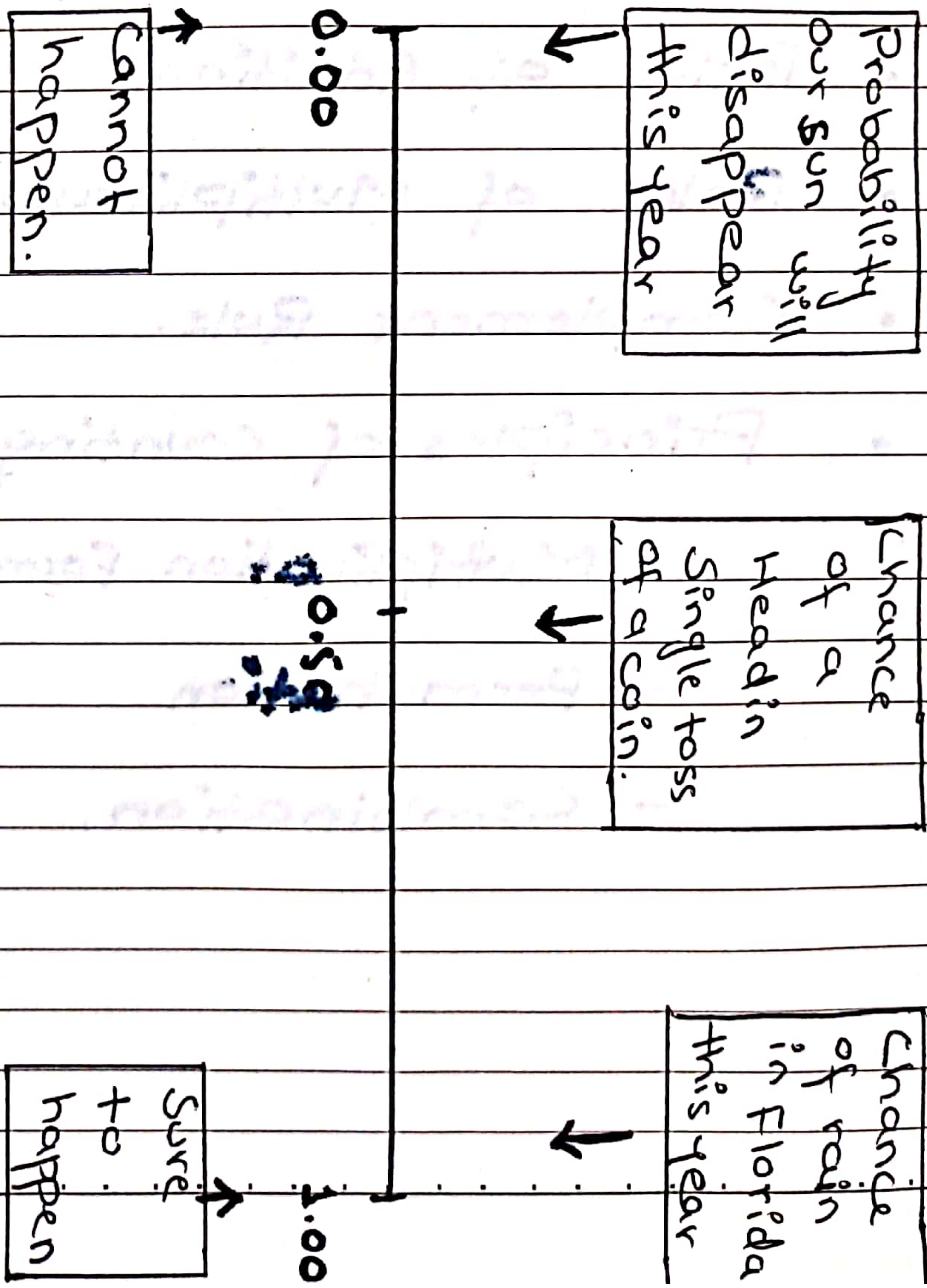
PROBABILITY

- Empirical Probability.
- Classical Probability.
- Rules of Addition.
- Rules of Multiplication.
- Complement Rule.
- Principles of Counting.
  - Multiplication Formula.
  - Permutation
  - Combination.

→ Probability:

Probability is a value b/w '0' & '1' inclusive that represents the possibility/chance a particular event will happen.



Three key words are used in the study of probability.

- Experiment.
- Outcome.
- Event.

Experiment:

A process that generates possible observations.

Outcome:

A particular result of an experiment.

Event:

A collection of one or more outcomes of an experiment.

Experiment	Roll a die
All Possible Outcomes	• Observe a 1 • " " 2 • " " 3 • " " 4 • " " 5 • " " 6
Some Possible events.	• Observe an even No. • " a No. greater than 4 • Observe a No. 3 or less

Approaches to Assigning Probabilities:

⟹ Classical Prob.:

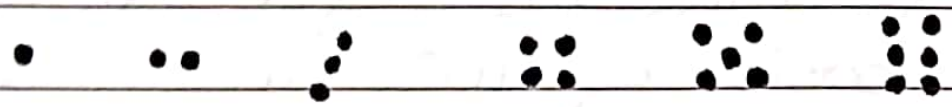
Classical probability is based on the assumption that the outcomes of an experiment are equally likely.

$$\text{Probability of an event} = \frac{\text{No. of favorable outcomes}}{\text{Total No. of possible outcomes.}}$$

⟹ Example:

Consider an experiment of rolling a six-sided die. What is the probability of the event "an even No. of spots appear face up"?

The possible outcomes are:



$$\text{Probability of an even No.} = \frac{3}{6}$$

$$= \boxed{0.5}$$

→ EXAMPLE :

A fair coin is tossed. Find the probability

1 → Head appears
2 → tail "
3 → No head appears

All possible outcomes = H , T

A = Head appears = H
B = Tail " = T
C = No head appears = T

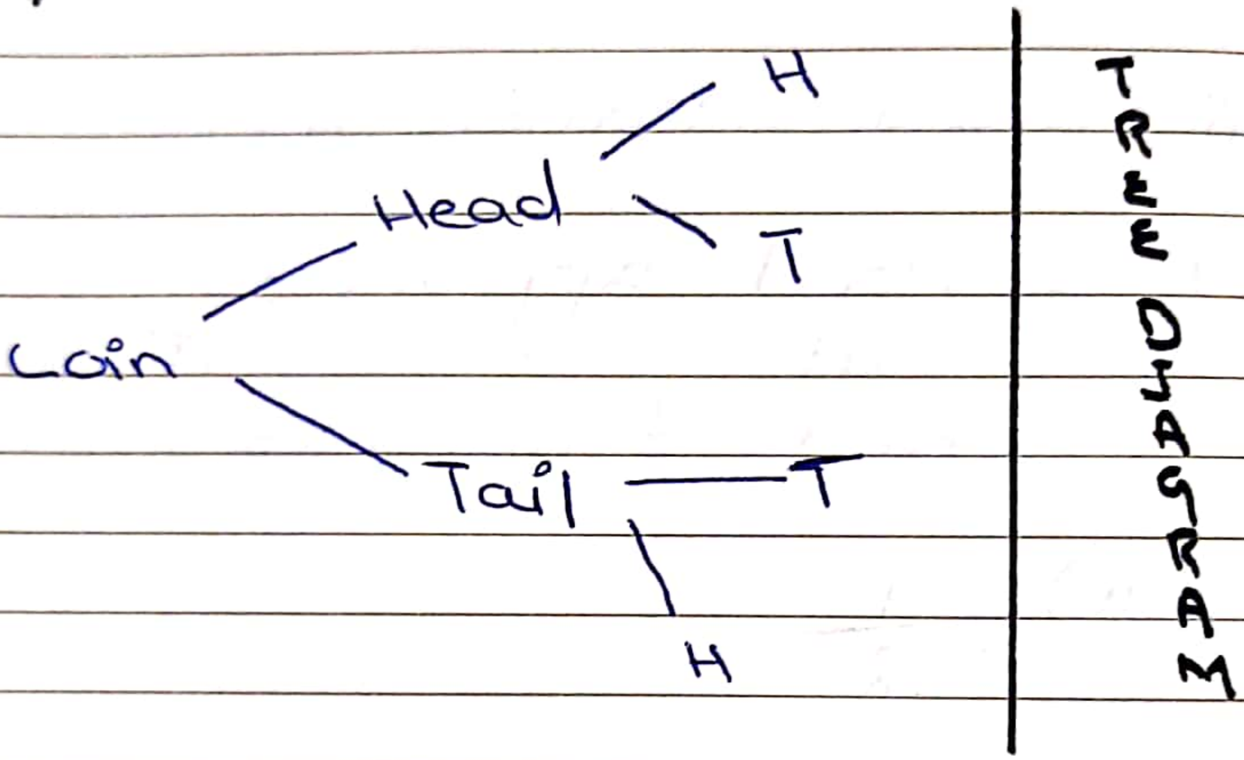
$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2}$

$P(C) = \frac{1}{2}$

→ Two coins are tossed. Find the prob.

1 → No head appears.
2 → 1 " "
3 → 2 " "
4 → 1 tail & 1 head appear.
5 → 1 or more than 1 head ".



All possible outcomes = {HH, HT, TH, TT}

A = No Head appears = TT

B = 1 head appears = HT, TH

C = 2 head appears = HH

D = 1 tail & 1 head appear.

= TH, HT

E = 1 or more than 1 head appear.

= HT, TH, HH

$\boxed{P(A) = 1/4}$

$P(B) = \frac{2}{4} = 1/2$

$\boxed{P(C) = 1/4}$

$P(D) = \frac{2}{4} = 1/2$

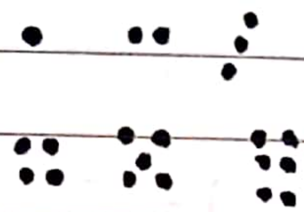
$\boxed{P(E) = \frac{3}{4}}$

⟹ EXAMPLE:

If a die is rolled. What is the probability.

1. Greater than 4 appear.
2. Multiple of 3 " .
3. More than 7 " .

all possible outcomes =



A = Greater than 4 appear = 5, 6

B = Multiple of 3 appear = 3, 6

C = More than 7 appear = 0

$$P(A) = \frac{2}{6} = 1/3$$

$$P(B) = \frac{2}{6} = 1/3$$

$$\boxed{P(C) = 0}$$

⟹ Empirical Probability:

The probability of an event happening is the fraction of the time similar events happened in the past.

$$\text{Empirical probability} = \frac{\text{No. of times the event occurs.}}{\text{Total No. of observations.}}$$

⟹ Example:

On Feburary 1, 2003, the Space Shuttle Columbia exploded. This was the second disaster in 113 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

$$\text{Probability of a Successful flight} = \frac{\text{No. of Successful flights}}{\text{Total No. of flights.}}$$

$$P(A) = \frac{111}{113}$$

$$\boxed{P(A) = 0.98}$$

Based on past experience, the probability is 0.98 that a future space shuttle mission will be safely completed.

⟹ MUTUALLY EXCUSIVE EVENTS:

Two or more events are said to be mutually exclusive if they can not occure together. i.e., they have no common point.
e.g.,

'A' & 'B' are mutually exclusive.
A die is rolled & 'A' = Even No.
B = Odd No.

→ COLLECTIVELY EXHAUSTIVE:

Two or more events are said

to be Exhaustive events, if they are mutually exclusive & their union is a complete sample set of outcomes. e.g.,

$A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$.

⟹ SOME RULES FOR COMPUTING PROBABILITIES :

- Rules of Addition.
  - Special Rule of addition.
  - General Rule of addition.
- Rules of Multiplication.
  - Special Rule of Multiplication.
  - General Rule of Multiplication.

⇒ SPECIAL RULE OF ADDITION:

To apply the special rule of addition, the events must be mutually exclusive.

$$P(A \text{ or } B) = P(A) + P(B).$$

For three mutually exclusive events designated A, B, & C, the rule is written:

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

⇒ COMPLEMENT RULE:

$$P(A) + P(\bar{A}) = 1$$

$$P(A) = 1 - P(\bar{A})$$

$$P(\bar{A}) = 1 - P(A)$$

It is used to determine the prob. of an event occuring by subtracting the prob. of the event not occuring from 1.

⟹ GENERAL RULE OF ADDITION:

The outcomes of an experiment may not be mutually exclusive. The rule for two Non-Mutually exclusive events designated A & B is written:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B).$$

⟹ EXAMPLE :

A machine fills plastic bags with a mixture of beans, broccoli and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. A check of 4000 packages filled in the past month revealed:

Weight	Event	No. of Packages	Prob. of Occurance
Underweight	A	100	100/4000 = 0.025
Satisfactory	B	3600	0.900
Overweight.	C	300	0.075
		4000	1.00

1) What is the prob. that a parti-cular package will be either underweight or overweight?

$P(A \text{ or } C) = P(A) + P(C)$

$= 0.025 + 0.075$

$= \boxed{0.10}$

2) Use the complement rule to show the prob. of a satis-factory bag is 0.900.

$P(B) = 1 - \{P(A) + P(C)\}$

$$P(B) = 1 - \{0.025 + 0.075\}$$

$$\boxed{P(B) = 0.900}$$

EXAMPLE:

A marble is drawn at random from a box containing 10 red, 30 White, 20 blue and 15 orange marbles. Find the Prob. that it is:

1. Orange or red.
2. Not red or orange.
3. Not blue.
4. Red, White or blue.

All possible outcomes = 10 R + 30 W + 20 B + 15 Orange

= 75

A = orange = 15

B = Red = 10

C = Blue = 20

D = White = 30

1. p(orange or red) = p(A or B)

$= P(A) + P(B)$

$= \frac{15}{75} + \frac{10}{75}$

$= \frac{25}{75} = \boxed{1/3}$

2. p(Not red or orange) = 1 − p(red or orange)

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

3. $P(\text{Not blue}) = 1 - P(\text{Blue})$

$= 1 - P(C)$

$= 1 - \frac{20}{75}$

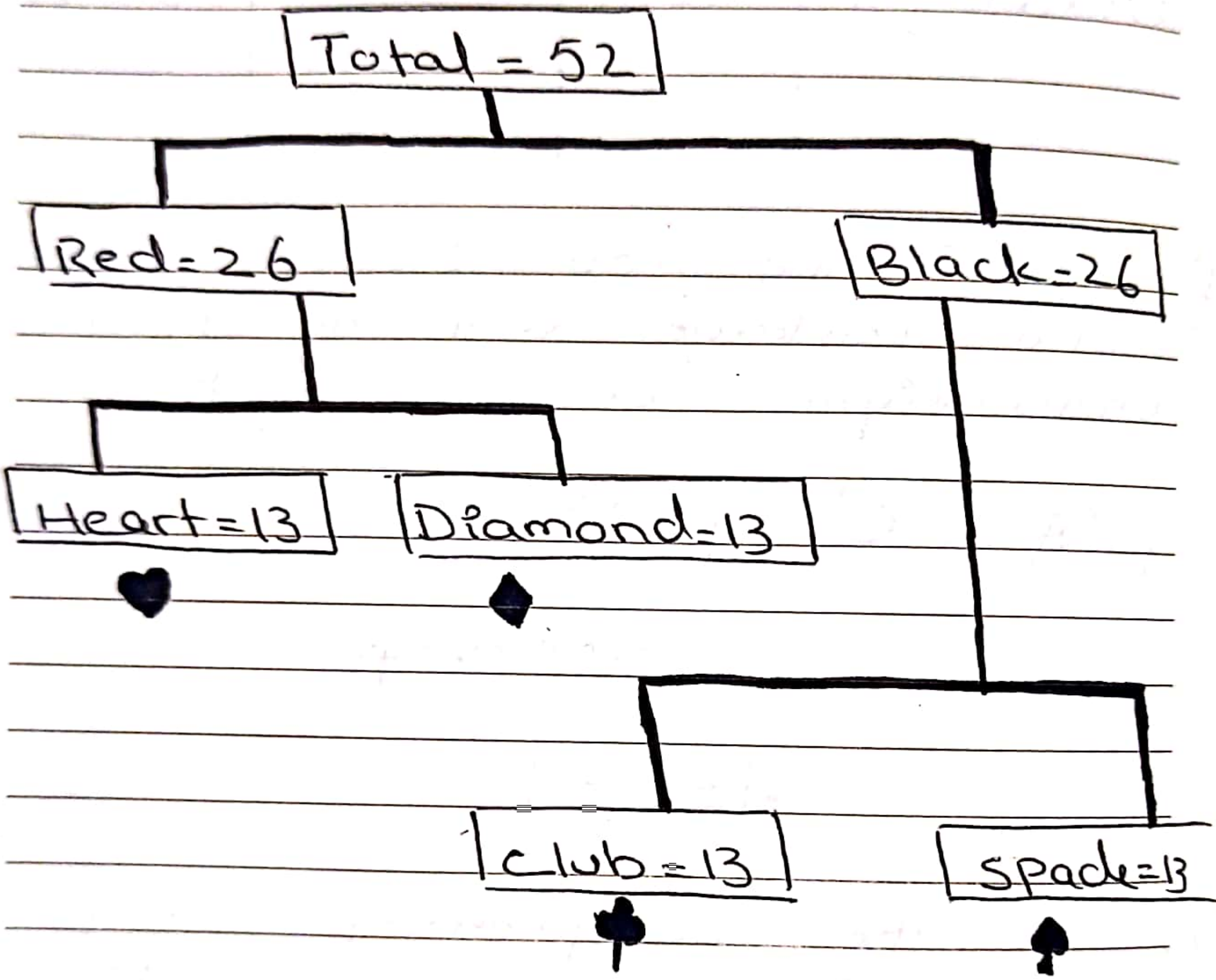
4. $P(\text{red}, \text{white}, \text{Blue}) = P(B) + P(D) + P(C).$

$$= \frac{10}{75} + \frac{30}{75} + \frac{20}{75}$$

$$= \frac{60}{75}$$

$$= \boxed{\frac{4}{5}}$$

DIVISION OF CARDS:



→ EXAMPLE:

What is the prob. that a card chosen at random from a standard deck of cards will be either a king or a heart?

A = king
B = Heart

$P(A) = 4/52$

$P(B) = 13/52$

$P(A \cap B) = 1/52$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$\boxed{P(A \text{ or } B) = \frac{16}{52}}$$

⟹ SPECIAL RULE OF MULTIPLICATION:

⟹ Independent Events:

Two or more events are said to be independent if they happen together & do not effect each other. The selection of events will be With Replacement.

⟹ Dependent Events:

Two or more events are said to be dependent if they happen one after another & the prob. of 2nd event is effected due to first. The selection of events will be Without Replacement.

The special rule of multiplication requires that two events A & B are indep. It is written symbolically as:

$P(A \text{ and } B) = P(A).P(B)$

For three independent events, A, B, and C, the special rule of multiplication used to determine the probability that all three events will occur is:

$P(A \text{ and } B \text{ and } C) = P(A).P(B).P(C).$

⟹ CONDITIONAL PROB.:

It is the probability that an event will happen given that another event has already happened.

⟹ GENERAL RULE OF MULTIPLICATION:

If two events A & B are dependent then;

$$P(A \text{ and } B) = P(A) \cdot P(B/A).$$

$\Rightarrow$ **NOTE:**

- $P(A \text{ or } B) = P(A \cup B)$.
- $P(A \text{ and } B) = P(A \cap B)$. $\rightarrow$ Joint prob.
- $P(A \cap B) = P(A)P(B/A)$.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

- Equally likely outcomes are those outcomes who have same chance of occurance.
- Joint prob. is the prob. /possibility that two or more events will happen at the same time.

⟹ EXAMPLE:

Two cards are drawn from a well-shuffled pack of 52 cards. Find the probability that they are both aces if the first card drawn is

1 → Replaced.

2 → Not Replaced.

1 → $p(A \text{ and } B) = p(A) \cdot p(B)$

$$p(A \cap B) = \frac{4}{52} \cdot \frac{4}{52}$$

$$= \boxed{\frac{1}{169}}$$

2 → $p(A \cap B) = p(A) \cdot p(B|A)$.

$$= \frac{4}{52} \cdot \frac{3}{51}$$

$$= 1/221$$

⟹ EXAMPLE:

For two indep. events A & B, $P(A) = 0.25$ & $P(B) = 0.40$

Find.

- $P(A \cap B)$

$$\text{Prob}(A \text{ and } B) = P(A) \cdot P(B)$$

$$= (0.25)(0.40)$$

$$= \boxed{0.10}$$

→ EXAMPLE :

A and B can solve 60% & 80% of the problems in a book respectively. What is the prob. that either A or B can solve a problem chosen at random?

$$P(A) = \frac{60}{100}$$

$$P(B) = \frac{80}{100}$$

$$P(A \text{ and } B) = P(A \cap B)$$

$$= P(A) \cdot P(B).$$

$$= \frac{60}{100} \cdot \frac{80}{100}$$

$$= \frac{4800}{10000} = 0.48$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{60}{100} + \frac{80}{100} - 0.48$$

$$\boxed{P(A \cup B) = 0.92}$$

⟹ EXAMPLE:

If two cards are drawn from an ordinary deck of 52 cards. What is the prob. that both will be diamonds, if the drawing is without replacement?

A = First card is diamond

B = 2nd " " "

Date

$$P(A \text{ and } B) = P(A \cap B)$$

$$= P(A)\,P(B|A)$$

$$= \frac{13}{52} \cdot \frac{12}{51}$$

$$= \frac{156}{2652}$$

$$\boxed{P(A \text{ and } B) = 0.06}$$

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⇒ EXAMPLE:

Suppose $P(A) = 0.40$ & $P(B|A) = 0.30$ what is the joint prob. of A and B?

$P(A \text{ and } B) = P(A)\,P(B|A)$.

$= (0.40)(0.30)$

$= \boxed{0.12}$

→ PRINCIPLES OF COUNTING:

If the number of possible outcomes in an experiment is small, it is relatively easy to count them. If however, there are a large No. of possible outcomes, it would be difficult to count all the possibilities.

To facilitate counting, we dis-cuss three formulas

- Multiplication formula.
- Permutation ″ .
- Combination ″ .

→ MULTIPLICATION FORMULA:

If there are 'm' ways of doing one thing & 'n' ways of doing another thing, there are m×n ways of doing both.

In terms of a formula:

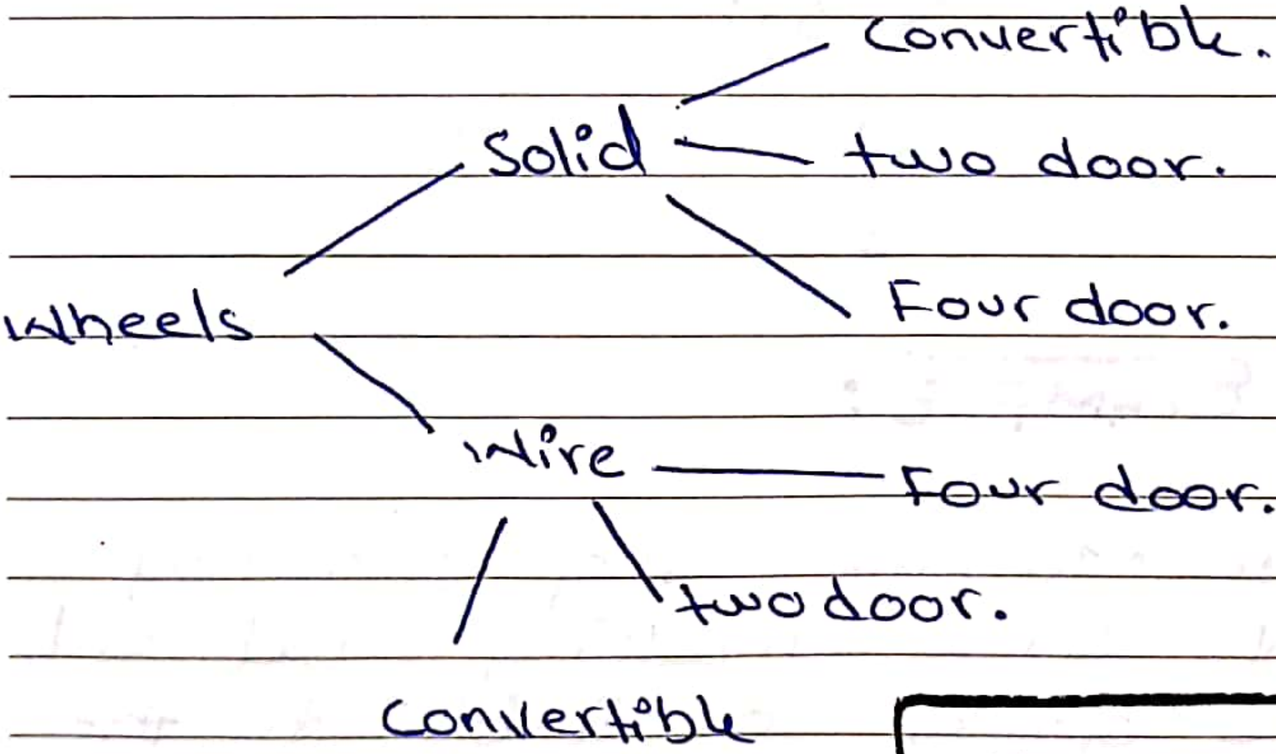
Total No. of arrangements = $m \times n$

This can be extended to more than two events. For three events m, n & k:

Total No. of arrangements = $m \times n \times k$

⟹ EXAMPLE:

An automobile dealer wants to advertise that for $29,999 you can by a convertible, a two-door sedan or a four-door model with your choice of either wire wheel covers or solid wheel covers. How many different arrangements of models & wheel covers can the dealer offer?



Tree Diagram

$m = 3$

$n = 2$

Total Possible arrangements $= (3)(2) = 6$

⟹ EXAMPLE:

A developer of a new subdivision offers home buyers a choice of Tudor, rustic, colonial and traditional exterior styling in ranch, two-story and split level floor plans. In how many diff. ways can a buyer order one of these homes?

$m = 4$, $n = 3$

Total possible arrangements = $(4)(3) = 12$

⇒ PERMUTATION FORMULA :

An arrangement of 'r' objects selected from a single group of 'n' possible objects. In it order of the objects selected from a group is important.

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

EXAMPLE:

Referring to the group of three electronic parts that are to be assembled in any order, in how many diff. ways can they be assembled?

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^3P_3 = \frac{3!}{(3-3)!}$$

$$= \frac{3!}{1} = 6$$

EXAMPLE :

Betts Machine shop Inc. has eight screw machines but only three spaces available in the production area for the machines. In how many different ways can the eight machines be arranged in the three spaces available?

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$^{8}P_{3} = \frac{8!}{(8-3)!}$$

$$= \boxed{336}$$

⟹ COMBINATION FORMULA:

If the order of the selected objects is not important, any

selection is called a combi-
-nation. The formula to
count the number of 'r'
object combinations from
a set of objects is:

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}.$$

⟹ EXAMPLE:

A pollster randomly selected 4 of 10 available people. How many diff groups of 4 are possible?

$$^{10}C_4 = 210.$$

$\because$ $n = 10$

$\because$ $r = 4$

$\Rightarrow$ **EXAMPLE:**

Find ${}^7C_3$.

$${}^7C_3 = \frac{n!}{r!\,(n-r)!}$$

$$= \frac{7!}{3!\,(7-3)!}$$

$$= 35$$

⟹ EXAMPLE :

In a lottery game, three numbers are randomly selected from a tumbler of balls numbered 1 through 50.

a. How many permutations are possible

b. How many combinations are possible

$n = 50$

$r = 3$

$^{n}P_{r} = {}^{50}P_{3}$

$= 17600$

$^{n}C_{r} = {}^{50}C_{3}$

$= 19600$