize his/ her limited income in such a way that, total utility of the purchased goods becomes maximum. Mathematics approach in this respect is

(i) To take first derivative of (U) total utility function.
(ii) To keep it equal to zero and find the quantity at which Tu is maximized.
(iii) To test it in 2nd derivative of the said function [it must be –tive (less than zero)]

Two conditions of maxima must be satisfied.

(i) $\frac{dTu}{dQ} = 0$ (ii) $\frac{d^2Tu}{dQ^2} < 0$

Example 1:

Utility function is given. Find quantity (Q) at which total utility(U) is maximum.

$$U = 16Q - 4Q^2$$

First derivative $\frac{du}{dQ} = 16 - 8Q$

Second derivative $\frac{d^2u}{dQ^2} = 8 < 0$

Keeping Ist derivative equal to zero

$$16 - 8Q = 0$$
$$8Q = 16$$
$$Q = 2$$

Thus, at Q = 2 total utility (U) will to maximum.

Putting Q = 2 in first derivative of the funciton

$$16 - 8Q = 0$$
$$16 - 8(2) = 0$$
$$16 - 16 = 0$$
$$0 = 0$$

At Q = 2 both the conditions of maxima are being satisfied.

Example 2:

Total utility function (U) is given. Find the quantity (Q) at which (U) is maximum.

$$U = 20Q - 20Q^2$$

First derivative $\frac{du}{dQ} = 20 - 40Q$

Second derivative $\frac{d^2u}{dQ^2} = -40 < 0$

Putting Ist derivative equal to zero

$$20 - 40Q = 0$$
$$-40Q = -20$$
$$40Q = 20$$
$$Q = \frac{20}{40} = 0.5$$

At $Q = 0.5$, $20 - 40Q = 0$

$$20 - 40(0.5) = 0$$
$$20 - 20 = 0$$
$$0 = 0$$

As both the conditions of maxima are satisfied thus, at Q = 0.5 total utility (U) will be maximum.

Example 3:

Utility function is given: Find quantity (Q) at which U is maximum. Point out total utility (U) also.

$$U = 8Q - 5Q^2$$

Ist derivative $\frac{du}{dQ} = \frac{d}{dQ}(8Q - 5Q^2) = 8 - 10Q$

Keeping first derivative equal to zero.

$$8 - 10Q = 0$$
$$-10Q = -8$$
$$10Q = 8$$
$$Q = 8,10 = 0.8$$

2nd derivative $\frac{d^2u}{dQ^2} = \frac{d}{dq}(8 - 10Q)$

$$= -10 < 0$$

Thus Q = 0.8 at which total utility (U) is maximum.

$$Q = 0.8$$

Total utility $U = 8(0.8) - 5(0.8)^2$

$$U = 6.4 - 3.2 = 3.2$$
$$U = 3.2$$

$U = 337.5$

EXERCISE

Find the quantity (Q) at which total utility (U) is maximum. Calcualte total utility (u) also;

1. $U = 50Q - 5Q^2$
2. $U = 45Q - 9Q^2$
3. $U = 90Q - 0.9Q^2$
4. $U = 36Q - 0.6Q^2$
5. $U = 48Q - 0.8Q^2$
6. $U = 100Q - 10Q^2$
7. $U = 70Q - 0.7Q^2$
8. $U = 66Q - 0.3Q^2$
9. $U = 33Q - 0.3Q^2$
10. $U = 75Q - 15Q^2$

MAXIMIZATION OF REVENUE

Firm's main aim is to produce such a quantity at which her total revenue is maximized. To achieve this aim first derivative of total revenue function (TR or R is kept equal to zero). In this respect two conditions of 'Maxima' must be satisfied. That is:

(i) $\frac{dy}{dx} = 0$ (First derivative of the function is equal to zero)

(ii) $\frac{d^2y}{dx^2} < 0$ (2nd derivative of the function must be –tive)

As total revenue function = TR or R and quantity is represented by Q.

Thus, (i) $\frac{dTR}{dQ} = 0$ and $\frac{d^2TR}{dQ^2} < 0$

Example 1:

TR function is given. Find the quantity (Q) at which TR is maximum.

$$TR = 120Q - 4Q^2$$

Ist derivative $\frac{dTR}{dQ} = 120 - 8Q$

Keeping it equal to zero

$$120 - 8Q = 0$$
$$-8Q = -120$$
$$8Q = 120$$
$$Q = \frac{120}{8}$$
$$Q = 15$$

At Q = 15. Ist and 2nd both conditions of maxima are satisfied.

1. First derivative of the function is equal to zero

$$120 - 8Q = 0$$
$$120 - 8(15) = 0$$
$$120 - 120 = 0$$
$$0 = 0$$

2. Second derivative of the function is less than zero (–tive)

$$\frac{d^2TR}{dQ^2} = -8 < 0 \quad (\text{–tive})$$

Example 2:

AR function is given. Find quantity (q) at which TR is maximum.

$$AR = 100 - 0.01q$$

TR function $TR = AR\,(q) = (100 - 0.01\,q)q$

$$= 100q - 0.01q^2$$

Ist derivative $\frac{dTR}{dq} = 100 - 0.02\,q$

2nd derivative $\frac{d^2TR}{dq^2} = -0.02 < 0$ (–tive)

Putting Ist derivative equal to zero.

$$100 - 0.02q = 0$$

$$0.02\,q = 100$$

$$q = \frac{100}{0.02} = \frac{10000}{2} = 5000$$

$$q = 5000$$

Thus, at q = 5000, TR is maxmum.

Example 3:

Average Revenue (AR) function is given. Find the quantity (Q) at which TR is maximum.

$$AR = 60 - 3Q$$

$$TR = AR\,(Q) = (60 - 3Q)Q$$

$$TR = 60Q - 3Q^2$$

Ist derivative $\frac{dTR}{dQ} = \frac{d}{dQ}(60Q - 3Q^2)$

$$= 60 - 6Q$$

Putting Ist derivative equal to zero.

$$60 - 6Q = 0$$

$$6Q = 60$$

$$Q = 10$$

2nd derivative $\frac{d^2TR}{dQ^2} = \frac{d}{dQ}(60 - 6Q)$

$$= -6 < 0 \quad \text{(negative)}$$

TR at Q = 10 $\quad TR = 60Q - 3Q^2$

$$= 60\,(10) - 3(10)^2$$

$$= 600 - 3(100)$$

$$= 600 - 300 = 300$$

$$TR = 300$$

Thus, TR = 300 is maximum at Q = 10.

Example 4:

At the given demand function

Find the quantity (q) at which TR is maximum.

$$q = 20 - 2p$$

Taking AR function

As, $\quad p = AR$

$$2p = 20 - q$$

Thus, $p = \frac{20-q}{2} = \frac{20}{2} - \frac{q}{2} = 10 - 0.5q$

$AR = 10 - 0.5q$

Converting AR function into TR function:

$R = AR(q) = (10 - 0.5q)q$

$= 10q - 0.5q^2$

$R = 10q - 0.5q^2$

Ist derivative $\frac{dR}{dq} = 10 - q$

2^{nd} derivative $\frac{d^2R}{dq^2} = -1 < 0$

Keeping Ist derivative equal to zero.

$10 - q = 0$

$q = 10$

Test/Proof: Thus, at $q = 10$, TR will be maximum

$10 - q = 0$

$10 - 10 = 0$

$0 = 0$

$\frac{d^2R}{dq^2} = -1 < 0$

Thus, at $q = 10$, TR is maximum

At which TR will be maximum.

EXERCISE

(a) Find maximum Total Revenue (TR) from following functions.

(i) $TR = 32Q - Q^2$

(ii) $AR = 60 - 3Q$

(iii) $AR = 15 - Q$

(iv) $Tr = 125Q - 2.5Q^2$

(v) $P = 142 - 2Q$

(b) Find maximum TR from the following demand functions.

(i) $Q = 16 - 2p$

(ii) $Q - 45 + 5p = 0$

(ii) $44 - 4p - Q = 0$

Thus, MC = AC

EXERCISE

1. Find minimum average cost.

(i) $AC = Q^2 - 4Q + 214$

(ii) $TC = 5Q - 2Q^2 + 2Q^3$

ANSWERS

(i) $Q = 2$ $AC = 210$

(ii) $Q = 0.5$ $AC = 4.5$

2. Find minimum marginal cost.

(i) $AC = Q^2 - 5Q + 14$

(ii) $AC = Q^2 - 8Q + 57 + \frac{2}{Q}$

(iii) $Tc = Q^3 - 12Q^2 + 60Q$

ANSWERS

(i) $Q = 1.67$ $MC = 5.67$

(ii) $Q = 2.67$ $MC = 35.66$

(iii) $Q = 4$ $MC = 12$

3. Find minimum total cost

(i) $TC = 31 + 24Q - 5.5Q^2 + \frac{1}{3}Q^3$

(ii) $TC = \frac{1}{3}Q^3 - 8.5Q^2 + 60Q + 27$

(iii) $TC = \frac{1}{3}Q^3 - 4.5Q^2 + 14Q + 22$

PROFIT MAXIMIZATION

A firm my produce that level of output which maximizes it's profit. Because this is the main aim of the firm. The process goes on as under:

Two conditions must be fulfilled in this respect.

1. The level of output at which the difference between Total Revenue (TR) and Total Cost (Tc) is maximum.
2. The level at which MR = MC Normally, R represents TR, which depends upon quantity produced (Q)

Thus, R = R (Q)

Similarly, C represents Tc which also depends upon Q. Hence, C = C (Q) while, the profit function (π) is:

$$\pi = \pi(Q) = R(Q) - C(Q) \text{ OR } \pi = R - C$$

It must be remembered that:

(a) Derivative of TR′ = MR

(b) Derivative of TC′ = MC

Thus, TR′ = TC′ or MR = MC and this the prime conditions of profit maximization

We know that profit is maximum where,

$$\frac{d\pi}{dQ} = 0 \text{ Ist derivative of profit function.}$$

OR

$$\frac{d\pi}{dQ} = \pi'(Q) = R'(Q) - C'(Q) = 0$$

And it may be possible when

$$R'(Q) = C'(Q)$$

$$MR = MC$$

The second condition is that:

$$\frac{d^2\pi}{dQ^2} = \pi''(Q) = R''Q) - C''(Q) < 0 \text{ } 2^{nd} \text{ derivative.}$$

OR $R''(Q) < C''(Q)$

Hence at $Q = 4$, $P = 5$ and $MC = MR = 2$

EXAMPLE – 5. The AC and P of a firm are given. Find quantity and price where profits are maximum. Also prove at equilibrium (1) MC = MR, (2) Slope of MC > Slope of MR. [(UOP:2013)(UOS:2010)(UAJK:2014,2015)(GCUF:2016)]

$$AC = \frac{30}{Q} + 9 + 0.3\,Q\,,\; P = 30 - 0.75Q.$$

Now $R = P \times Q = (30 - 0.75\,Q)Q = 30Q - 0.75Q^2$

$C = (AC)\,Q = \left(\frac{30}{Q} + 9 + 0.3\,Q\right)Q = 30 + 9Q + 0.3Q^2$

$$\pi = R - C = (30Q - 0.75Q^2) - \left(30 + 9Q + 0.3Q^2\right)$$

$$= 30Q - 0.75Q^2 - 30 - 9Q - 0.3Q^2 = -1.05Q^2 + 21Q - 30$$

$$\frac{d\pi}{dQ} = -2.1Q + 21 = 0 \Rightarrow Q = \frac{21}{2.1} = 10$$

Again $\frac{d\pi}{dQ} = -2.1Q + 21 \Rightarrow \frac{d^2\pi}{dQ^2} = -2.1 < 0$

At $Q = 10$, $\frac{d^2\pi}{dQ^2} = -2.1 < 0$

Thus at $Q = 10$, $\frac{d^2\pi}{dQ^2} < 0$.

Hence profits are maximum when $Q = 10$.

To find amount of profits, we put Q = 10 in profit function, then

$$\pi = -1.05Q^2 + 21Q^2 - 30 = -1.05(10)^2 + 21(10)^2 - 30 = 75$$

At $Q = 10$, $P = 30 - 0.75Q = 30 - 0.75(10) = 30 - 7.5 = 22.5$

Again, $C = 30 + 9Q + 0.3Q^2 \Rightarrow MC = \frac{dC}{dQ} = 9 + 0.6Q$

At $Q = 10$, $MC = 9 + 0.6(10) = 9 + 6 = 15$

Now $R = 30Q - 0.75Q^2 \Rightarrow MR = \frac{dR}{dQ} = 30 + 1.5Q$

Art, $Q = 4$, $MR = 30 + 1.5(10) = 30 - 15 = 15$

Hence at $Q = 4, P = 3$ and $MC = MR = 15$.

$MC = 9 + 0.6Q \Rightarrow$ Slope of $MC = \frac{d(MC)}{dQ} = 0.6$

$MR = 30 - 1.5Q \Rightarrow$ Slope of $MR = \frac{d(MR)}{dQ} = -1.5$

Showing that $\frac{d(MR)}{dQ} < \frac{d(MC)}{dQ}$ as $-1.5 < 0.6$.

EXAMPLE – 7. With revenue and cost equation given below, find Q and P where π are maximized. Also check that in such situation

(1) $MC = MR$ (2) Slope of MC > Slope of MR :

$$R = 1000Q - 2Q^2, \; C = Q^3 - 59Q^2 + 1315Q + 2000.$$

$$\pi = R - C = (1000Q - 2Q^2) - (Q^3 - 59Q^2 + 1315Q + 2000)$$

$$= 1000Q - 2Q^2 - Q^3 + 59Q^2 - 1315Q - 2000 = -Q^3 + 57Q^2 - 315Q - 2000$$

$$\frac{d\pi}{dQ} = -3Q^2 + 114Q - 315 = 0 \Rightarrow \quad -3Q^2 + 114Q - 315 = 0$$

$$Q = \frac{-114 \pm \sqrt{12996 - 3780}}{-6} = \frac{-114 \pm \sqrt{9216}}{-6} = \frac{-114 \pm 96}{-6}$$

$$Q = \frac{-18}{-6} = 3, \; \frac{-210}{-6} = 35$$

Again $\frac{d\pi}{dQ} = -3Q^2 + 114Q - 315 \quad \Rightarrow \quad \frac{d^2\pi}{dQ^2} = -6Q + 114$

At $Q = 35$, $\quad \frac{d^2\pi}{dQ^2} = -6Q + 114 = -6(35) + 114 = -96 < 0$

At $Q = 3$, $\quad \frac{d^2\pi}{dQ^2} = -6Q + 114 = -6(3) + 114 = 96 > 0$

Thus at $Q = 35$, $\frac{d^2\pi}{dQ^2} < 0$. Hence profits are maximum when $Q = 35$.

The price level is found : $\quad P = \frac{R}{Q} = \frac{1000Q - 2Q^2}{Q} = 1000 - 2Q$

At $Q = 3$, $P = 1000 - 2(35) = 1000 - 70 = 930$

The MR is found thus: Now $R = 1000Q - 2Q^2 \Rightarrow MR = \frac{dR}{dQ} = 1000 - 4Q$

At, $Q = 35$, $\quad MR = 1000 - 4Q = 1000 - 140 = 860$

The MC is found thus:

Again, $C = Q^3 - 59Q^2 + 1315Q + 2000 \quad \Rightarrow \quad MC = \frac{dC}{dQ} = 3Q^2 - 118Q + 1315$

At $Q = 35$, $\quad MC = 3(35)^2 - 118(35) + 1315 = 3675 - 4130 + 1315 = 860$

Slope of MC and MR are found thus:

$$MC = 3Q^2 - 118Q + 1315 \quad \Rightarrow \quad \text{Slope of } MC = \frac{d(MC)}{dQ} = 6Q - 118$$

At $Q = 35$, $\dfrac{d(MC)}{dQ} = 6Q - 118 = 6(35) - 118 = 210 - 118 = 92$

$$MR = 1000 - 4Q \quad \Rightarrow \quad \text{Slope of } MR = \frac{d(MR)}{dQ} = -4$$

Showing that $\dfrac{d(MR)}{dQ} < \dfrac{d(MC)}{dQ}$ as $-4 < 92$

Thus the equilibrium quantity of the firm is 35 ($Q = 35$) and the equilibrium price is 930 ($P = 930$).. Putting Q = 35 in π function we can find the amount of profits.

At $Q = 200, \pi = R - C - T$ ...

EXAMPLE – 11. The demand and cost of the firm are given, find (1) equilibrium quantity and price by first order and second order conditions (2) equilibrium price and quantity if Govt, imposes Rs.8/- as tax per unit.

(QAU:2016)

This is the example 10 without tax.

$$P = 29 - 3Q,\ TC = Q^2 + 5Q.$$

Solution. $TR = PQ = (29 - 3Q)Q = 29Q - 3Q^2$

$$\pi = TR - TC = (29Q - 3Q^2) - (Q^2 + 5Q)$$

$$= 29Q - 3Q^2 - Q^2 - 5Q = -4Q^2 + 24Q$$

$$\frac{d\pi}{dQ} = -8Q + 24 = 0 \Rightarrow -8Q = -24 \Rightarrow Q = \frac{24}{8} = 3,\ \frac{d^2Q}{dQ^2} = -8 < 0$$

Thus at $Q = 3$, the firm is in equilibrium. $TC = Q^2 + 5Q \Rightarrow MC = \frac{d(TC)}{dQ} = 2Q + 5$

At $Q = 3$, $MC = 2Q = 5 = 2(3) + 5 = 6 + 5 = 11$

$$TR = 29Q - 3Q^2 \quad \Rightarrow \quad MR = \frac{d(TR)}{dQ} = 29 - 6Q$$

At $Q = 3$, $MR = 29 - 6Q = 29 - 6(3) = 29 - 18 = 11$

$$MC = 2Q + 5 \Rightarrow \text{Slope of } MC = \frac{d(MC)}{dQ} = 2$$

$$MR = 29 - 6Q \Rightarrow \text{Slope of } MR = \frac{d(MR)}{dQ} = -6$$

Thus at $Q = 3$, $MC = MR$ and Slope of $MC >$ Slope of MR, as $2 > -6$

Putting $Q = 3$ in P and π, we get $P = 29 - 3Q = 29 - 3(3) = 29 - 9 = 20$

$$\pi = -4Q^2 + 24Q = -4(3)^2 + 24(3) = -36 + 72 = 36.$$

Imposition of taxes. $T = 8Q$

$$\pi = TR - TC - T = (29Q - 3Q^2) - (Q^2 + 5Q) - 8Q$$

$$= 29Q - 3Q^2 - Q^2 - 5Q - 8Q = -4Q^2 + 16Q$$

$$\frac{d\pi}{dQ} = -8Q + 16 = 0 \Rightarrow \quad -8Q = -16 \Rightarrow Q = \frac{16}{8} = 2, \quad \frac{d^2\pi}{dQ^2} = -8 < 0.$$

Thus at $Q = 2$, profits are maximized when Rs.8/- per unit tax is imposed.

EXAMPLE – 12. If $P = 150 - 0.5\,Q$, $C = 100+3Q+7Q^2$, $S = 3Q$ where S = subsidies, find P and Q where profits are maximum. What will happen if subsidies (S) are not given to the firm?

Solution. Profit maximization with subsidies :

$$R = PQ = (150 - 0.5Q)Q = 150Q - 0.5Q^2$$

$$\pi = R - C + S = (150Q - 0.5Q^2) - (100 + 3Q + 7Q^2) + 3Q$$

$$= 150Q - 0.5Q^2 - 100 - 3Q - 7Q^2 + 3Q \quad = -7.5Q^2 + 150Q - 100$$

$$\frac{d\pi}{dQ} = -15Q + 150 = 0 \Rightarrow -15Q = -150 \Rightarrow Q = 10, \quad \frac{d^2\pi}{dQ^2} = -15 < 0$$

Thus at Q = 10, profits are maximized in the presence of subsidies.

At Q = 10, $P = 150 - 0.5Q = 150(- 0.5(10) = 150 - 5 = 145$

$$R = 150Q - 0.5Q^2 = 150(10) + 0.5(10)^2 = 1500 - 50 = 1450$$

$$C = 100 + 3Q + 7Q^2 = 100 + 3(10) + 7(10)^2 = 100 + 30 + 700 = 830$$

$$\pi = R - C + S = 1450 - 830 + 30 = 650, \text{ as } S = 3Q = 3(10) = 30.$$

Profit maximization without subsidies. (IUBWR:2016)

$$R = PQ = (150 - 0.5Q)Q = 150Q - 0.05Q^2$$

$$\pi = R - C = (150Q - 0.05Q^2) - (100 + 3Q + 7Q^2)$$

$$= 150Q - 0.5Q^2 - 100 - 3Q - 7Q^2 = -7.5Q^2 + 147Q - 100$$

$$\frac{d\pi}{dQ} = -15Q + 147 = 0 \Rightarrow 15Q = 147 = 9.8 \Rightarrow Q = 9.8, \quad \frac{d^2\pi}{dQ^2} = -15 < 0$$

Thus at Q = 9.8, profits are maximized without subsidies.

At Q = 9.8. $P = 150 - 0.5Q = 150 - 0.5(9.8) = 145.10$

$$\pi = -7.5Q^2 + 147Q - 100 = -7.5(9.8)^2 + 147(9.8) - 100 = 620.30$$

Hence in this case $\pi = 620.30$, $P = 145.10$ and $Q = 9.8$.

EXAMPLE – 13. If $P = 100 - 0.25\,Q$, $C = 10 - 70Q + 19.75Q^2$, (1) we find P and Q where profits are maximum. (2) If govt. imposesRs.2/- as excise duty, what would be the new level of quantity and price ? (3) If govt. imposes Rs.2/- as subsidy what will be the new level of quantity and price ?

Solution. $R = PQ = (100 - 0.25Q)Q = 100Q - 0.25Q^2$

$$\pi = R - C = (100Q - 0.25Q^2) - (10 - 70Q + 19.75Q^2)$$

$$= 100Q - 0.25Q^2 - 10 + 70Q - 19.75Q^2 \quad = -20Q^2 + 170Q - 10$$

$$\frac{d\pi}{dQ} = -40Q + 170 = 0 \Rightarrow -40Q = -170 \Rightarrow Q = 4.25, \quad \frac{d^2\pi}{dQ^2} = -40 < 0$$

Thus at Q = 4.25, profits are maximized.

At Q = 4.25, $P = 100 - 0.25Q = 100 - 0.25(4.25) = 100 - 1.0625 = 98.9375$

$C = 19.75Q^2 - 70Q + 10 = 19.75(4.25)^2 - 70(4.25) + 10$

$= 356.734375 - 297.5 + 10 = 69.234375$

$= 356.734375 - 297.5 + 10 = 69.234375 \quad = 425 - 4.515625 = 420.484375$

$R = 100Q - 0.25Q^2 = 100(4.25) - 0.25(4.25)^2 = 425 - 4.515625 = 420.484375$

$\pi = R - C = 420.84375 - 69.234375 = 351.25$

Imposition of Excise Duty of Rs. 2/- per unit:.

$\pi = R - C - T = (100Q - 0.25Q^2) - (10 - 70Q + 19.75Q^2) - (2Q)$

$= 100Q - 0.25Q^2 - 10 + 70Q - 19.75Q^2 - 2Q = -20Q^2 + 168Q - 10$

$\frac{d\pi}{dQ} = -40Q + 168 = 0 \Rightarrow -40Q = -168 \Rightarrow Q = 4.20, \quad \frac{d^2\pi}{dQ^2} = -40 < 0$

Thus at Q = 4.20, profits are maximized in the presence of taxes.

At Q = 4.20, $\pi = -20Q^2 + 168Q - 10 = -20(4.20)^2 + 168(4.20) - 10$

$= -352.8 + 705.6 - 10 = 342.80$

$P = 100 - 0.25Q = 100 - 0.25(4.20) = 98.95$

Granting of subsidies of Rs. 2/- per unit:.

$\pi = R - C + S = (100Q - 0.25Q^2) - (10 - 70Q + 19.75Q^2) + (2Q)$

$= 100Q - 0.25Q^2 - 10 + 70Q - 19.75Q^2 + 2Q \quad = -20Q^2 + 172Q - 10$

$\frac{d\pi}{dQ} = -40Q + 172 = 0 \Rightarrow -40Q = -172 \Rightarrow Q = 4.30, \quad \frac{d^2\pi}{dQ^2} = -40 < 0$

Thus at Q = 4.30, profits are maximized in the presence of subsidies.

At Q = 4.30, $P = 100 - 0.25Q = 100 - 0.25(4.30) = 98.92$

$\pi = -20Q^2 - 172Q - 10 = -20(4.30)^2 + 168(4.30) - 10 = 359.80$

The above discussion shows that where firm's profits are maximum, MC = MR.

Example-15: A monopolist faces following demand function and total cost function: $P = 18 - 2q$, $C = 2q^2 + 6q$, (a) Find his profit maximizing output (q) and price (p), (b) If govt. imposes a tax of Rs. 6 per unit of output sold, what will happen to price of product, quantity sold and total profit. **(GCUF:2012)**

$$P = 18 - 2q \Rightarrow R = pq = (18 - 2q)\,q = 18q - 2q^2, C = 2q^2 + 6q$$

$$\pi = R - C = (18q - 2q^2) - (2q^2+6q) = 18q - 2q^2 - 2q^2 - 6q \Rightarrow \pi = -4q^2+12q$$

$$\pi = -4q^2 + 12q, \frac{d\pi}{dq} = -8q + 12 = 0 \Rightarrow -8q = -12 \Rightarrow q = 1.5$$

$$\pi = -4q^2 + 12, \frac{d^2q}{dq^2} = -8 < 0, p = 18 - 2q = 18 - 2(1.5) = 15$$

$$\pi = -4q^2 + 12q = -4(1.5)^2 + 12(1.5) = -9 + 18 = 9$$

Tax = T(q) = 6q. Thus $\pi = R - C - T$

$$\pi = (18q - 2q^2) - (2q^2 + 6q) - (6q) = 18q - 2q^2 - 2q^2 - 6q - 6q$$

$$\pi = -4q^2 + 6q, \frac{d\pi}{dq} = -8q + 6 = 0 \Rightarrow -8q = -6 \Rightarrow q = \frac{6}{8} = \frac{3}{4} = 0.75$$

$$\frac{d\pi}{dq} = -8q + 6, \frac{d^2\pi}{dq^2} = -8 < 0, P\ 18 - 2q = 18 - 2(0.75) = 16.5$$

$$\pi = -4q^2 + 6q - 4(0.75)^2 + 6(0.75) = -2.25 + 4 = 2.25$$

Example-16: Given the Demand and Total Cost Functions of a Firm:

EXERCISE – 18

Find equilibrium quantity and price level where profits are maximum. Moreover, also find amount of the profits :

No.	Equations	Ans. P	Q	π
1.	$P = 30 - 0.75Q$ $AC = \frac{30}{Q} - 9 + 0.3Q$	10	22.5	75
2.	$22 - 0.5Q - P = 0$ $T = 20Q$ $AC = \frac{1}{3}Q^2 - 8.5Q + 50 + \frac{90}{Q}$ (UOS : 2016) (UAJK : 2016)	14	15	171.33
3.	$TR = 45Q - 0.5Q^2$ $TC = Q^3 - 39.5Q^2 + 120Q + 125$	25	32.5	6750
4.	$C = 50 + 40Q$ $P = 100 - 2Q$	15	70	400
5.	$P = 200 - 0.01Q$ $C = 10Q + 20000$	9500	105	882500
6.	$P = 50 - 2Q$ $C = 75 + 14Q$ $T = 10Q$	6.5	37	9.5
7.	$TC = Q^3 - 9Q^2 + 30Q + 10$ $TR = 70Q - 8Q^2$	4	38	102
8.	$TC = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$ $Q = 100 - P$ (UAJK : 2015 / S(QAU : 2014)	11	89	111.30