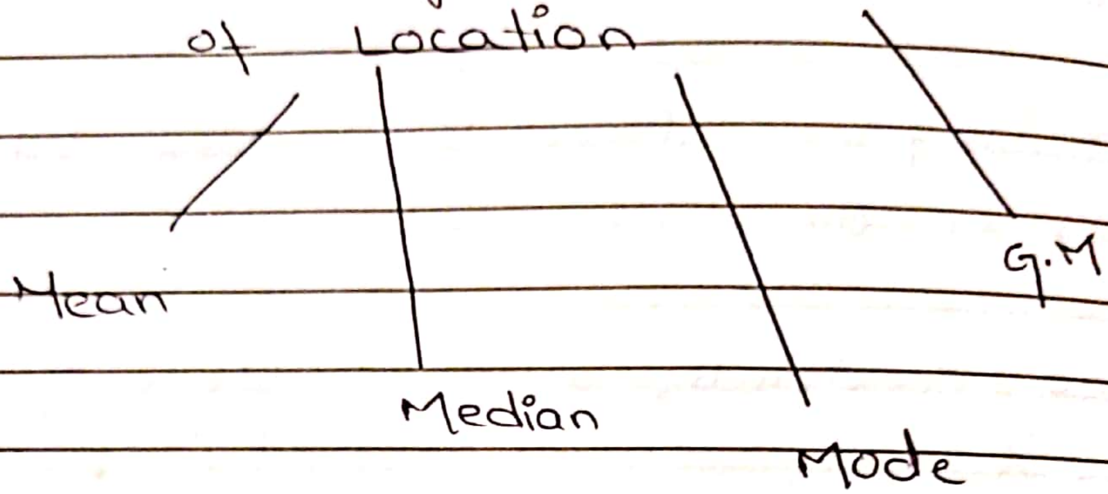
Measures of Central Tendency OR Measures of Location



⟹ Measures of Central Tendency/Location:

Measures of location are often referred as averages. The purpose of a measure of location is to point out the center of data. An Average is a numerical value that shows the central value of data.

⟹ Mean:

The ratio b/w the sum of the observations and the total number of observation is called mean. i.e.,

$$\text{Mean} = \frac{\text{Sum of all the values}}{\text{Total No. of values}}.$$

Mean

Pop. Mean

$$\mu = \frac{\Sigma X}{N}$$

Sample Mean

$$\bar{x} = \frac{\Sigma x}{n}$$

⇒ Properties of Mean:

- Every set of data has a mean.
- All the values are included in computing the mean.
- $\Sigma(x-\bar{x}) = 0$. For example, the mean of

  3, 8, 4

  is '5' Then:

$$\Sigma(x-\bar{x}) = (3-5)+(8-5)+(4-5)$$
$$= -2+3-1$$
$$\boxed{\Sigma(x-\bar{x}) = 0}$$

⟹ Merits of Mean:

- It is easy to calculate.
- " " " " understand.
- " " based upon all the values.

⟶ Demerits of Mean:

- It can't be used for qualitative data.
- It is affected by Extreme values.

→ Example:

There are 42 exits on I-75 through the state of Kentucky. Listed below are the distances b/w exits (in miles).

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

$$\mu = \frac{\Sigma X}{N} = \boxed{4.57} = \frac{192}{42}$$

→ Example:

SunCom is studying the number of minutes used by clients in a particular cell phone rate plan. A random sample of 12 clients showed the following number of minutes used last month.

90	77	94	89	119	112
91	110	92	100	113	83

$$\bar{X} = \frac{\Sigma X}{n} = \frac{1170}{12} = \boxed{97.5}$$

⟹ Example:

The marks obtained by 9 students are given below.

45, 32, 37, 46, 39, 36, 41, 48, 36.

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{360}{9} = \boxed{40 \text{ marks}}$$

⟹ Example:

Following are the hieghts (cms) of students

87, 91, 89, 88, 89, 91, 87, 92, 90, 98

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{902}{10} = \boxed{90.20 \text{ cm}}$$

→ Weighted Mean:

It is a special case of the arithmetic mean. Arithmetic mean is used when all the observations are given equal importance but there are certain situations in which the diff. observations get diff. weights. In this situation weighted mean is preferred. It is denoted by '$\bar{X}_w$'

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W}.$$

⇒ Example :

The following data is about the % kill (x_i) & the no. of insects (w_i) used in a study, the interest is to calculate the mean of the % kill.

X	W	WX
88	44	88 x 44
85.7	42	85.7 x 42
52.1	24	52.1 x 24
33.1	16	33.1 x 16
12.0	6	12.0 x 6
		9326.6

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W}$$

$$= \frac{9326.6}{132}$$

$$\boxed{\bar{X}_w = 70.65606}$$

→ Example :

The Carter construction company pays its hourly employees \$16.50, \$19.00 or \$25.00 per hour. There are 26 hourly employees, 14 of which are paid at the \$16.50 rate, 10 at the \$19.00 rate, & 2 at the \$25.00 rate. What is the mean hourly rate paid the 26 employees?

$$\bar{X}_w = \frac{14(\$16.50) + 10(\$19.00) + 2(\$25.00)}{26}$$

$$= \frac{\$471.00}{26}$$

$$= \boxed{\$18.1154}$$

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⟹ Example :

Item	Expenditure (Rs)	Weights
Food	290	7.5
Rent	54	2.0
Clothing	98	1.5
Fuel & Light	75	1.0
other items	75	0.5

$$\bar{X}_w = \frac{\Sigma w x}{\Sigma w}$$

$$= \frac{2542.5}{12.5}$$

$$\boxed{\bar{X}_w = 203.4 \text{ Rs}}$$

→ Example :

An examination candidate's % are

English 73 , French 82 , Math 57, Science 62 , History 60,

Find candidate's weighted Mean if weights of 4, 3, 3, 1, 1 respectively are allotted to the subjects,

$$\bar{X}_{W} = \frac{\sum WX}{\sum W}$$

$$\bar{X}_{W} = \boxed{\quad}$$

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Example 3.1: Find arithmetic mean of the following data:

102, 104, 106, 108, 110.

Solution:

Y
102
104
106
108
110
$\Sigma Y = 530$

3.25 Salman obtained the following marks in a certain examination. Find the weighted mean if weights 4, 3, 3, 2 and 2 respectively are allotted to the subjects.

English	Urdu	Math	Stat	Physics
73	82	80	57	62

Solution:

Subjects	(Y)	W	WY
English	73	4	292
Urdu	82	3	246
Math	80	3	240
Stat	57	2	114
Physics	62	2	124
Σ	--------	14	1016

WEIGHTED MEAN:

$$\bar{Y}_w = \frac{\Sigma WY}{\Sigma W} = \frac{1016}{14} = 72.57$$

3.26 Calculate weighted mean for the following items:

Items	Expenditures	Weights
Food	290	7.5
Rent	54	2
Clothing	98	1.5
Fuel and light	75	1.0
Other Items	75	0.5

Geometric Mean:

It is defined as the n^{th} root of the product of 'n' valuey. It is useful in finding the average change of percentages, ratios or growth rates over time.

It has a wide application in business because we are often interested in finding the percentage changey in sales, salaries etc.

$$GM = \sqrt[n]{(X_1)(X_2)\cdots\cdots(X_n)}$$

⟹ EXAMPLE:

Calculate G.M for the observations:
0.5, 10.0, 2.7, 3.48 & 4.7.

$$G.M = \sqrt[n]{(x_1)(x_2)\cdots(x_n)}$$

$$= \sqrt[5]{(0.5)(10)(2.7)(3.48)(4.7)}$$

$$\boxed{G.M = 2.94}$$

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$\Rightarrow$ Example :

calculate GM for the following data of the percentage change in the weight of eight animals.

45, 30, 35, 40, 44, 32, 42, 37

$$GM = \sqrt[8]{(45)(30)(35)(40)(44)(32)(42)(37)}$$

$$= \left(\boxed{} \right)^{1/8}$$

$$\boxed{GM = 37.7616}$$

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⟹ Example :

Find the GM of

45, 32, 37, 46, 39, 36, 41, 48 & 36.

$$GM = \sqrt[9]{(32)(37)(46)(39)(36)(41)(48)(36)(45)}$$

$$= (\quad)^{1/9}$$

$$\boxed{GM = 39.68}$$

Exercises

27. Compute the geometric mean of the following percent increases: 8, 12, 14, 26, and 5.
28. Compute the geometric mean of the following percent increases: 2, 8, 6, 4, 10, 6, 8, and 4.
29. Listed below is the percent increase in sales for the MG Corporation over the last 5 years. Determine the geometric mean percent increase in sales over the period.

9.4	13.8	11.7	11.9	14.7

ple 3.8: A man gets a rise of 10% in salary at the end of his first year of service and further of 20% and 25% at the end of the second and third years respectively. The rise in each case being calculated on his salary at the beginning of the year. To what annual percentage increase in this equivalent?

on:

Suppose the initial salary of the man = 100

At the end of	Increment	Salary(Y)
First year	10%	110
Second year	20%	120
Third Year	25%	125

$$\text{G.M.} = \sqrt[3]{110 \times 120 \times 125}$$

$$= 118.16$$

Annual percentage increase = 118.16 – 100 = 18.16%

Mode:

The most frequent value in a dataset is called Mode. If data has only one mode then it is called unimodal. The data may have two mode (bimodal) and more than two mode (multimodal).

⇒ Merits of Mode:

- It is very quick to find
- " " not affected by extremely high or low values.

⇒ Demerits of Mode:

- There may be more than one mode in a data set.
- For many data sets there is no mode.

Example

Recall the data regarding the distance in miles between exits on I-75 through Kentucky. The information is repeated below.

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

What is the modal distance?

Solution

The first step is to organize the distances into a frequency table. This will help us determine the distance that occurs most frequently.

Distance in Miles between Exits	Frequency
1	8
2	7
3	7
4	3
5	4
6	1
7	3
8	2
9	1
10	4
11	1
14	1
Total	42

The distance that occurs most often is one mile. This happens eight times—that is, there are eight exits that are one mile apart. So the modal distance between exits is one mile.

For Exercises 18–20, determine the (a) mean, (b) median, and (c) mode.

18. The following is the number of oil changes for the last 7 days at the Jiffy Lube located at the corner of Elm Street and Pennsylvania Avenue.

41	15	39	54	31	15	33

19. The following is the percent change in net income from last year to this year for a sample of 12 construction companies in Denver.

5	1	−10	−6	5	12	7	8	2	5	−1	11

20. The following are the ages of the 10 people in the video arcade at the Southwyck Shopping Mall at 10 A.M.

12	8	17	6	11	14	8	17	10	8

Example 3.23: Find mode for the following data:

91, 89, 88, 87, 89, 91, 87, 92, 90, 98, 95, 97, 96, 100, 101, 96, 98, 99, 98, 100, 102, 99, 101, 105, 103, 107, 105, 106, 107, 112.

Solution:

91, 89, 88, 87, 89, 91, 87, 92, 90, 98, 95, 97, 96, 100, 101, 96, 98, 99, 98, 100, 102, 99, 101, 105, 103, 107, 105, 106, 107, 112.

Since the most frequent value of the data is 98.

Therefore, Mode = 98

Median:

For data containing one or two very large or very small values (extreme values), the mean may not be represen-tative. The center for such data can be better described by a measure of location called the Median.

The midpoint of the values after they have been ordered from the smallest to the largest, or the largest to the smallest.

→ Properties of Median:

- It is not affected by extremely large or small values
- It can be computed for ordinal-level data/ranked data/ordered data.

Example

Facebook is a popular social networking website. Users can add friends and send them messages, and update their personal profiles to notify friends about themselves and their activities. A sample of 10 adults revealed they spent the following number of hours last month using Facebook.

3	5	7	5	9	1	3	9	17	10

Find the median number of hours.

Solution

Note that the number of adults sampled is even (10). The first step, as before, is to order the hours using Facebook from low to high. Then identify the two middle times. The arithmetic mean of the two middle observations gives us the median hours. Arranging the values from low to high:

1	3	3	5	5	7	9	9	10	17

The median is found by averaging the two middle values. The middle values are 5 hours and 7 hours, and the mean of these two values is 6. We conclude that the typical Facebook user spends 6 hours per month at the website. Notice that the median is not one of the values. Also, half of the times are below the median and half are above it.

⟹ Example:

Prices data:

70,000 , 80,000 60,000 , 65000 , 275000

Prices ordered from low to high.

60,000

65,000

70,000

80,000

275,000

$$\boxed{\tilde{x} = 70,000}$$

⟶ Example:

heights of students (cms)

87, 91, 89, 88, 89, 91, 87, 92 , 90, 98

ordered observations are:

87, 87, 88, 89, 89, 90, 91, 91, 92, 98

$$\tilde{x} = \frac{89+90}{2}$$

$$= \boxed{89.5 \text{ cm}}$$

→ Example :

Given below are the marks obtained by 9 students

45, 32, 37, 46, 39, 36, 41, 48 & 36.

32, 36, 36, 37, 39, 41, 45, 46, 48

$$\boxed{\tilde{x} = 39 \text{ marks}}$$