Variance:

It is defined as mean of squared deviations of all observations from their mean.

When it is calculated from the population, the variance is called population variance & is denoted by σ^2.

When it is calculated from the sample, the variance is called sample variance and is denoted by s^2.

$$\sigma^2 = \frac{\Sigma (x-\mu)^2}{N} \rightarrow \text{pop. var.}$$

$$s^2 = \frac{\Sigma (x-\bar{x})^2}{n-1} \rightarrow \text{Sample var.}$$

Here μ is population mean.

→ Merits of variance:

- It is based on all the observations of data.
- It is easy to calculate.
- " " " " understand.

→ Demerit of variance:

- It is affected by extreme values.
- It is difficult to interpret. (i.e, If you have data of dollars & you find variance = $10. Here variance is not in terms of dollars but squared dollars. So it is difficult to interpret)

⟹ Example 1: (pop. variance)

X = 19, 17, 22, 18, 28, 34, 45, 39, 38, 44, 34, 10

$$\mu = \frac{\Sigma(x)}{N}$$

$$= \frac{19+17+22+18+28+34+45+39+38+44+34+10}{12}$$

$$\mu = \frac{348}{12}$$

$$\boxed{\mu = 29}$$

X	$X-\mu$	$(X-\mu)^2$
19	−10	100
17	−12	144
22	−7	49
18	−11	121
28	−1	1
34	5	25
45	16	256
39	10	100
38	9	81
44	15	225
34	5	25
10	−19	361
		1488

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

$$= \frac{1488}{12}$$

$$\boxed{\sigma^2 = 124}$$

Scanned with CamScanner

⇒ Example 2: (Sample var.)

The hourly wages for a sample of part-time employees at Home Depot are:

\$12, \$20, \$16, \$18 & \$19.

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{12+20+16+18+19}{5}$$

$$= \frac{85}{5}$$

$$\boxed{\bar{x} = \$17}$$ \$ ~~is dollars squared.~~

$\bar{X}$	$X - \bar{X}$	$(X - \bar{X})^2$
12	-5	25
20	3	9
16	-1	1
18	1	1
19	2	4
		40

$$s^2 = \frac{\Sigma (X - \bar{X})^2}{n-1}$$

$$= \frac{40}{5-1}$$

$$s^2 = \frac{40}{4}$$

$\boxed{s^2 = 10}$ in dollars squared.

→ Example 3: (Pop. var.)

3, 6, 2, 1, 7, 5.

$$\mu = \frac{\Sigma X}{N}$$

$$= \frac{3+6+2+1+7+5}{6}$$

$$= \frac{24}{6}$$

$$\boxed{\mu = 4}$$

X	$(X-\mu)$	$(X-\mu)^2$
3	-1	1
6	2	4
2	-2	4
1	-3	9
7	3	9
5	1	1
		28

$$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$

$$= \frac{28}{6}$$

$$\boxed{\sigma^2 = 4.67}$$

$\Rightarrow$ Example 4 :

A population of 10 has the observations

7, 8, 10, 13, 14, 19, 20, 25, 26 & 28.

Find its variance.

$$\mu = \frac{\Sigma x}{N}$$

$$= \frac{170}{10}$$

$$\boxed{\mu = 17}$$

Scanned with CamScanner

x	$x-\mu$	$(x-\mu)^2$	
7	-10	100	
8	-9	81	
10	-7	49	
13	-4	16	
14	-3	9	
19	2	4	
20	3	9	
25	8	64	
26	9	81	
28	11	121	
		534	

$$\sigma^2 = \frac{\Sigma (x-\mu)^2}{N}$$

$$= \frac{534}{10}$$

$$\boxed{\sigma^2 = 53.4}$$

→ Example 5: (Sample var.)

45, 32, 37, 46, 39, 36, 41, 48, 36

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{45+32+37+46+39+36+41+48+36}{9}$$

$$= \frac{360}{9}$$

$$\boxed{\bar{x} = 40}$$

Scanned with CamScanner

X	$(X-\bar{X})$	$(X-\bar{X})^2$
45	5	25
32	-8	64
37	-3	9
46	6	36
39	-1	1
36	-4	16
41	1	1
48	8	64
36	-4	16
		232

$$S^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}$$

$$= \frac{232}{9-1}$$

$$\boxed{S^2 = 29}$$

Scanned with CamScanner

⟹ Example 6: (Sample var.)

102, 104, 106, 108, 110.

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{102 + 104 + 106 + 108 + 110}{5}$$

$$= \frac{530}{5}$$

$$\boxed{\bar{x} = 106}$$

x	$x - \bar{x}$	$(x - \bar{x})^2$	
102	-4	16	
104	-2	4	
106	0	0	
108	2	4	
110	4	16	
		40	

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$= \frac{40}{5-1}$$

$$= \frac{40}{4}$$

$$\boxed{S^2 = 10}$$

Standard Deviation:

As varianc is difficult to interpret. i.e., variance = 10 squared dollars.

There is a way out of this difficulty. By taking taking the square root of variance, we can transform it to the same unit of measurement used for the original data. The square root of 10 squared dollars is 3.16 dollars.

The square root of the varianc is called standard deviation.

$$\sigma = \sqrt{\frac{\Sigma(x-\mu)^2}{N}} \quad \text{(pop SD)}$$

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} \quad \text{(Sample SD)}$$

⟹ Merits of SD :

- It is based on all observations of data.

- It is easy to calculate & simple to understand by formula.

- It is better measure of dispersion than other measures of dispersions.

Scanned with CamScanner

⟹ Example: (pop. SD)

Use Example 1 of variance to find SD.

$$\sigma^2 = \frac{1488}{12}$$

$$\sigma^2 = 124$$

$$SD = \sigma = \sqrt{124}$$

$$\boxed{SD = 11.135}$$

Scanned with CamScanner

⇒ Example: (Sample SD)

use Example 2 of variance to find SD.

$$s^2 = \frac{40}{4}$$

$s^2 = 10$ in dollars squared.

$$SD = s = \sqrt{10}$$

$$\boxed{s = \$3.16}$$

Example: (pop. SD)

use Example 3 of variance to find SD:

$$\sigma^2 = \frac{28}{6}$$

$$\sigma^2 = 4.67$$

$$\sigma = \sqrt{4.67}$$

$$\boxed{\sigma = 2.16}$$

Scanned with CamScanner

→ Example : (pop. SD).

use example 4 of variance

to find SD :

$$\sigma^2 = \frac{534}{10}$$

$$\sigma^2 = 53.4$$

$$\sigma = \sqrt{53.4}$$

$$\boxed{\sigma = 7.31}$$

Example : (Sample SD)

use Example 5 of variance to find SD.

$$s^2 = \frac{232}{8}$$

$$s^2 = 29$$

$$s = \sqrt{29}$$

$$\boxed{s = 5.4}$$

⇒ Example: (Sample SD).

use example 6 of var. & find SD.

$$s^2 = \frac{40}{4}$$

$$S^2 = 10$$

$$S = \sqrt{10}$$

$$\boxed{S = 3.16}$$