Date:

Now we can calculate the fraction of gas molecule $\frac{n(E)dE}{N}$ having energy b/w E and E+dE because

n(E) is known. The total number of molecule is given as $N = \int_0^{\infty} n(E)\,dE$

⇒ Internal Energy :-

Consider there are n(E)dE molecules with energy b/w E and E+dE. Their contribution to internal energy of gas will $E = n(E)\,dE$

$$E = n(E)\,dE$$

The total of all such contribution gives the internal energy of the gas

$$E_{int} = \int_0^{\infty} E\,n(E)\,dE$$

$$= \int_0^{\infty} \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \cdot E^{3/2} e^{\frac{-E}{kT}}\,dE \quad \text{putting } n(E)$$

putting $\frac{E}{kT} = u$

$$\Rightarrow E = kTu$$

$$dE = kT\,du$$

So above eqn become

$$= \frac{2N}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \int_0^{\infty} (kT)^{3/2} u^{3/2} e^{-u} (kT\,du)$$

Internal

$$E_{int} = \frac{2N}{\sqrt{\pi}} kT \int_0^{\infty} u^{3/2} \cdot e^{-u} \, du$$

To convert this into standard integral by putting $u = x^2$

$$E_{int} = \frac{2N}{\sqrt{\pi}} kT \int_0^{\infty} x^3 e^{-x^2} (2x \, dx)$$

$$= \frac{4N}{\sqrt{\pi}} kT \int_0^{\infty} x^4 e^{-x^2} \, dx$$

using

$$\int_0^{\infty} x^{2n} e^{ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

here $a = 1$, $n = 2$

$$E_{int} = \frac{4N}{\sqrt{\pi}} kT \frac{(2 \times 2 - 1)}{2^{2+1}} \sqrt{\frac{\pi}{1}}$$

$$= \frac{\cancel{4}N}{\cancel{\sqrt{\pi}}} kT \frac{3}{\cancel{8}_2} \cancel{\sqrt{\pi}}$$

$$= \frac{3}{2} NkT$$

MEAN FREE ENERGY:

The mean free energy is given as

$$\langle E \rangle = \frac{1}{N}\int_0^\infty E\,n(E)\,dE$$

$$= \frac{1}{N} E_{int}$$

$$= \frac{1}{N}\frac{3NkT}{2}$$

$$= \frac{3}{2}KT$$

Most probable energy:

The Maxwell's Boltzman energy distribution

$$n(E) = \frac{2N}{\sqrt{\pi}}\cdot\frac{1}{(KT)^{3/2}} E^{1/2} e^{-\frac{E}{KT}}$$

$$\frac{dn(E)}{dE} = \frac{2N}{\sqrt{\pi}}\frac{1}{(KT)^{3/2}}\frac{d}{dE}\left(E^{1/2}\cdot e^{-\frac{E}{KT}}\right)$$

for most probable energy, putting $\frac{dn(E)}{dE} = 0$ so above eq becomes

$$0 = \frac{2N}{\sqrt{\pi}}\frac{1}{(KT)^{3/2}}\frac{d}{dE}\left(E^{1/2}\cdot e^{-\frac{E}{KT}}\right)$$

$$\frac{d}{dE}\left(E^{1/2}\cdot e^{-\frac{E}{KT}}\right) = 0$$

$$E^{1/2}\cdot\left(\frac{-1}{KT}\right)e^{-\frac{E}{KT}} + \left(\frac{1}{2}E^{-1/2}\right)e^{-\frac{E}{KT}} = 0$$

$$\frac{E^{1/2}}{KT} = \frac{1}{2E^{1/2}}$$

Important:

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$E = \frac{1KT}{2K}$ or $\boxed{E_P = \frac{1KT}{2}}$

This is called most probable energy.

⇒ LOW TEMPERATURE PHYSICS:

The branch of physics which deal with production of low temperature is called low temperature physics

The temperature below or around $0^\circ C$ is called low temperature.

The temp upto $-120^\circ C$ can be produced by freezing mixtures consisting potassium nitrate or sodium chloride with ice.

The liquid gases are used to produce still more low temperature.

The Temp upto 1k can be produced with liquid helium.

Different methods used to produce low temperatures are:

1. Cascade process of liquifying gases
2. Refrigeration and air conditioning
3. Cooling by Joule Thomson effect
4. Cooling by adiabatic expansion
5. Cooling by adiabatic demagnetization