Log-linear Models in Contingency Table

Log-linear Model

- Poisson log-linear model is used when Y = counts and all of predictors X = categories.
- Frequently, variables Y , X , and cell counts are in a form of contingency tables.
- Thus, Log-linear model is often used for modeling cell count data in contingency tables.

Objective:

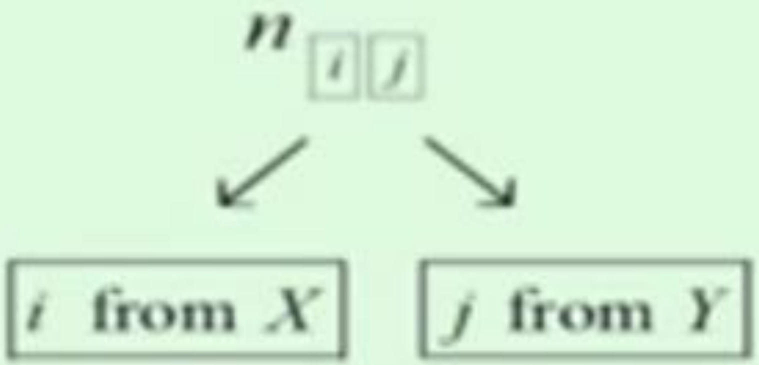
To analyze association and interaction patterns among variables in contingency table

Log-linear Models
for Two-way Table

A Log-linear Modeling Framework

Data:

X \ Y	1	0
1	n_{11}	n_{10}
0	n_{01}	n_{00}



Variables X and Y are determined for association

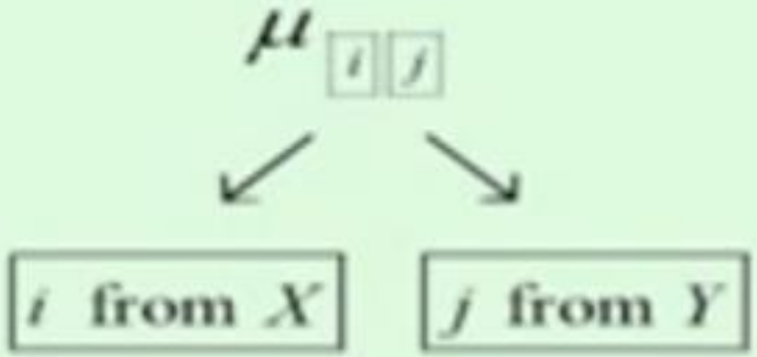
Data n_{ij} are in a form of cell counts

n_{ij} = Number of observations for cell $X = i$, $Y = j$

A Log-linear Modeling Framework

Means:

		Y: 1	Y: 0
X	1	$E(n_{11}) = \mu_{11}$	$E(n_{10}) = \mu_{10}$
	0	$E(n_{01}) = \mu_{01}$	$E(n_{00}) = \mu_{00}$



n_{ij} is assumed to be a $Poisson\,(\mu_{ij})$

Independent Model for Two Variables

Independent model: X and Y are independent

Independence: $P(X \cap Y) = P(X) \times P(Y)$

Probabilities: $p_{ij} = p_{i+} \, p_{+j}$

Let $N = n_{11} + n_{10} + n_{01} + n_{00}$

Expected counts: $\mu_{ij} = Np_{ij} = Np_{i+} \, p_{+j}$

$\log \mu_{ij} = \log N + \log p_{i+} + \log p_{+j}$

$\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y$

Independent model of two variables:

$$\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y$$

$$\log \mu_{00} = \lambda$$

$$\log \mu_{01} = \lambda + \lambda_1^Y$$

$$\log \mu_{10} = \lambda + \lambda_1^X$$

$$\log \mu_{11} = \lambda + \lambda_1^X + \lambda_1^Y$$

		Y = 1	Y = 0
X	1	$\log \mu_{11} = \lambda + \lambda_1^X + \lambda_1^Y$	$\log \mu_{10} = \lambda + \lambda_1^X$
	0	$\log \mu_{01} = \lambda + \lambda_1^Y$	$\log \mu_{00} = \lambda$

Association (Interaction) Model

- Usually, interaction model is used for indicating association between the row and the column factors in the two-way table

- Interaction model: $\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_{ij}^{XY}$

		Y	
		1	0
X	1	$\log \mu_{11} = \lambda + \lambda_1^X + \lambda_1^Y + \lambda_{11}^{XY}$	$\log \mu_{10} = \lambda + \lambda_1^X$
	0	$\log \mu_{01} = \lambda + \lambda_1^Y$	$\log \mu_{00} = \lambda$

Association (Interaction) model of two variables is obtained by adding Interaction term

$$\log \mu_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_{ij}^{XY}$$

Association Model Ignoring Indicator Variables:

$$\log \mu_{00} = \lambda$$

$$\log \mu_{01} = \lambda + \lambda_1^Y$$

$$\log \mu_{10} = \lambda + \lambda_1^X$$

$$\log \mu_{11} = \lambda + \lambda_1^X + \lambda_1^Y + \lambda_{11}^{XY}$$

- Need to estimate 4 parameters: λ, $\lambda_1^X, \lambda_1^Y, \lambda_{11}^{XY}$
- Based on 4 observed cell counts: $n_{00}, n_{10}, n_{01}, n_{11}$

Association Model for Two Variables

$$\log \mu_{00} = \lambda \quad \rightarrow \mu_{00} = E(n_{00}) = \exp(\lambda)$$

$$\log \mu_{01} = \lambda + \lambda_1^Y \quad \rightarrow \mu_{01} = E(n_{01}) = \exp(\lambda + \lambda_1^Y)$$

$$\log \mu_{10} = \lambda + \lambda_1^X \quad \rightarrow \mu_{10} = E(n_{10}) = \exp(\lambda + \lambda_1^X)$$

$$\log \mu_{11} = \lambda + \lambda_1^X + \lambda_1^Y + \lambda_{11}^{XY}$$

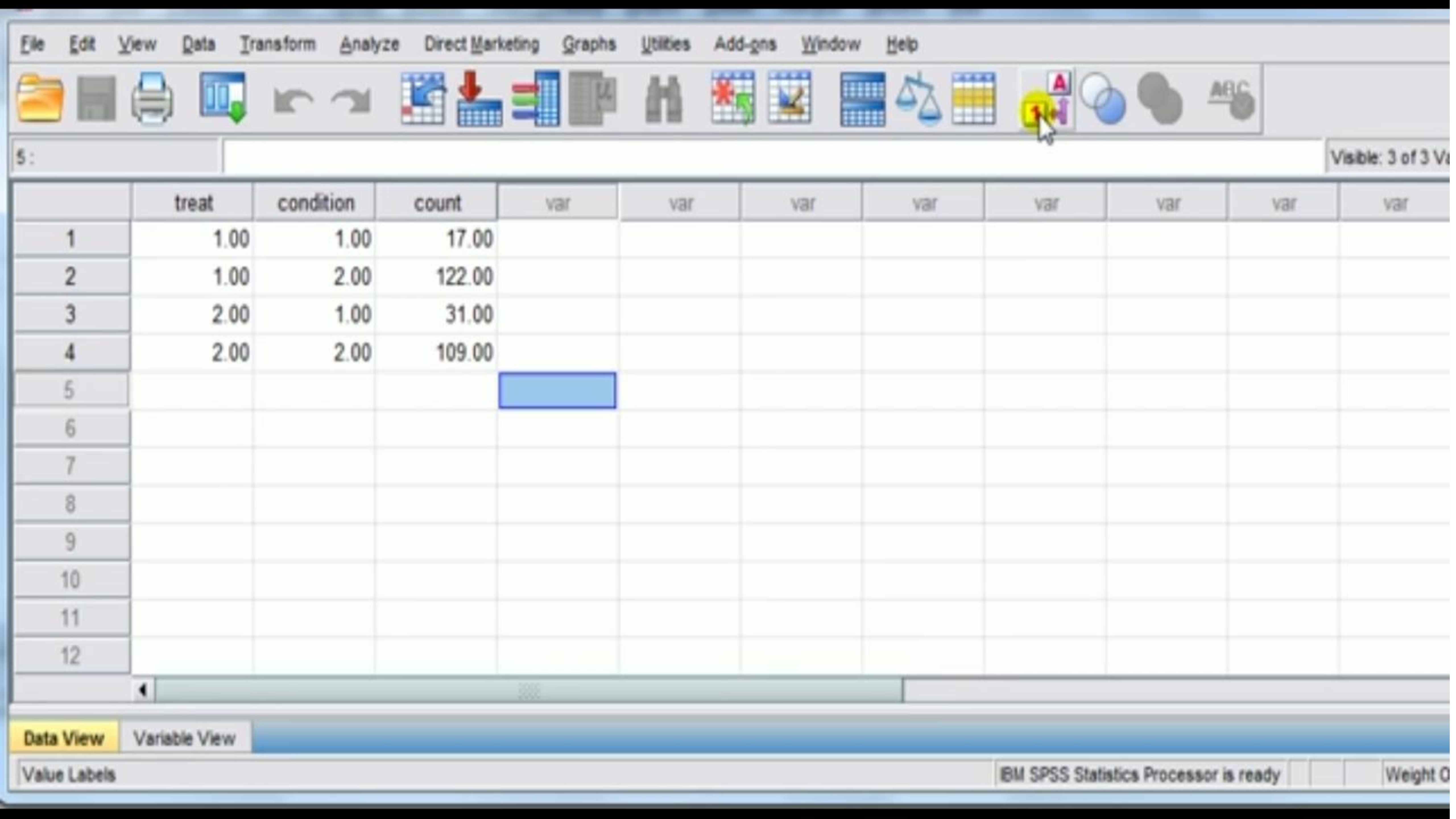
$$\rightarrow \mu_{11} = E(n_{11}) = \exp(\lambda + \lambda_1^X + \lambda_1^Y + \lambda_{11}^{XY})$$

Means:

		Y = 1	Y = 0
X	1	$\mu_{11} = \exp(\lambda + \lambda_1^X + \lambda_1^Y + \lambda_{11}^{XY})$	$\mu_{10} = \exp(\lambda + \lambda_1^X)$
	0	$\mu_{01} = \exp(\lambda + \lambda_1^Y)$	$\mu_{00} = \exp(\lambda)$

Treatment

Outcome		Vitamin C	Placebo	
	Cold	17	31	48
	no cold	122	109	231
		139	109	279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2 1.00 2.00 122.00
3 2.00 1.00 31.00
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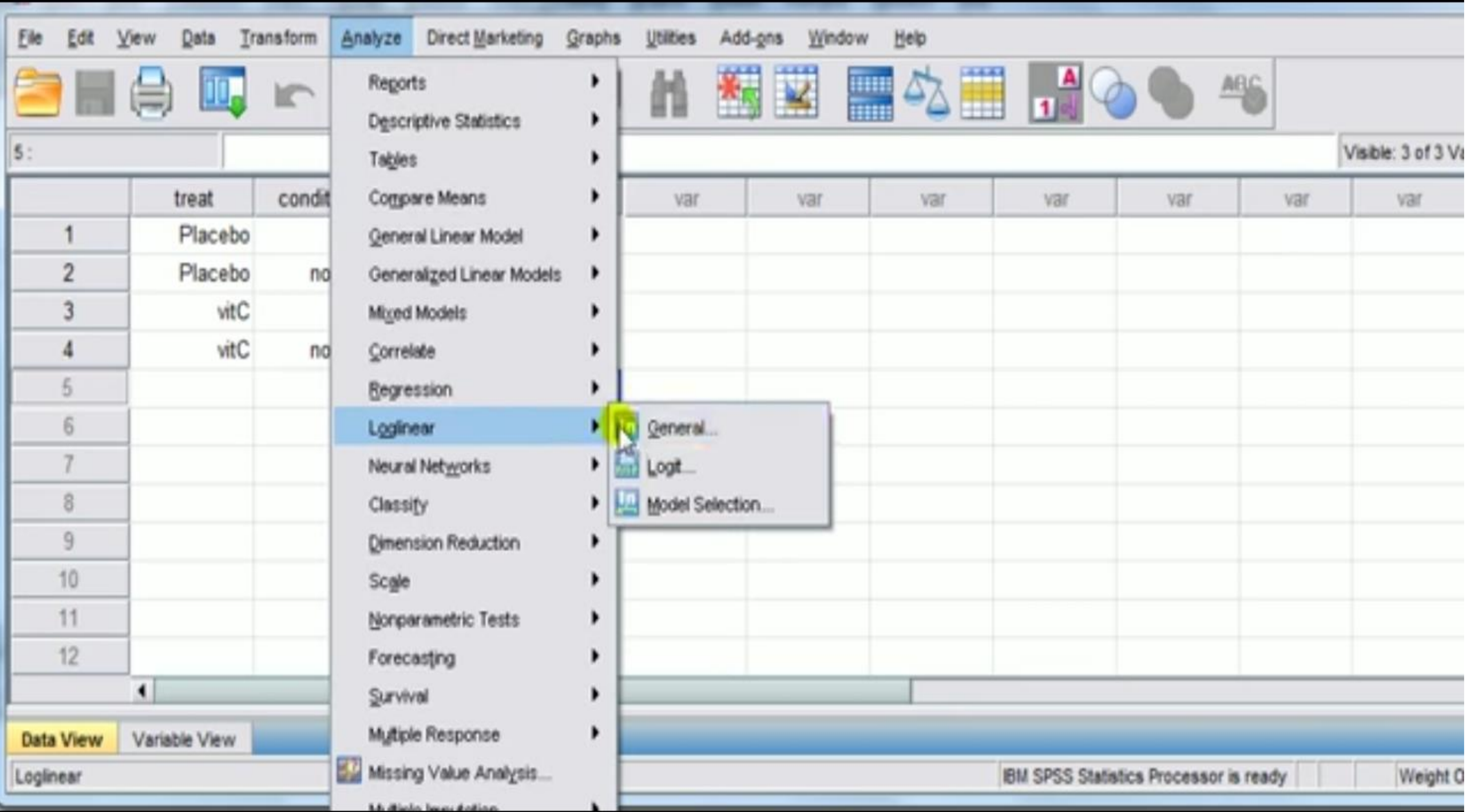
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Tables
Compare Means
General Linear Model
Generalized Linear Models
Mixed Models
Correlate
Regression
Loglinear
Neural Networks
Classify
Dimension Reduction
Scale
Nonparametric Tests
Forecasting
Survival
Multiple Response
Missing Value Analysis...
General...
Logit...
Model Selection...
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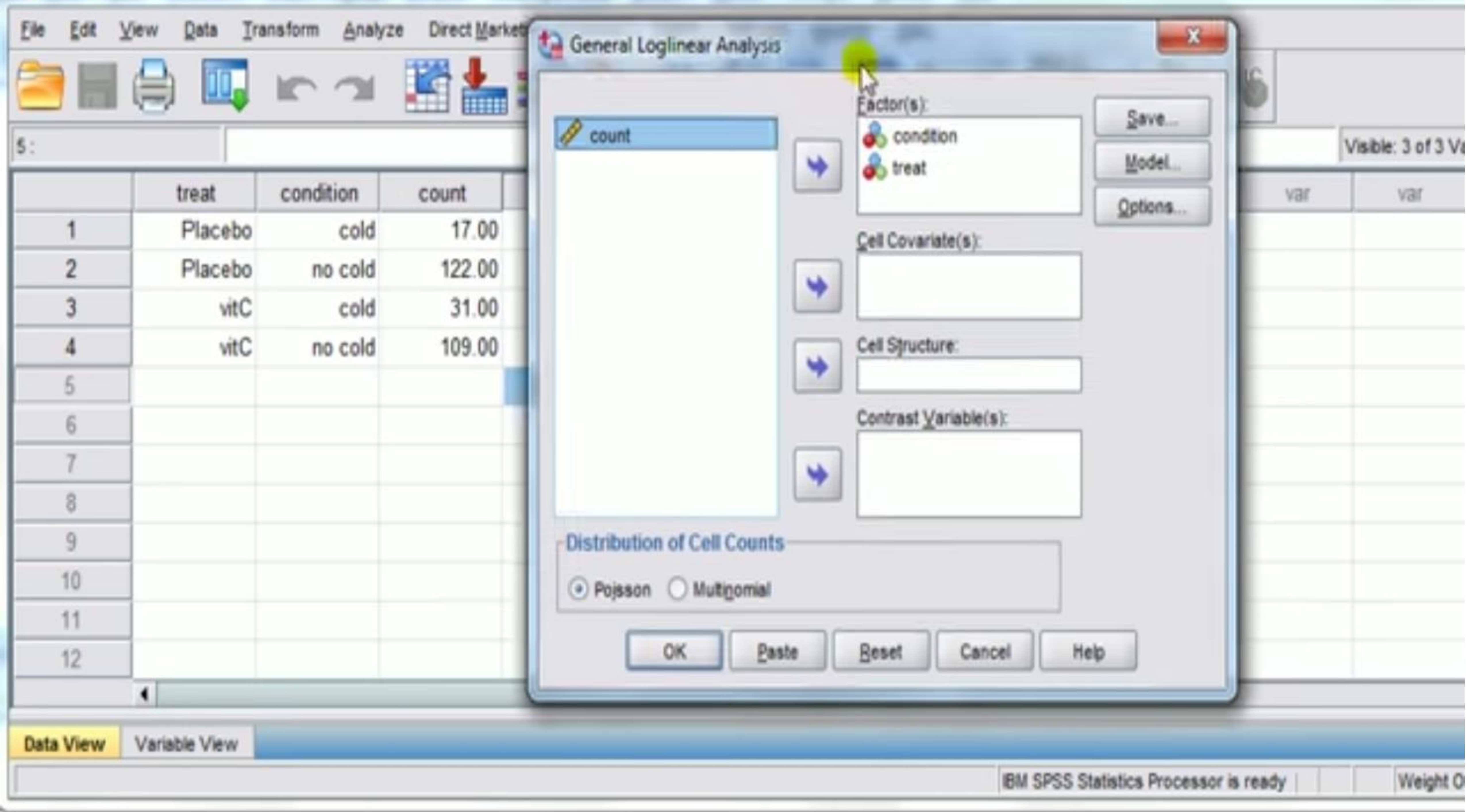
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Save...
Model...
Options...
Cell Covariate(s):
Cell Structure:
Contrast Variable(s):
Distribution of Cell Counts
Poisson
Multinomial
OK
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Reset
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Variable View
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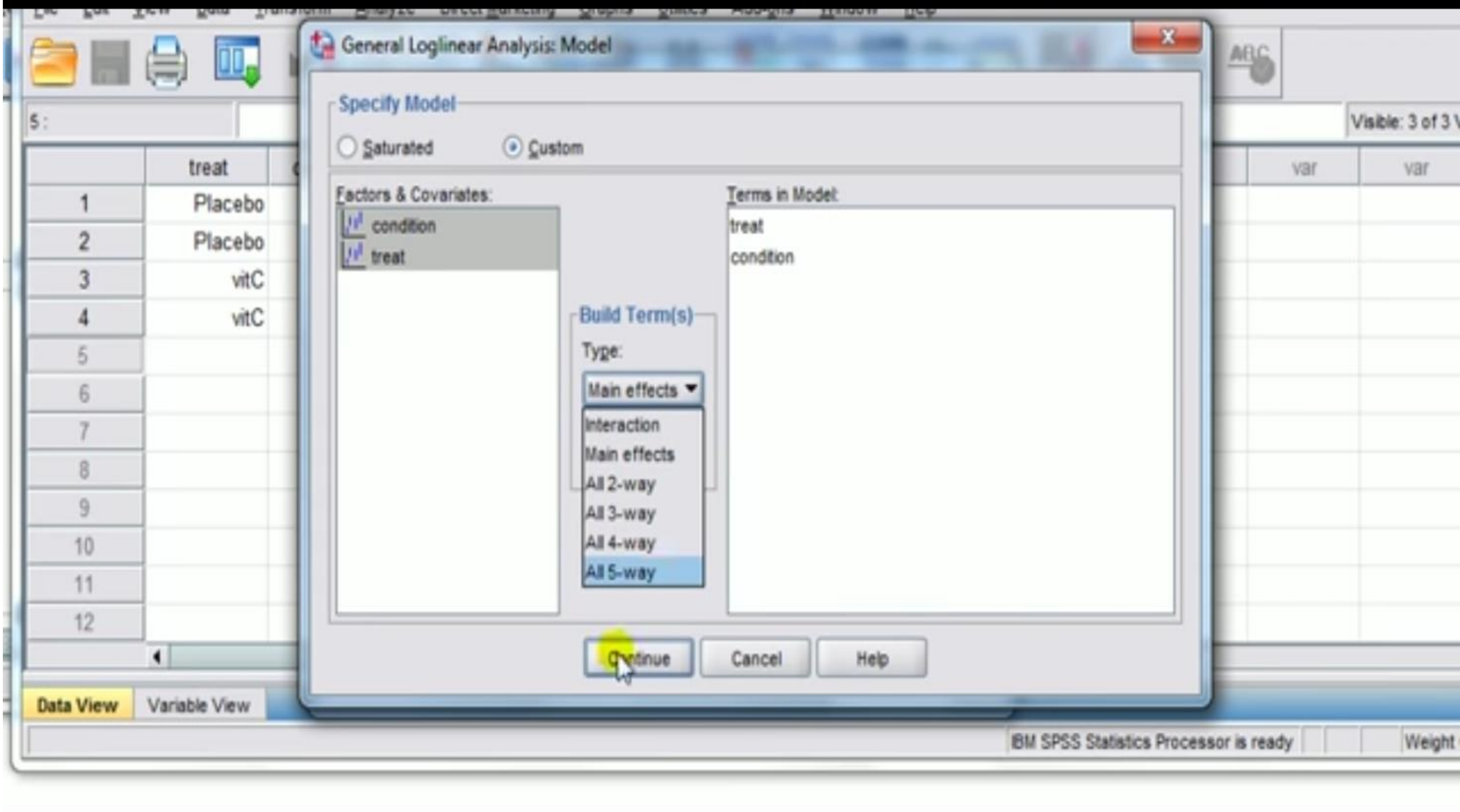
General Loglinear Analysis: Model
Specify Model
Saturated
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Factors & Covariates:
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treat
Terms in Model:
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Build Term(s)
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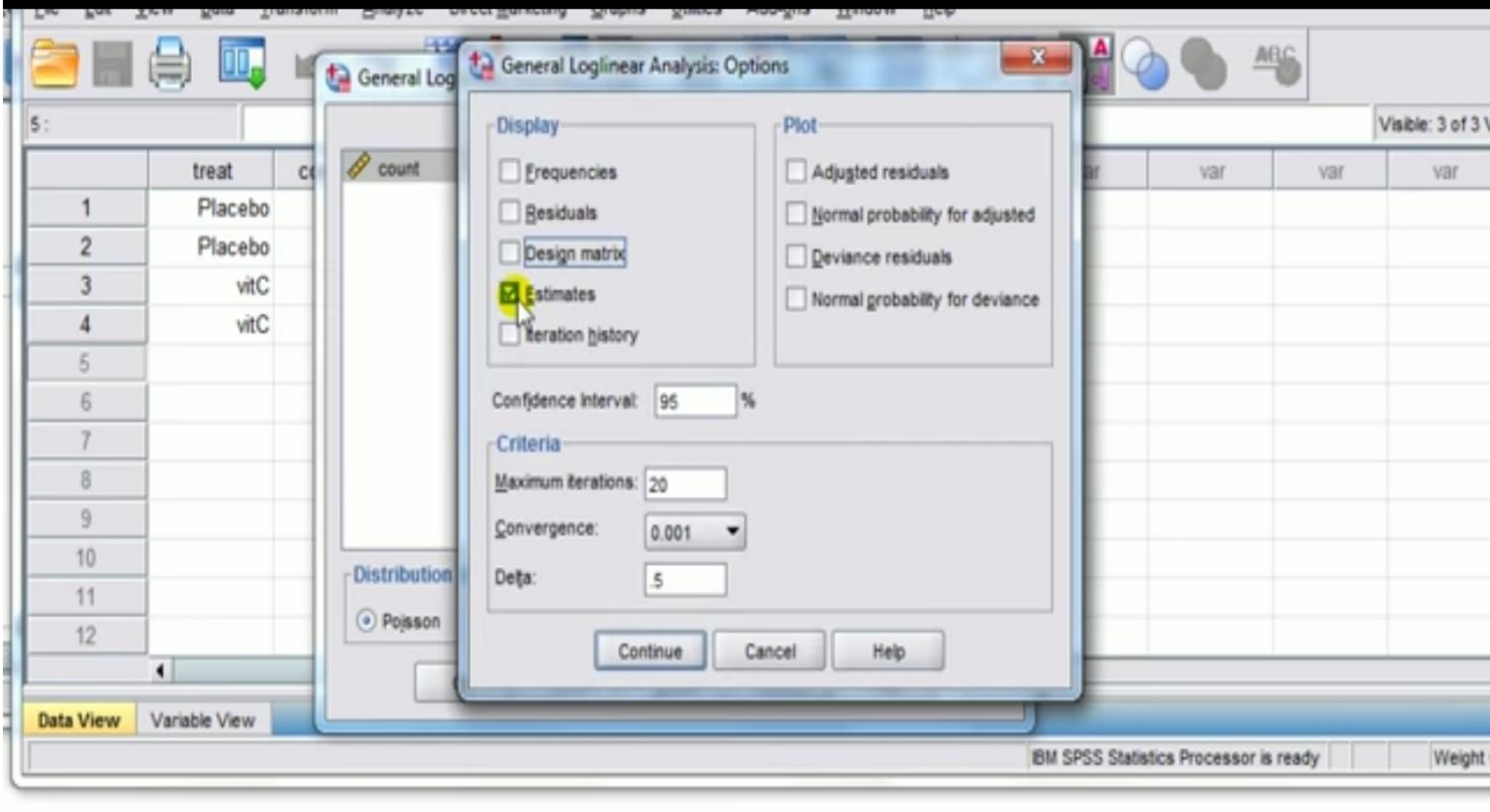
General Loglinear Analysis: Options
Display
Frequencies
Residuals
Design matrix
Estimates
Iteration history
Plot
Adjusted residuals
Normal probability for adjusted
Deviance residuals
Normal probability for deviance
Confidence Interval: 95 %
Criteria
Maximum iterations: 20
Convergence: 0.001
Delta: .5
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General Log
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Data View
Variable View
IBM SPSS Statistics Processor is ready
Weight

Likelihood Ratio	4.872	1	.027
Pearson Chi-Square	4.811	1	.028

a. Model: Poisson

b. Design: Constant + treat + condition

Parameter Estimates[b,c]

Parameter	Estimate	Std. Error	Z	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Constant	4.753	.089	53.516	.000	4.579	4.927
[treat = 1.00]	-.007	.120	-.060	.952	-.242	.228
[treat = 2.00]	0[a]	.	.	.	.	.
[condition = 1.00]	-1.571	.159	-9.905	.000	-1.882	-1.260
[condition = 2.00]	0[a]	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

b. Model: Poisson