Mo Tu We Th Fr Sa Su

Memo no. _____

Date / /

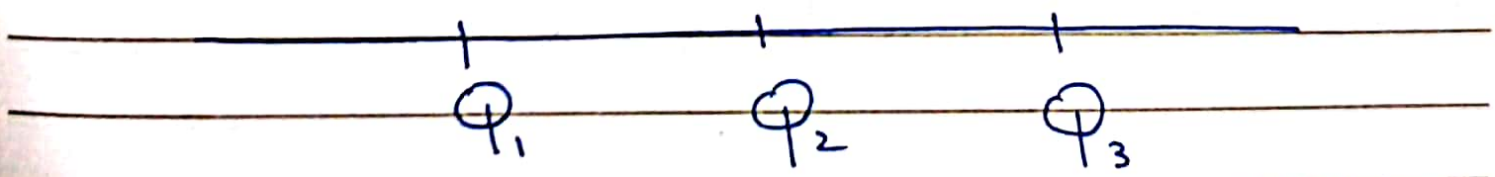
Quantiles OR Measures of Position.

- Quartiles.
- Deciles
- Percentiles.

Sometimes our interest is to know the position of an observation relative to the others in a dataset. The measures used for this purpose are called Quantiles or measures of position.

⟹ Quartiles:

Quartiles divide a set of observations into four equal parts.



Here:

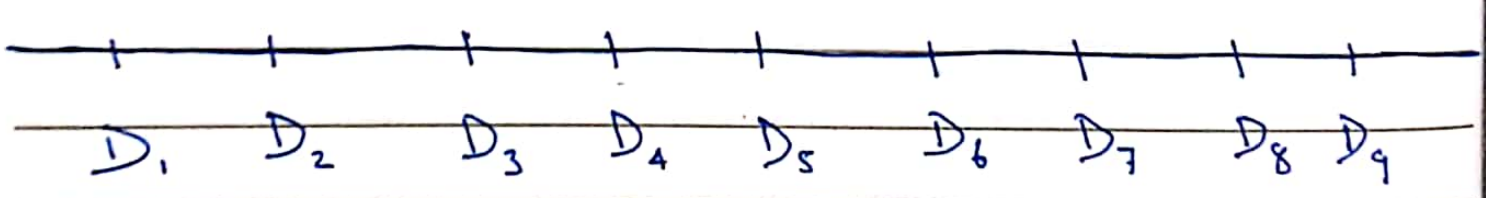
Q_1 : First / Lower Quartile. It is the value below which 25 percent of the observations occur.

Q_2 : Second Quartile / Median.

Q_3 : Third / Upper Quartile. It is the value below which 75 percent of the observations occur

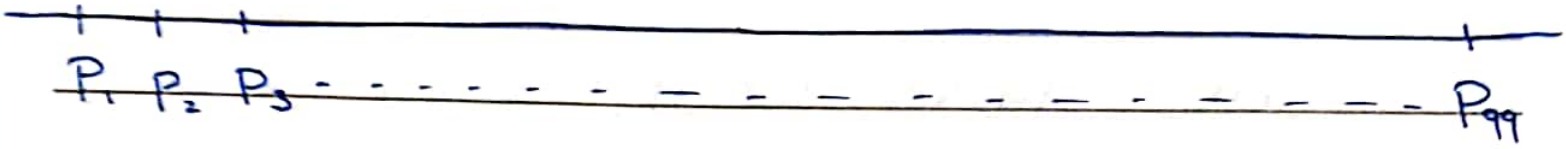
Deciles:

Deciles divide a set of observations into 10 equal parts.



⟹ Percentiles:

Percentiles divide a set of observations into 100 equal parts.



⟹ Formula:

$$P_L = (n+1)\frac{L}{100} \text{ th value of arranged data.}$$

i.e.,

$$P_1 = (n+1)\frac{1}{100} \text{ "}$$

$$P_{50} = (n+1)\frac{50}{100} \text{ "}$$

$$P_{75} = (n+1)\frac{75}{100} \text{ "}$$

$$P_{25} = (n+1)\frac{25}{100} \text{ " etc.}$$

⟹ Relationship b/w Quartiles, Deciles & Percentiles:

- Median = $Q_2 = D_5 = P_{50}$
- $Q_1 = P_{25}$
- $Q_3 = P_{75}$
- $D_1 = P_{10}$
- $D_7 = P_{70}$
- $D_9 = P_{90}$
- $D_2 = P_{20}$
- $D_3 = P_{30}$
- $D_4 = P_{40}$
- $D_6 = P_{60}$
- $D_8 = P_{80}$

⟹ Example:

X = 10, 12, 20, 15, 9, 8, 8, 7, 19, 25

$Q_1 = ?$

$Q_3 = ?$

$Q_2 = ?$

$D_4 = ?$

Arranged data:

X = 7, 8, 8, 9, 10, 12, 15, 19, 20, 25.

$n = 10$

$Q_1 = P_{25} = (n+1)\frac{L}{100}$ th value of arranged Data.

$= (10+1)\frac{25}{100}$ "

$= 2.75$ "

$= 2^{nd} + 0.75(3^{rd} - 2nd)$

$= 8 + 0.75(8-8)$

$= 8 + 0.75(0)$

$$\boxed{Q_1 = 8}$$

$Q_3 = P_{75} = (n+1)\frac{L}{100}$ th value of arranged data.

$Q_3 = P_{75} = (10+1)\frac{75}{100}$ "

$= \frac{(11)(75)}{100}$

$= 8.25$ "

$= 8^{th} + 0.25\left(9^{th} - 8^{th}\right)$

$= 19 + 0.25(20-19)$

$$\boxed{Q_3 = P_{75} = 19.25}$$

$Q_2 = P_{50} = (n+1)\frac{L}{100}$ th value of arranged data

$= (10+1)\frac{50}{100}$ "

$= \frac{(11)50}{100}$ "

$= 5.5$ "

$= 5^{th} + 0.5\left(6^{th} - 5^{th}\right)$

$= 10 + 0.5\left(12 - 10\right)$

$\boxed{Q_2 = P_{50} = 11}$

$D_9 = P_{90} = (n+1)\frac{L}{100}$ th value of arranged Data.

$= (10+1)\frac{90}{100}$ "

$= 9.9$ "

$$= 9^{th} + 0.9\left(10^{th} - 9^{th}\right)$$

$$= 20 + 0.9\left(25 - 20\right)$$

$$\boxed{D_9 = 24.5}$$

→ Example:

X = 20, 22, 18, 15, 20, 25, 38, 37, 29, 26, 42,
7

$Q_1 = ?$

$Q_3 = ?$

$D_4 = ?$

$P_{10} = ?$

Arranged data:

X = 7, 15, 18, 20, 20, 22, 25, 26, 29, 37,
38, 42

n = 12

$Q_1 = P_{25}$

$Q_1 = \frac{(n+1)\,L}{100}^{th}$ value of arranged Data

$Q_1 = (12+1)\frac{25}{100}^{th}$ value of arranged data

$= 3.25$ "

$= 3^{rd} + 0.25\left(4^{th} - 3^{rd}\right)$

$= 18 + 0.25\left(20 - 18\right)$

$\boxed{Q_1 = 18.5}$

$Q_3 = P_{75}$

$Q_3 = (n+1)\frac{75}{100}^{th}$ value of arranged data.

$= (12+1)\frac{75}{100}$ "

$= 9.75$ "

$= 9^{th} + 0.75\left(10^{th} - 9^{th}\right)$

$= 29 + 0.75 (37 - 29)$

$\boxed{Q_3 = 35}$

$D_4 = P_{40}$

$= (n+1) \frac{L}{100}$ th value of arranged Data

$= (12+1) \frac{40}{100}$ "

$= 5.2$ "

$= 5^{th} + 0.2 (6^{th} - 5^{th})$

$= 20 + 0.2 (22 - 20)$

$\boxed{D_4 = 20.4}$

$P_{10} = (n+1)\frac{1}{100}$ th value of arranged Data

$= (12+1)\frac{10}{100}$ "

$= 1.3$ "

$= 1^{st} + 0.3\left(2^{nd} - 1^{st}\right)$

$= 7 + 0.3(15 - 7)$

$\boxed{P_{10} = 9.4}$

→ Example:

For the following dataset find P_{25}, P_{75}.

20, 18, 19, 25, 15, 23, 29, 15

arranged Data:

15, 15, 18, 19, 20, 23, 25, 29.

P_{25} = $\frac{(n+1)}{100}$ $^{-th}$ value of arranged Data.

$= \frac{(8+1)}{100} \frac{25}{}$ "

$= 2.25$ "

$= 2^{nd} + 0.25 \left(3^{rd} - 2^{nd}\right)$

$= 15 + 0.25 \left(18 - 15\right)$

$\boxed{P_{25} = 15.75}$

$P_{75} = (n+1)\frac{L}{100}^{th}$ value of arranged data.

$= (8+1)\frac{75}{100}$

$= 6.75$ "

$= 6^{th} + 0.75\left(7^{th} - 6^{th}\right)$

$= 23 + 0.75(25-23)$

$\boxed{P_{75} = 24.50}$

⟹ Example:

Find Q_1, Q_2 & Q_3 of following dataset:

95.05, 94.50, 88.03, 84.60, 94.90

Arranged data:

84.60, 88.03, 94.50, 94.90, 95.05

$Q_1 = P_{25}$

$= (n+1)\frac{L}{100}^{th}$ value of arranged data

$= (5+1)\frac{25}{100}$ "

$= 1.5$ "

$= 1^{st} + 0.5(2^{nd} - 1^{st})$

$= 84.60 + 0.5(88.03 - 84.60)$

$\boxed{Q_1 = 86.315}$

$Q_2 = P_{50}$

$= (5+1)\frac{50}{100}^{th}$ value of arranged Data.

$= 3^{rd}$ value of arranged Data.

$\boxed{Q_2 = 94.50}$

$Q_3 = P_{75}$

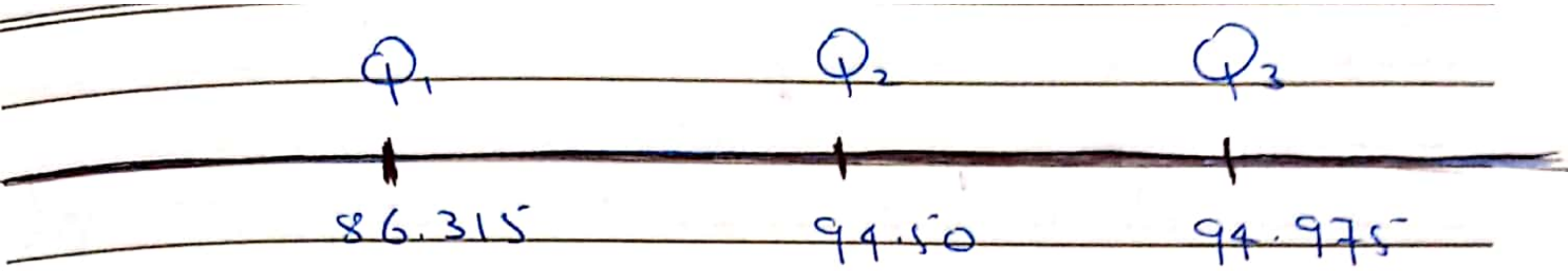
$= (5+1)\frac{75}{100}^{th}$ value of arranged Data.

$= 4.5^{th}$ "

$= 4^{th} + 0.5(5^{th} - 4^{th})$

$= 94.90 + 0.5(95.05 - 94.90)$

$\boxed{Q_3 = 94.975}$



→ Example:

Given below are the marks obtained by 9 students

45, 32, 37, 46, 39, 36, 41, 48 & 36

Find Median & quartiles.

arranged data:

32, 36, 36, 37, 39, 41, 45, 46, 48.

$Q_1 = P_{25}$

$= (9+1)\ \frac{25}{100}^{th}$ value of arranged Data.

$= 2.5$ "

$= 2^{nd} + 0.5\,(3^{rd} - 2nd)$

$= 36 + 0.5\,(36 - 36)$

$Q_1 = 36$ marks

$Q_2 = P_{50}$

$= (9+1)\frac{50}{100}^{th}$ value of arranged data

$= 5^{th}$ "

$\boxed{Q_2 = 39 \text{ marks}}$

$Q_3 = P_{75}$

$= (9+1)\frac{75}{100}^{th}$ value of arranged data

$= 7.5$ "

$= 7^{th} + 0.5\,(8^{th} - 7^{th})$

$= 45 + 0.5\,(46 - 45)$

$\boxed{Q_3 = 45.5}$